Jawapan

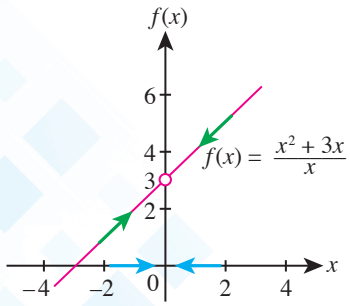
BAB 2 PEMBEZAAN

Aktiviti Penerokaan 1 (Halaman 30)

2. Nilai bagi $f(0)$ tidak dapat ditentukan kerana pada $x = 0$, fungsi $f(x) = \frac{x^2 + 3x}{x}$ adalah tidak tertakrif.

3.

					→ ←				
x	−0.1	−0.01	−0.001	−0.0001	…	0.0001	0.001	0.01	0.1
$f(x)$	2.9	2.99	2.999	2.9999	…	3.0001	3.001	3.01	3.1



Daripada jadual dan graf, nilai had bagi $f(x) = \frac{x^2 + 3x}{x}$ apabila x menghampiri sifar ialah 3, iaitu $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^2 + 3x}{x} = 3$.

4. $\lim_{x \to 0} f(x) \neq f(0)$

Latihan Kendiri 2.1

1. (a) $\lim_{x \to 0} (x^2 + x - 3) = 0^2 + 0 - 3$
$= -3$

(b) $\lim_{x \to 0} \sqrt{x + 1} = \sqrt{0 + 1}$
$= \sqrt{1}$
$= 1$

(c) $\lim_{x \to 0} \left(\frac{x + 4}{x - 2}\right) = \frac{0 + 4}{0 - 2}$
$= -2$

(d) $\lim_{x \to 0} \left(\frac{a}{ax + a}\right) = \frac{a}{a(0) + a}$
$= \frac{a}{a}$
$= 1$

2. (a) $\lim_{x \to 0} (3x - 1) = 3(0) - 1$
$= -1$

(b) $\lim_{x \to -3} \left(\sqrt{10 - 2x}\right) = \sqrt{10 - 2(-3)}$
$= \sqrt{16}$
$= 4$

(c) $\underset{x \to -3}{\text{had}} \left(\frac{x^2 + x - 6}{x + 3}\right) = \underset{x \to -3}{\text{had}} \left(\frac{(x + 3)(x - 2)}{x + 3}\right)$

$= \underset{x \to -3}{\text{had}} (x - 2)$

$= -3 - 2$

$= -5$

(d) $\underset{x \to 6}{\text{had}} \left(\frac{x - 6}{x^2 - 36}\right) = \underset{x \to 6}{\text{had}} \left(\frac{x - 6}{(x - 6)(x + 6)}\right)$

$= \underset{x \to 6}{\text{had}} \left(\frac{1}{x + 6}\right)$

$= \frac{1}{6 + 6}$

$= \frac{1}{12}$

(e) $\underset{x \to 2}{\text{had}} \left(\frac{x^2 - 3x + 2}{x^2 - 4}\right) = \underset{x \to 2}{\text{had}} \left(\frac{(x - 2)(x - 1)}{(x + 2)(x - 2)}\right)$

$= \underset{x \to 2}{\text{had}} \left(\frac{x - 1}{x + 2}\right)$

$= \frac{2 - 1}{2 + 2}$

$= \frac{1}{4}$

(f) $\underset{x \to 0}{\text{had}} \left(\frac{1 - \sqrt{2x + 1}}{2x^2 - x}\right) = \underset{x \to 0}{\text{had}} \left(\frac{1 - \sqrt{2x + 1}}{2x^2 - x}\right)\left(\frac{1 + \sqrt{2x + 1}}{1 + \sqrt{2x + 1}}\right)$

$= \underset{x \to 0}{\text{had}} \left(\frac{1 - (2x + 1)}{x(2x - 1)\left(1 + \sqrt{2x + 1}\right)}\right)$

$= \underset{x \to 0}{\text{had}} \left(\frac{-2x}{x(2x - 1)\left(1 + \sqrt{2x + 1}\right)}\right)$

$= \underset{x \to 0}{\text{had}} \left(\frac{-2}{(2x - 1)\left(1 + \sqrt{2x + 1}\right)}\right)$

$= \frac{-2}{(2(0) - 1)\left(1 + \sqrt{2(0) + 1}\right)}$

$= \frac{-2}{-2}$

$= 1$

(g) $\underset{x \to 4}{\text{had}} \left(\frac{x - 4}{\sqrt{x} - 2}\right) = \underset{x \to 4}{\text{had}} \left(\frac{x - 4}{\sqrt{x} - 2}\right)\left(\frac{\sqrt{x} + 2}{\sqrt{x} + 2}\right)$

$= \underset{x \to 4}{\text{had}} \left(\frac{(x - 4)(\sqrt{x} + 2)}{x - 4}\right)$

$= \underset{x \to 4}{\text{had}} \left(\sqrt{x} + 2\right)$

$= \sqrt{4} + 2$

$= 2 + 2$

$= 4$

(h) $\underset{x \to 3}{\text{had}} \dfrac{3 - \sqrt{2x + 3}}{x - 3} = \underset{x \to 3}{\text{had}} \left(\dfrac{3 - \sqrt{2x + 3}}{x - 3}\right)\left(\dfrac{3 + \sqrt{2x + 3}}{3 + \sqrt{2x + 3}}\right)$

$= \underset{x \to 3}{\text{had}} \dfrac{9 - (2x + 3)}{(x - 3)\left(3 + \sqrt{2x + 3}\right)}$

$= \underset{x \to 3}{\text{had}} \dfrac{6 - 2x}{(x - 3)\left(3 + \sqrt{2x + 3}\right)}$

$= \underset{x \to 3}{\text{had}} \dfrac{-2(x - 3)}{(x - 3)\left(3 + \sqrt{2x + 3}\right)}$

$= \underset{x \to 3}{\text{had}} \dfrac{-2}{3 + \sqrt{2x + 3}}$

$= \dfrac{-2}{3 + \sqrt{2(3) + 3}}$

$= \dfrac{-2}{3 + \sqrt{9}}$

$= \dfrac{-2}{6}$

$= -\dfrac{1}{3}$

(i) $\underset{x \to -2}{\text{had}} \dfrac{x + 2}{\sqrt{5x + 14} - 2} = \underset{x \to -2}{\text{had}} \left(\dfrac{x + 2}{\sqrt{5x + 14} - 2}\right)\left(\dfrac{\sqrt{5x + 14} + 2}{\sqrt{5x + 14} + 2}\right)$

$= \underset{x \to -2}{\text{had}} \dfrac{(x + 2)(\sqrt{5x + 14} + 2)}{(5x + 14) - 4}$

$= \underset{x \to -2}{\text{had}} \dfrac{(x + 2)(\sqrt{5x + 14} + 2)}{5x + 10}$

$= \underset{x \to -2}{\text{had}} \dfrac{(x + 2)(\sqrt{5x + 14} + 2)}{5(x + 2)}$

$= \underset{x \to -2}{\text{had}} \dfrac{\sqrt{5x + 14} + 2}{5}$

$= \dfrac{\sqrt{5(-2) + 14} + 2}{5}$

$= \dfrac{\sqrt{4} + 2}{5}$

$= \dfrac{4}{5}$

3. (a) $\underset{x \to 0}{\text{had}} \dfrac{x^2 - 2x}{x^3 - 4x} = \underset{x \to 0}{\text{had}} \dfrac{x(x - 2)}{x(x^2 - 4)}$

$= \underset{x \to 0}{\text{had}} \dfrac{x(x - 2)}{x(x + 2)(x - 2)}$

$= \underset{x \to 0}{\text{had}} \dfrac{1}{x + 2}$

$= \dfrac{1}{2}$

(b) $\underset{x \to 3}{\text{had}} \dfrac{x^2 - 4x + 3}{2x^2 - 5x - 3} = \underset{x \to 3}{\text{had}} \dfrac{(x - 3)(x - 1)}{(2x + 1)(x - 3)}$

$= \underset{x \to 3}{\text{had}} \dfrac{x - 1}{2x + 1}$

$= \dfrac{3 - 1}{2(3) + 1}$

$= \dfrac{2}{7}$

(c) $\underset{x \to 3}{\text{had}} \dfrac{x^3 - 5x^2 + 6x}{x^2 - 3x} = \underset{x \to 3}{\text{had}} \dfrac{x(x^2 - 5x + 6)}{x(x - 3)}$

$= \underset{x \to 3}{\text{had}} \dfrac{x(x - 2)(x - 3)}{x(x - 3)}$

$= \underset{x \to 3}{\text{had}}\ x - 2$

$= 3 - 2$

$= 1$

(d) $\underset{x \to 0}{\text{had}} \dfrac{5x}{3 - \sqrt{x + 9}} = \underset{x \to 0}{\text{had}} \left(\dfrac{5x}{3 - \sqrt{x + 9}}\right)\left(\dfrac{3 + \sqrt{x + 9}}{3 + \sqrt{x + 9}}\right)$

$= \underset{x \to 0}{\text{had}} \dfrac{5x(3 + \sqrt{x + 9})}{9 - (x + 9)}$

$= \underset{x \to 0}{\text{had}} \dfrac{5x(3 + \sqrt{x + 9})}{-x}$

$= \underset{x \to 0}{\text{had}} -5(3 + \sqrt{x + 9})$

$= -5(3 + \sqrt{0 + 9})$

$= -5(6)$

$= -30$

(e) $\underset{x \to 4}{\text{had}} \dfrac{x - 4}{2 - \sqrt{8 - x}} = \underset{x \to 4}{\text{had}} \left(\dfrac{x - 4}{2 - \sqrt{8 - x}}\right)\left(\dfrac{2 + \sqrt{8 - x}}{2 + \sqrt{8 - x}}\right)$

$= \underset{x \to 4}{\text{had}} \dfrac{(x - 4)(2 + \sqrt{8 - x})}{4 - (8 - x)}$

$= \underset{x \to 4}{\text{had}} \dfrac{(x - 4)(2 + \sqrt{8 - x})}{x - 4}$

$= \underset{x \to 4}{\text{had}}\ 2 + \sqrt{8 - x}$

$= 2 + \sqrt{8 - 4}$

$= 2 + \sqrt{4}$

$= 4$

(f) $\underset{x \to 7}{\text{had}} \dfrac{\sqrt{x + 2} - 3}{x - 7} = \underset{x \to 7}{\text{had}} \left(\dfrac{\sqrt{x + 2} - 3}{x - 7}\right)\left(\dfrac{\sqrt{x + 2} + 3}{\sqrt{x + 2} + 3}\right)$

$= \underset{x \to 7}{\text{had}} \dfrac{(x + 2) - 9}{(x - 7)\left(\sqrt{x + 2} + 3\right)}$

$= \underset{x \to 7}{\text{had}} \dfrac{x - 7}{(x - 7)\left(\sqrt{x + 2} + 3\right)}$

$= \underset{x \to 7}{\text{had}} \dfrac{1}{\sqrt{x + 2} + 3}$

$= \dfrac{1}{\sqrt{7 + 2} + 3}$

$= \dfrac{1}{\sqrt{9} + 3}$

$= \dfrac{1}{6}$

4. (a) (i) 4

(ii) Apabila x menghampiri sifar dari arah kiri, $\underset{x \to 0}{\text{had}}\ f(x) = 1$ dan apabila x menghampiri sifar dari arah kanan, $\underset{x \to 0}{\text{had}}\ f(x) = 4$. Oleh sebab had kiri tidak sama dengan had kanan, maka $\underset{x \to 0}{\text{had}}\ f(x)$ tidak wujud.

(i) $\underset{x \to -1}{\text{had}}\ f(x) = 2$

(ii) $\underset{x \to 5}{\text{had}}\ f(x) = 3$

Aktiviti Penerokaan 2 (Halaman 34)

5. Apabila titik C menghampiri titik B, nilai m menghampiri 6.

6.

δx	$x + \delta x$	$y + \delta y$	δy	$\frac{\delta y}{\delta x}$
1	4	16	7	7
0.5	3.5	12.25	3.25	6.5
0.05	3.05	9.3025	0.3025	6.05
0.005	3.005	9.030025	0.030025	6.005

7. Apabila δx menghampiri sifar, nilai $\frac{\delta y}{\delta x}$ menghampiri 6, iaitu:

Nilai $\underset{\delta x \to 0}{\text{had}} \frac{\delta y}{\delta x}$ = Kecerunan garis tangen di titik B
$= 6$

Latihan Kendiri 2.2

1. (a) $y = x$

$$\delta y = x + \delta x - x$$
$$= \delta x$$
$$\frac{\delta y}{\delta x} = 1$$
$$\text{Maka, } \frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \frac{\delta y}{\delta x}$$
$$= 1$$

(b) $y = 5x$

$$\delta y = 5(x + \delta x) - 5x$$
$$= 5x + 5\delta x - 5x$$
$$= 5\delta x$$
$$\frac{\delta y}{\delta x} = 5$$
$$\text{Maka, } \frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \frac{\delta y}{\delta x}$$
$$= 5$$

(c) $y = -4x$

$$\delta y = -4(x + \delta x) - (-4x)$$
$$= -4x - 4\delta x + 4x$$
$$= -4\delta x$$
$$\frac{\delta y}{\delta x} = -4$$
$$\text{Maka, } \frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \frac{\delta y}{\delta x}$$
$$= -4$$

(d) $y = 6x^2$

$$\delta y = 6(x + \delta x)^2 - 6x^2$$
$$= 6(x^2 + 2x\delta x + (\delta x)^2) - 6x^2$$
$$= 6x^2 + 12x\delta x + 6(\delta x)^2 - 6x^2$$
$$= 12x\delta x + 6(\delta x)^2$$
$$\frac{\delta y}{\delta x} = 12x + 6\delta x$$
$$\text{Maka, } \frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} (12x + 6\delta x)$$
$$= 12x$$

(e) $y = -x^2$

$$\delta y = -(x + \delta x)^2 - (-x^2)$$
$$= -(x^2 + 2x\delta x + (\delta x)^2) + x^2$$
$$= -x^2 - 2x\delta x - (\delta x)^2 + x^2$$
$$= -2x\delta x - (\delta x)^2$$
$$\frac{\delta y}{\delta x} = -2x - \delta x$$

Maka, $\frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \ (-2x - \delta x)$

$$= -2x$$

(f) $y = 2x^3$

$$\delta y = 2(x + \delta x)^3 - 2x^3$$
$$= 2(x + \delta x)(x + \delta x)^2 - 2x^3$$
$$= 2(x + \delta x)(x^2 + 2x\delta x + (\delta x)^2) - 2x^3$$
$$= 2[x^3 + 2x^2\delta x + x(\delta x)^2 + x^2\delta x + 2x(\delta x)^2 + (\delta x)^3] - 2x^3$$
$$= 2[x^3 + 3x^2\delta x + 3x(\delta x)^2 + (\delta x)^3] - 2x^3$$
$$= 2x^3 + 6x^2\delta x + 6x(\delta x)^2 + 2(\delta x)^3 - 2x^3$$
$$= 6x^2\delta x + 6x(\delta x)^2 + 2(\delta x)^3$$
$$\frac{\delta y}{\delta x} = 6x^2 + 6x\delta x + 2(\delta x)^2$$

Maka, $\frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \ \frac{\delta y}{\delta x}$

$$= \underset{\delta x \to 0}{\text{had}} \ (6x^2 + 6x\delta x + 2(\delta x^2))$$
$$= 6x^2$$

(g) $y = \frac{1}{2}x^2$

$$\delta y = \frac{1}{2}(x + \delta x)^2 - \frac{1}{2}x^2$$
$$= \frac{1}{2}(x^2 + 2x\delta x + (\delta x)^2) - \frac{1}{2}x^2$$
$$= \frac{1}{2}x^2 + x\delta x + \frac{1}{2}(\delta x)^2 - \frac{1}{2}x^2$$
$$= x\delta x + \frac{1}{2}(\delta x)^2$$
$$\frac{\delta y}{\delta x} = x + \frac{1}{2}\delta x$$

Maka, $\frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \ \frac{\delta y}{\delta x}$

$$= \underset{\delta x \to 0}{\text{had}} \ (x + \frac{1}{2}\delta x)$$
$$= x$$

(h) $y = \frac{1}{x}$

$$\delta y = \frac{1}{x + \delta x} - \frac{1}{x}$$
$$= \frac{x - (x + \delta x)}{x(x + \delta x)}$$
$$= \frac{x - x - \delta x}{x(x + \delta x)}$$
$$= \frac{-\delta x}{x(x + \delta x)}$$
$$\frac{\delta y}{\delta x} = -\frac{1}{x(x + \delta x)}$$

Maka, $\frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \ \frac{\delta y}{\delta x}$

$$= \underset{\delta x \to 0}{\text{had}} \ \left[-\frac{1}{x(x + \delta x)}\right]$$
$$= -\frac{1}{x^2}$$

2. $y = 2x^2 - x + 7$

$$\delta y = 2(x + \delta x)^2 - (x + \delta x) + 7 - (2x^2 - x + 7)$$
$$= 2(x^2 + 2x\delta x + (\delta x)^2) - x - \delta x + 7 - 2x^2 + x - 7$$
$$= 2x^2 + 4x\delta x + 2(\delta x)^2 - x - \delta x + 7 - 2x^2 + x - 7$$
$$= 4x\delta x + 2(\delta x)^2 - \delta x$$
$$\frac{\delta y}{\delta x} = 4x + 2\delta x - 1$$

Maka, $\frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \frac{\delta y}{\delta x}$

$$= \underset{\delta x \to 0}{\text{had}} (4x + 2\delta x - 1)$$
$$= 4x - 1$$

3. $y = 3 + x - x^2$

$$\delta y = 3 + x + \delta x - (x + \delta x)^2 - (3 + x - x^2)$$
$$= 3 + x + \delta x - (x^2 + 2x\delta x + (\delta x)^2) - 3 - x + x^2$$
$$= \delta x - 2x\delta x - (\delta x)^2$$
$$\frac{\delta y}{\delta x} = 1 - 2x - \delta x$$

Maka, $\frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \frac{\delta y}{\delta x}$

$$= \underset{\delta x \to 0}{\text{had}} (1 - 2x - \delta x)$$
$$= 1 - 2x$$

Latihan Formatif 2.1

1. (a) (i) 8 (ii) 3

(iii) 0 (iv) -1

(v) 0 (vi) $\underset{x \to 4}{\text{had}}\ x^2 - 4x + 3 = 4^2 - 4(4) + 3$
$= 3$

(b) $\underset{x \to a}{\text{had}}\ x^2 - 4x + 3 = 8$

$a^2 - 4a + 3 = 8$

$a^2 - 4a - 5 = 0$

$(a + 1)(a - 5) = 0$

$a = -1$ atau $a = 5$

(c) (i) $\frac{dy}{dx} = 2x - 4$

(ii) $2(4) - 4 = 4$

2. (a) $\underset{x \to 0}{\text{had}}\ (x^2 - 6x + 9) = 0^2 - 6(0) + 9$
$= 9$

(b) $\underset{x \to 2}{\text{had}}\ \sqrt[3]{x^4 - 2x^2} = \sqrt[3]{2^4 - 2(2)^2}$
$= \sqrt[3]{8}$
$= 2$

(c) $\underset{x \to 9}{\text{had}} \left[\frac{9 - x}{x^2 - 81}\right] = \underset{x \to 9}{\text{had}} \left[\frac{-(x - 9)}{(x - 9)(x + 9)}\right]$
$= -\frac{1}{9 + 9}$
$= -\frac{1}{18}$

(d) $\underset{x \to 2}{\text{had}} \left[\frac{x^2 - x - 2}{x - 2}\right] = \underset{x \to 2}{\text{had}} \left[\frac{(x - 2)(x + 1)}{x - 2}\right]$
$= \underset{x \to 2}{\text{had}} (x + 1)$
$= 2 + 1$
$= 3$

(e) $\underset{x \to 1}{\text{had}} \left[\frac{x^3 - x}{x - 1}\right] = \underset{x \to 1}{\text{had}} \left[\frac{x(x^2 - 1)}{x - 1}\right]$

$= \underset{x \to 1}{\text{had}} \left[\frac{x(x - 1)(x + 1)}{x - 1}\right]$

$= \underset{x \to 1}{\text{had}}\ x(x + 1)$

$= 1(1 + 1)$

$= 2$

(f) $\underset{x \to 5}{\text{had}} \left[\frac{x^2 - 7x + 10}{x^2 - 25}\right] = \underset{x \to 5}{\text{had}} \left[\frac{(x - 2)(x - 5)}{(x - 5)(x + 5)}\right]$

$= \underset{x \to 5}{\text{had}} \left[\frac{x - 2}{x + 5}\right]$

$= \frac{5 - 2}{5 + 5}$

$= \frac{3}{10}$

3. (a) $\underset{x \to 0}{\text{had}} \frac{\sqrt{1 + 2x} - \sqrt{1 - 2x}}{x} = \underset{x \to 0}{\text{had}} \left(\frac{\sqrt{1 + 2x} - \sqrt{1 - 2x}}{x}\right)\left(\frac{\sqrt{1 + 2x} + \sqrt{1 - 2x}}{\sqrt{1 + 2x} + \sqrt{1 - 2x}\, x}\right)$

$= \underset{x \to 0}{\text{had}} \frac{1 + 2x - (1 - 2x)}{x\left(\sqrt{1 + 2x} + \sqrt{1 - 2x}\right)}$

$= \underset{x \to 0}{\text{had}} \frac{4x}{x\left(\sqrt{1 + 2x} + \sqrt{1 - 2x}\right)}$

$= \underset{x \to 0}{\text{had}} \frac{4}{\sqrt{1 + 2x} + \sqrt{1 - 2x}}$

$= \frac{4}{\sqrt{1 + 2(0)} + \sqrt{1 - 2(0)}}$

$= \frac{4}{\sqrt{1} + \sqrt{1}}$

$= \frac{4}{2}$

$= 2$

(b) $\underset{x \to 4}{\text{had}} \frac{3 - \sqrt{x + 5}}{x - 4} = \underset{x \to 4}{\text{had}} \left(\frac{3 - \sqrt{x + 5}}{x - 4}\right)\left(\frac{3 + \sqrt{x + 5}}{3 + \sqrt{x + 5}}\right)$

$= \underset{x \to 4}{\text{had}} \frac{9 - (x + 5)}{(x - 4)\left(3 + \sqrt{x + 5}\right)}$

$= \underset{x \to 4}{\text{had}} \frac{4 - x}{(x - 4)\left(3 + \sqrt{x + 5}\right)}$

$= \underset{x \to 4}{\text{had}} \frac{-(x - 4)}{(x - 4)\left(3 + \sqrt{x + 5}\right)}$

$= \underset{x \to 4}{\text{had}} \frac{-1}{3 + \sqrt{x + 5}}$

$= \frac{-1}{3 + \sqrt{4 + 5}}$

$= \frac{-1}{3 + \sqrt{9}}$

$= -\frac{1}{6}$

(c) $\underset{x \to 3}{\text{had}} \dfrac{x^2 - 5x + 6}{2 - \sqrt{x+1}} = \underset{x \to 3}{\text{had}} \left(\dfrac{x^2 - 5x + 6}{2 - \sqrt{x+1}}\right)\left(\dfrac{2 + \sqrt{x+1}}{2 + \sqrt{x+1}}\right)$

$= \underset{x \to 3}{\text{had}} \dfrac{(x^2 - 5x + 6)\left(2 + \sqrt{x+1}\right)}{4 - (x+1)}$

$= \underset{x \to 3}{\text{had}} \dfrac{(x-2)(x-3)\left(2 + \sqrt{x+1}\right)}{3 - x}$

$= \underset{x \to 3}{\text{had}} \dfrac{(x-2)(x-3)\left(2 + \sqrt{x+1}\right)}{-(x-3)}$

$= \underset{x \to 3}{\text{had}} -(x-2)\left(2 + \sqrt{x+1}\right)$

$= -(3-2)\left(2 + \sqrt{3+1}\right)$

$= -\left(2 + \sqrt{4}\right)$

$= -4$

4. (a) Secara penggantian langsung menghasilkan bentuk tak tentu $\dfrac{0}{0}$.

Jadi, $2^2 - k = 0$

$4 - k = 0$

$k = 4$

(b) $(-1)^2 - 2(-1) - h = 0$

$3 - h = 0$

$h = 3$

$k(-1) + 2 = 0$

$-k + 2 = 0$

$k = 2$

Jadi, $h + k = 3 + 2$

$= 5$

5. (a) $y = 5x - 8$

$\delta y = 5(x + \delta x) - 8 - (5x - 8)$

$= 5x + 5\delta x - 8 - 5x + 8$

$= 5\delta x$

$\dfrac{\delta y}{\delta x} = 5$

Maka, $\dfrac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \dfrac{\delta y}{\delta x}$

$= \underset{\delta x \to 0}{\text{had}} 5$

$= 5$

(b) $y = x^2 - x$

$\delta y = (x + \delta x)^2 - (x + \delta x) - (x^2 - x)$

$= x^2 + 2x\delta x + (\delta x)^2 - x - \delta x - x^2 + x$

$= 2x\delta x + (\delta x)^2 - \delta x$

$\dfrac{\delta y}{\delta x} = 2x + \delta x - 1$

Maka, $\dfrac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \dfrac{\delta y}{\delta x}$

$= \underset{\delta x \to 0}{\text{had}} (2x + \delta x - 1)$

$= 2x - 1$

(c) $y = (x + 1)^2$

$= x^2 + 2x + 1$

$\delta y = (x + \delta x)^2 + 2(x + \delta x) + 1 - (x^2 + 2x + 1)$

$= x^2 + 2x\delta x + (\delta x)^2 + 2x + 2\delta x + 1 - x^2 - 2x - 1$

$= 2x\delta x + (\delta x)^2 + 2\delta x$

$$\frac{\delta y}{\delta x} = 2x + \delta x + 2$$

Maka, $\frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \frac{\delta y}{\delta x}$

$$= \underset{\delta x \to 0}{\text{had}} (2x + \delta x + 2)$$

$$= 2x + 2$$

(d) $y = \frac{1}{4x}$

$$\delta y = \frac{1}{4(x + \delta x)} - \frac{1}{4x}$$

$$= \frac{4x - 4x - 4\delta x}{16x(x + \delta x)}$$

$$= -\frac{4\delta x}{16x(x + \delta x)}$$

$$\frac{\delta y}{\delta x} = -\frac{1}{4x(x + \delta x)}$$

Maka, $\frac{dy}{dx} = \underset{\delta x \to 0}{\text{had}} \frac{\delta y}{\delta x}$

$$= \underset{\delta x \to 0}{\text{had}} \left[-\frac{1}{4x(x + \delta x)}\right]$$

$$= -\frac{1}{4x^2}$$

6. $\delta s = (t + \delta t)^2 - 3(t + \delta t) - (t^2 - 3t)$

$= t^2 + 2t\delta t + (\delta t)^2 - 3t - 3\delta t - t^2 + 3t$

$= 2t\delta t + (\delta t)^2 - 3\delta t$

$\frac{\delta s}{\delta t} = 2t + \delta t - 3$

$\frac{ds}{dt} = \underset{\delta t \to 0}{\text{had}} \frac{\delta s}{\delta t}$

$= \underset{\delta t \to 0}{\text{had}} (2t + \delta t - 3)$

$= 2t - 3$

Apabila $t = 5$, $\frac{ds}{dt} = 2(5) - 3$

$= 7$

Maka, halaju tupai pada $t = 5$ ialah 7 ms^{-1}.

Aktiviti Penerokaan 3 (Halaman 39)

6. Bentuk graf bagi $y = f(x)$ ialah parabola manakala bentuk graf bagi fungsi kecerunannya $y = f'(x)$ merupakan suatu garis lurus.
Apabila nilai a berubah, garis tangen pada graf $y = f(x) = x^2$ juga turut berubah.

7.

Koordinat-x	–3	–2	–1	0	1	2	3
Kecerunan lengkung	–6	–4	–2	0	2	4	6

8. $f(x) = x^2$

$f'(x) = 2x^{2-1}$

$= 2x$

$f'(-3) = 2(-3) = -6$ $\quad f'(-2) = 2(-2) = -4$ $\quad f'(-1) = 2(-1) = -2$

$f'(0) = 2(0) = 0$ $\quad f'(1) = 2(1) = 2$ $\quad f'(2) = 2(2) = 4$ $\quad f'(3) = 2(3) = 6$

9. Bentuk graf bagi fungsi kubik $y = f(x) = x^3$ ialah lengkung kubik manakala bentuk graf bagi fungsi kecerunannya $y = f'(x) = 3x^2$ merupakan parabola.

Latihan Kendiri 2.3

1. (a) $\frac{d}{dx}\left(\frac{4}{5}x^{10}\right) = \frac{4}{5}(10x^{10-1})$

$= \frac{4}{5}(10x^9)$

$= 8x^9$

(b) $\frac{d}{dx}(-2x^4) = -2(4x^{4-1})$

$= -2(4x^3)$

$= -8x^3$

(c) $\frac{d}{dx}\left(\frac{3}{4x^8}\right) = \frac{d}{dx}\left(\frac{3}{4}x^{-8}\right)$

$= \frac{3}{4}(-8x^{-8-1})$

$= \frac{3}{4}(-8x^{-9})$

$= -\frac{6}{x^9}$

(d) $\frac{d}{dx}\left(\frac{6}{\sqrt[3]{x}}\right) = \frac{d}{dx}\left(6x^{-\frac{1}{3}}\right)$

$= 6\left(-\frac{1}{3}x^{-\frac{1}{3}-1}\right)$

$= -2x^{-\frac{4}{3}}$

$= -\frac{2}{x^{\frac{4}{3}}}$

$= -\frac{2}{\sqrt[3]{x^4}}$

(e) $\frac{d}{dx}\left(-12\sqrt[3]{x^2}\right) = \frac{d}{dx}\left(-12x^{\frac{2}{3}}\right)$

$= -12\left(\frac{2}{3}x^{\frac{2}{3}-1}\right)$

$= -8x^{-\frac{1}{3}}$

$= -\frac{8}{\sqrt[3]{x}}$

2. (a) $\frac{d}{dx}(4x^2 + 6x - 1) = \frac{d}{dx}(4x^2) + \frac{d}{dx}(6x) - \frac{d}{dx}(1) \leftarrow 1 = x^0$

$= 4(2x^{2-1}) + 6(1x^{1-1}) - 0x^{0-1}$

$= 8x + 6$

(b) $\frac{d}{dx}\left(\frac{4}{5}\sqrt{x} + \frac{2}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{4}{5}x^{\frac{1}{2}}\right) + \frac{d}{dx}\left(2x^{-\frac{1}{2}}\right)$

$= \frac{4}{5}\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) + 2\left(-\frac{1}{2}x^{-\frac{1}{2}-1}\right)$

$= \frac{2}{5}x^{-\frac{1}{2}} - x^{-\frac{3}{2}}$

$= \frac{2}{5\sqrt{x}} - \frac{1}{\sqrt{x^3}}$

(c) $\frac{d}{dx}(9 - 4x)^2 = \frac{d}{dx}(81 - 72x + 16x^2)$

$= \frac{d}{dx}(81) - \frac{d}{dx}(72x) + \frac{d}{dx}(16x^2)$

$= 32x - 72$

3. (a) $\frac{d}{dx}\left(4x^2(5-\sqrt{x}\right) = \frac{d}{dx}\left(20x^2 - 4x^{\frac{5}{2}}\right)$

$= 20(2x^{2-1}) - 4\left(\frac{5}{2}x^{\frac{5}{2}-1}\right)$

$= 40x - 10x^{\frac{3}{2}}$

$= 40x - 10\sqrt{x^3}$

(b) $\frac{d}{dx}\left(x^2 + \frac{4}{x}\right)^2 = \frac{d}{dx}(x^4 + 8x + 16x^{-2})$

$= 4x^{4-1} + 8(1x^{1-1}) + 16(-2x^{-2-1})$

$= 4x^3 + 8 - \frac{32}{x^3}$

(c) $\frac{d}{dx}\left(\frac{(4x-1)(1-x)}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{4x - 4x^2 - 1 + x}{x^{\frac{1}{2}}}\right)$

$= \frac{d}{dx}\left(4x^{\frac{1}{2}} - 4x^{\frac{3}{2}} - x^{-\frac{1}{2}} + x^{\frac{1}{2}}\right)$

$= \frac{d}{dx}\left(5x^{\frac{1}{2}} - 4x^{\frac{3}{2}} - x^{-\frac{1}{2}}\right)$

$= 5\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) - 4\left(\frac{3}{2}x^{\frac{3}{2}-1}\right) - \left(-\frac{1}{2}x^{-\frac{1}{2}-1}\right)$

$= \frac{5}{2\sqrt{x}} - 6\sqrt{x} + \frac{1}{2\sqrt{x^3}}$

4. (a) $y = x^2 - 2x$

$\frac{dy}{dx} = 2x - 2$

Apabila $x = \frac{1}{2}$, $\frac{dy}{dx} = 2\left(\frac{1}{2}\right) - 2$

$= -1$

(b) $y = \sqrt{x}(2 - x)$

$= 2x^{\frac{1}{2}} - x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{x}} - \frac{3}{2}\sqrt{x}$

Apabila $x = 9$, $\frac{dy}{dx} = \frac{1}{\sqrt{9}} - \frac{3}{2}\sqrt{9}$

$= \frac{1}{3} - \frac{9}{2}$

$= -\frac{25}{6}$

$= -4\frac{1}{6}$

(c) $y = \frac{x^2 + 4}{x^2}$

$= 1 + 4x^{-2}$

$\frac{dy}{dx} = -8x^{-3}$

$= -\frac{8}{x^3}$

Apabila $x = 2$, $\frac{dy}{dx} = -\frac{8}{2^3}$

$= -\frac{8}{8}$

$= -1$

Aktiviti Penerokaan 4 (Halaman 42)

2. $y = (2x + 3)^2$
$= (2x + 3)(2x + 3)$
$= 4x^2 + 12x + 9$
$\frac{dy}{dx} = 8x + 12$

3. (a) $y = u^2$

(b) $u = 2x + 3$, jadi $\frac{du}{dx} = 2$ dan $y = u^2$. Maka, $\frac{dy}{du} = 2u$

(c) $\frac{dy}{du} \times \frac{du}{dx} = 2u \times 2$
$= 4u$
$= 4(2x + 3)$
$= 8x + 12$

4. Jawapannya adalah sama.
Kaedah dalam langkah 3 menjadi pilihan kerana lebih mudah dan cepat untuk memperoleh terbitannya.

Latihan Kendiri 2.4

1. (a) $\frac{d}{dx}(x + 4)^5 = 5(x + 4)^{5-1}\frac{d}{dx}(x + 4)$
$= 5(x + 4)^4$

(b) $\frac{d}{dx}(2x - 3)^4 = 4(2x - 3)^{4-1}\frac{d}{dx}(2x - 3)$
$= 4(2x - 3)^3(2)$
$= 8(2x - 3)^3$

(c) $\frac{d}{dx}\left[\frac{1}{3}(6 - 3x)^6\right] = \frac{1}{3}[6(6 - 3x)^{6-1}]\frac{d}{dx}(6 - 3x)$
$= 2(6 - 3x)^5(-3)$
$= -6(6 - 3x)^5$

(d) $\frac{d}{dx}(4x^2 - 5)^7 = 7(4x^2 - 5)^{7-1}\frac{d}{dx}(4x^2 - 5)$
$= 7(4x^2 - 5)^6(8x)$
$= 56x(4x^2 - 5)^6$

(e) $\frac{d}{dx}\left(\frac{1}{6}x + 2\right)^8 = 8\left(\frac{1}{6}x + 2\right)^{8-1}\frac{d}{dx}\left(\frac{1}{6}x + 2\right)$
$= 8\left(\frac{1}{6}x + 2\right)^7\left(\frac{1}{6}\right)$
$= \frac{4}{3}\left(\frac{1}{6}x + 2\right)^7$

(f) $\frac{d}{dx}\left[\frac{2}{3}(5 - 2x)^9\right] = \frac{2}{3}[9(5 - 2x)^{9-1}]\frac{d}{dx}(5 - 2x)$
$= 6(5 - 2x)^8(-2)$
$= -12(5 - 2x)^8$

(g) $\frac{d}{dx}(1 - x - x^2)^3 = 3(1 - x - x^2)^{3-1}\frac{d}{dx}(1 - x - x^2)$
$= 3(1 - x - x^2)^2(-2x - 1)$
$= -3(2x + 1)(1 - x - x^2)^2$

(h) $\frac{d}{dx}(2x^3 - 4x + 1)^{-10} = -10(2x^3 - 4x + 1)^{-10-1}\frac{d}{dx}(2x^3 - 4x + 1)$
$= -10(2x^3 - 4x + 1)^{-11}(6x^2 - 4)$
$= -\frac{60x^2 - 40}{(2x^3 - 4x + 1)^{11}}$
$= -\frac{20(3x^2 - 2)}{(2x^3 - 4x + 1)^{11}}$

2. (a) $\frac{d}{dx}\left(\frac{1}{3x+2}\right) = \frac{d}{dx}[(3x+2)^{-1}]$

$= -1(3x+2)^{-1-1}\frac{d}{dx}(3x+2)$

$= -1(3x+2)^{-2}(3)$

$= -\frac{3}{(3x+2)^2}$

(b) $\frac{d}{dx}\left(\frac{1}{(2x-7)^3}\right) = \frac{d}{dx}[(2x-7)^{-3}]$

$= -3(2x-7)^{-3-1}\frac{d}{dx}(2x-7)$

$= -3(2x-7)^{-4}(2)$

$= -\frac{6}{(2x-7)^4}$

(c) $\frac{d}{dx}\left(\frac{5}{(3-4x)^5}\right) = \frac{d}{dx}[5(3-4x)^{-5}]$

$= 5[-5(3-4x)^{-5-1}]\frac{d}{dx}(3-4x)]$

$= -25(3-4x)^{-6}(-4)$

$= \frac{100}{(3-4x)^6}$

(d) $\frac{d}{dx}\left(\frac{3}{4(5x-6)^8}\right) = \frac{d}{dx}\left[\frac{3}{4}(5x-6)^{-8}\right]$

$= \frac{3}{4}\left[-8(5x-6)^{-8-1}\right]\frac{d}{dx}(5x-6)$

$= -6(5x-6)^{-9}(5)$

$= -\frac{30}{(5x-6)^9}$

(e) $\frac{d}{dx}\left(\sqrt{2x-7}\right) = \frac{d}{dx}(2x-7)^{\frac{1}{2}}$

$= \frac{1}{2}(2x-7)^{\frac{1}{2}-1}\frac{d}{dx}(2x-7)$

$= \frac{1}{2}(2x-7)^{-\frac{1}{2}}(2)$

$= \frac{1}{\sqrt{2x-7}}$

(f) $\frac{d}{dx}\left(\sqrt{6-3x}\right) = \frac{d}{dx}(6-3x)^{\frac{1}{2}}$

$= \frac{1}{2}(6-3x)^{\frac{1}{2}-1}\frac{d}{dx}(6-3x)$

$= \frac{1}{2}(6-3x)^{-\frac{1}{2}}(-3)$

$= -\frac{3}{2\sqrt{6-3x}}$

(g) $\frac{d}{dx}\left(\sqrt{3x^2+5}\right) = \frac{d}{dx}(3x^2+5)^{\frac{1}{2}}$

$= \frac{1}{2}(3x^2+5)^{\frac{1}{2}-1}\frac{d}{dx}(3x^2+5)$

$= \frac{1}{2}(3x^2+5)^{-\frac{1}{2}}(6x)$

$= \frac{3x}{\sqrt{3x^2+5}}$

(h) $\frac{d}{dx}\left(\sqrt{x^2 - x + 1}\right) = \frac{d}{dx}(x^2 - x + 1)^{\frac{1}{2}}$

$= \frac{1}{2}(x^2 - x + 1)^{\frac{1}{2}-1}\frac{d}{dx}(x^2 - x + 1)$

$= \frac{1}{2}(x^2 - x + 1)^{-\frac{1}{2}}(2x - 1)$

$= \frac{2x - 1}{2\sqrt{x^2 - x + 1}}$

3. (a) $y = (2x + 5)^4$

$\frac{dy}{dx} = 4(2x + 5)^{4-1}\frac{d}{dx}(2x + 5)$

$= 4(2x + 5)^3(2)$

$= 8(2x + 5)^3$

Apabila $x = 1$, $\frac{dy}{dx} = 8[2(1) + 5]^3$

$= 8(343)$

$= 2\ 744$

(b) $y = \sqrt{5 - 2x}$

$y = (5 - 2x)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(5 - 2x)^{\frac{1}{2}-1}\frac{d}{dx}(5 - 2x)$

$= \frac{1}{2}(5 - 2x)^{-\frac{1}{2}}(-2)$

$= -\frac{1}{\sqrt{5 - 2x}}$

Apabila $x = \frac{1}{2}$, $\frac{dy}{dx} = -\frac{1}{\sqrt{4}}$

$= -\frac{1}{2}$

(c) $y = \frac{1}{2x - 3}$

$= (2x - 3)^{-1}$

$\frac{dy}{dx} = -1(2x - 3)^{-1-1}\frac{d}{dx}(2x - 3)$

$= -1(2x - 3)^{-2}(2)$

$= -\frac{2}{(2x - 3)^2}$

Apabila $y = 1$, $1 = \frac{1}{2x - 3}$

$2x - 3 = 1$

$2x = 4$

$x = 2$

dan $\frac{dy}{dx} = -\frac{2}{[2(2) - 3]^2}$

$= -2$

Aktiviti Penerokaan 5 (Halaman 44)

2. $y = (x^2 + 1)(x - 4)^2$

$= (x^2 + 1)(x - 4)(x - 4)$

$= (x^2 + 1)(x^2 - 8x + 16)$

$= x^4 - 8x^3 + 16x^2 + x^2 - 8x + 16$

$= x^4 - 8x^3 + 17x^2 - 8x + 16$

$\frac{dy}{dx} = 4x^3 - 24x^2 + 34x - 8$

3. (a) $u = x^2 + 1$, jadi $\frac{du}{dx} = 2x$ dan $v = (x-4)^2$. Maka, $\frac{dv}{dx} = 2(x-4)$.

(b) $u\frac{dv}{dx} + v\frac{du}{dx} = (x^2+1) \times 2(x-4) + (x-4)^2 \times 2x$

$= 2(x^2+1)(x-4) + 2x(x-4)^2$

$= (x-4)[2(x^2+1) + 2x(x-4)]$

$= (x-4)(2x^2 + 2 + 2x^2 - 8x)$

$= (x-4)(4x^2 - 8x + 2)$

$= 4x^3 - 8x^2 + 2x - 16x^2 + 32x - 8$

$= 4x^3 - 24x^2 + 34x - 8$

4. Jawapannya adalah sama.
Kaedah dalam langkah 3 menjadi pilihan kerana tidak perlu mengembangkan dua ungkapan algebra yang diberi terlebih dahulu.

Aktiviti Penerokaan 6 (Halaman 45)

2. $y = x(x-1)^{-2}$

Menggunakan $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{dy}{dx} = x[-2(x-1)^{-3}] + (x-1)^{-2}(1)$

$= -\frac{2x}{(x-1)^3} + \frac{1}{(x-1)^2}$

$= \frac{-2x + x - 1}{(x-1)^3}$

$= \frac{-x-1}{(x-1)^3}$

$= -\frac{x+1}{(x-1)^3}$

3. (a) $u = x$, jadi $\frac{du}{dx} = 1$ dan $v = (x-1)^2$. Maka, $\frac{dv}{dx} = 2(x-1) = 2x - 2$.

(b) $\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(x-1)^2(1) - x(2x-2)}{(x-1)^4}$

$= \frac{x^2 - 2x + 1 - 2x^2 + 2x}{(x-1)^4}$

$= \frac{1 - x^2}{(x-1)^4}$

$= \frac{(1+x)(1-x)}{(x-1)^4}$

$= \frac{-(x-1)(1+x)}{(x-1)^4}$

$= -\frac{x+1}{(x-1)^3}$

4. Jawapannya adalah sama.
Kaedah dalam langkah 3 menjadi pilihan kerana boleh mencari $\frac{dy}{dx}$ secara langsung tanpa menggunakan petua hasil darab.

Perbincangan (Halaman 45)

Pertimbangkan hasil bahagi dua polinomial $y = \frac{u}{v}$, dengan $u = f(x)$ dan $v = g(x)$.

$$y = \frac{u}{v} \quad \ldots ❶$$

Daripada prinsip pertama: $y + \delta y = \frac{u + \delta u}{v + \delta v} \quad \ldots ❷$

❷ – ❶:

$$\delta y = \frac{u + \delta u}{v + \delta v} - \frac{u}{v}$$

$$= \frac{v(u + \delta u) - u(v + \delta v)}{v(v + \delta v)}$$

$$= \frac{v\delta u - u\delta v}{v(v + \delta v)} \ldots ❸$$

Bahagikan ❸ dengan δx: $$\frac{\delta y}{\delta x} = \frac{v\frac{\delta u}{\delta x} - u\frac{\delta v}{\delta x}}{v(v + \delta v)}$$

Ambil had di kedua-dua belah: $$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{v \lim\limits_{\delta x \to 0} \frac{\delta u}{\delta x} - u \lim\limits_{\delta x \to 0} \frac{\delta v}{\delta x}}{v \lim\limits_{\delta x \to 0} (v + \delta v)}$$

Oleh sebab $\delta x \to 0$, maka δv juga mendekati sifar, jadi $\lim\limits_{\delta x \to 0} (v + \delta v) = v$.

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{v \lim\limits_{\delta x \to 0} \frac{\delta u}{\delta x} - u \lim\limits_{\delta x \to 0} \frac{\delta v}{\delta x}}{v^2}$$

Oleh sebab $\lim\limits_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx}$, maka $\lim\limits_{\delta x \to 0} \frac{\delta u}{\delta x} = \frac{du}{dx}$ dan $\lim\limits_{\delta x \to 0} \frac{\delta v}{\delta x} = \frac{dv}{dx}$.

Maka, petua hasil bahagi menggunakan idea had ialah: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Perbincangan (Halaman 46)

1. Kaedah 1:

$y = x(1 - x^2)^2$

$= x(1 - 2x^2 + x^4)$

$= x - 2x^3 + x^5$

$\frac{dy}{dx} = 1 - 6x^2 + 5x^4$

Kaedah 2:

$y = x(1 - x^2)^2$

Menggunakan $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{dy}{dx} = x(2)(1 - x^2)(-2x) + (1 - x^2)^2(1)$

$= -4x^2(1 - x^2) + 1 - 2x^2 + x^4$

$= -4x^2 + 4x^4 + 1 - 2x^2 + x^4$

$= 1 - 6x^2 + 5x^4$

Ya, jawapannya adalah sama.

2. (a) Petua rantai:

$y = 3(2x - 1)^4$

$\frac{dy}{dx} = 3(4)(2x - 1)^{4-1}\frac{d}{dx}(2x - 1)$

$= 12(2x - 1)^3(2)$

$= 24(2x - 1)^3$

(b) Petua hasil darab:

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{dy}{dx} = 3(4)(2x - 1)^3(2) + (2x - 1)^4(0)$

$= 24(2x - 1)^3$

$\therefore$ Petua rantai

Latihan Kendiri 2.5

1. (a) $y = 4x^2(5x + 3)$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= 4x^2(5) + (5x + 3)(8x)$

$= 20x^2 + 40x^2 + 24x$

$= 60x^2 + 24x$

(b) $y = -2x^3(x + 1)$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= -2x^3(1) + (x + 1)(-6x^2)$$
$$= -2x^3 - 6x^3 - 6x^2$$
$$= -8x^3 - 6x^2$$

(c) $y = x^2(1 - 4x)^4$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= x^2[4(1 - 4x)^3(-4)] + (1 - 4x)^4(2x)$$
$$= -16x^2(1 - 4x)^3 + 2x(1 - 4x)^4$$
$$= 2x(1 - 4x)^3[-8x + (1 - 4x)]$$
$$= 2x(1 - 12x)(1 - 4x)^3$$

(d) $y = x^2\sqrt{1 - 2x^2}$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= x^2\left[\frac{1}{2}(1 - 2x^2)^{-\frac{1}{2}}(-4x)\right] + \left(\sqrt{1 - 2x^2}\right)(2x)$$
$$= -\frac{2x^3}{\sqrt{1 - 2x^2}} + 2x\sqrt{1 - 2x^2}$$
$$= \frac{-2x^3 + 2x(1 - 2x^2)}{\sqrt{1 - 2x^2}}$$
$$= \frac{-2x^3 + 2x - 4x^3}{\sqrt{1 - 2x^2}}$$
$$= \frac{-6x^3 + 2x}{\sqrt{1 - 2x^2}}$$
$$= \frac{2x(1 - 3x^2)}{\sqrt{1 - 2x^2}}$$

(e) $y = (4x - 3)(2x + 7)^6$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= (4x - 3)[6(2x + 7)^5(2)] + (2x + 7)^6(4)$$
$$= 12(4x - 3)(2x + 7)^5 + 4(2x + 7)^6$$
$$= 4(2x + 7)^5[3(4x - 3) + (2x + 7)]$$
$$= 4(2x + 7)^5[12x - 9 + 2x + 7]$$
$$= 4(14x - 2)(2x + 7)^5$$
$$= 8(7x - 1)(2x + 7)^5$$

(f) $y = (x + 5)^3(x - 4)^4$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= (x + 5)^3[4(x - 4)^3(1)] + (x - 4)^4[3(x + 5)^2(1)]$$
$$= 4(x + 5)^3(x - 4)^3 + 3(x - 4)^4(x + 5)^2$$
$$= (x + 5)^2(x - 4)^3[(4(x + 5) + 3(x - 4)]$$
$$= (x + 5)^2(x - 4)^3(4x + 20 + 3x - 12)$$
$$= (7x + 8)(x + 5)^2(x - 4)^3$$

2. (a) $y = (1 - x^2)(6x + 1)$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$= (1 - x^2)(6) + (6x + 1)(-2x)$$
$$= 6 - 6x^2 - 12x^2 - 2x$$
$$= 6 - 18x^2 - 2x$$
$$= -2(9x^2 + x - 3)$$

(b) $y = \left(x + \frac{2}{x}\right)\left(x^2 - \frac{1}{x}\right)$

$= x^3 - 1 + 2x - \frac{2}{x^2}$

$\frac{dy}{dx} = 3x^{3-1} + 2 + 4x^{-3}$

$= 3x^2 + 2 + \frac{4}{x^3}$

(c) $y = (x^3 - 5)(x^2 - 2x + 8)$

$= x^5 - 2x^4 + 8x^3 - 5x^2 + 10x - 40$

$\frac{dy}{dx} = 5x^4 - 8x^3 + 24x^2 - 10x + 10$

3. $f(x) = x\sqrt{x-1}$

$f'(x) = x\left[\left(\frac{1}{2}\right)(x-1)^{-\frac{1}{2}}(1)\right] + \left(\sqrt{x-1}\right)(1)$

$= \frac{x}{2\sqrt{x-1}} + \sqrt{x-1}$

$= \frac{x + 2(x-1)}{2\sqrt{x-1}}$

$= \frac{3x-2}{2\sqrt{x-1}}$

$f'(5) = \frac{3(5)-2}{2\sqrt{5-1}}$

$= \frac{13}{4}$

4. $y = x\sqrt{x^2+9}$

$\frac{dy}{dx} = x\left[\frac{1}{2}(x^2+9)^{-\frac{1}{2}}(2x)\right] + \left(\sqrt{x^2+9}\right)(1)$

$= \frac{x^2}{\sqrt{x^2+9}} + \sqrt{x^2+9}$

$= \frac{x^2 + x^2 + 9}{\sqrt{x^2+9}}$

$= \frac{2x^2+9}{\sqrt{x^2+9}}$

Pada $x = 4$, $\frac{dy}{dx} = \frac{2(4)^2+9}{\sqrt{4^2+9}}$

$= \frac{41}{5}$

5. (a) $\frac{d}{dx}\left(\frac{3}{2x-7}\right) = \frac{(2x-7)(0) - 3(2)}{(2x-7)^2}$

$= -\frac{6}{(2x-7)^2}$

(b) $\frac{d}{dx}\left(\frac{3x}{4x+6}\right) = \frac{(4x+6)(3) - 3x(4)}{(4x+6)^2}$

$= \frac{12x + 18 - 12x}{(4x+6)^2}$

$= \frac{18}{(4x+6)^2}$

(c) $\frac{d}{dx}\left(\frac{4x^2}{1-6x}\right) = \frac{(1-6x)(8x) - (4x^2)(-6)}{(1-6x)^2}$

$= \frac{8x - 48x^2 + 24x^2}{(1-6x)^2}$

$= \frac{8x - 24x^2}{(1-6x)^2}$

$= \frac{8x(1-3x)}{(1-6x)^2}$

(d) $\frac{d}{dx}\left(\frac{x^3+1}{2x-1}\right) = \frac{(2x-1)(3x^2)-(x^3+1)(2)}{(2x-1)^2}$

$= \frac{6x^3-3x^2-2x^3-2}{(2x-1)^2}$

$= \frac{4x^3-3x^2-2}{(2x-1)^2}$

(e) $\frac{d}{dx}\left(\frac{\sqrt{x}}{x+1}\right) = \frac{(x+1)\left(\frac{1}{2\sqrt{x}}\right)-(\sqrt{x})(1)}{(x+1)^2}$

$= \frac{\left(\frac{x+1-2x}{2\sqrt{x}}\right)}{(x+1)^2}$

$= \frac{1-x}{2\sqrt{x}\,(x+1)^2}$

(f) $\frac{d}{dx}\left(\frac{x}{\sqrt{x-1}}\right) = \frac{\left(\sqrt{x-1}\right)(1)-\left(\frac{1}{2\sqrt{x-1}}\right)(x)}{\left(\sqrt{x-1}\right)^2}$

$= \frac{\left(\frac{2(x-1)-x}{2\sqrt{x-1}}\right)}{x-1}$

$= \frac{x-2}{2(x-1)\sqrt{x-1}}$

$= \frac{x-2}{2\sqrt{(x-1)^3}}$

(g) $\frac{d}{dx}\left(\frac{3x^2}{\sqrt{2x^2+3}}\right) = \frac{\left(\sqrt{2x^2+3}\right)(6x)-\left(\frac{2x}{\sqrt{2x^2+3}}\right)(3x^2)}{\left(\sqrt{2x^2+3}\right)^2}$

$= \frac{\left(\frac{6x(2x^2+3)-6x^3}{\sqrt{2x^2+3}}\right)}{2x^2+3}$

$= \frac{12x^3+18x-6x^3}{\sqrt{2x^2+3}\,(2x^2+3)}$

$= \frac{6x^3+18x}{\sqrt{2x^2+3}\,(2x^2+3)}$

$= \frac{6x(x^2+3)}{\sqrt{(2x^2+3)^3}}$

(h) $\frac{d}{dx}\left(\sqrt{\frac{4x+1}{3x^2-7}}\right) = \frac{d}{dx}\left(\frac{\sqrt{4x+1}}{\sqrt{3x^2-7}}\right)$

$= \frac{\left(\sqrt{3x^2-7}\right)\left(\frac{2}{\sqrt{4x+1}}\right)-\left(\sqrt{4x+1}\right)\left(\frac{3x}{\sqrt{3x^2-7}}\right)}{\left(\sqrt{3x^2-7}\right)^2}$

$= \frac{\left(\frac{2(3x^2-7)-3x(4x+1)}{\left(\sqrt{4x+1}\right)\left(\sqrt{3x^2-7}\right)}\right)}{3x^2-7}$

$= \frac{6x^2-14-12x^2-3x}{(3x^2-7)\left(\sqrt{4x+1}\right)\left(\sqrt{3x^2-7}\right)}$

$= \frac{6x^2-14-12x^2-3x}{\left(\sqrt{4x+1}\right)\sqrt{(3x^2-7)^3}}$

$= -\frac{6x^2+3x+14}{\left(\sqrt{4x+1}\right)\sqrt{(3x^2-7)^3}}$

6. $\dfrac{2(x+5)-(2x-3)}{(x+5)^2} = \dfrac{2x+10-2x+3}{(x+5)^2}$

$= \dfrac{13}{(x+5)^2}$

Jadi, secara perbandingan, nilai pemalar r ialah 13.

Latihan Formatif 2.2

1. (a) $\dfrac{d}{dx}\left(9x^2 - \dfrac{3}{x^2}\right) = \dfrac{d}{dx}(9x^2 - 3x^{-2})$

$= 18x + 6x^{-3}$

$= 18x + \dfrac{6}{x^3}$

(b) $\dfrac{d}{dx}\left(\dfrac{6}{x^3} - \dfrac{1}{x} + 8\right) = \dfrac{d}{dx}(6x^{-3} - x^{-1} + 8)$

$= -18x^{-4} + x^{-2}$

$= \dfrac{1}{x^2} - \dfrac{18}{x^4}$

(c) $\dfrac{d}{dx}\left[5x + 4\sqrt{x} - 7\right] = \dfrac{d}{dx}(5x + 4x^{\frac{1}{2}} - 7)$

$= 5 + 2x^{-\frac{1}{2}}$

$= 5 + \dfrac{2}{\sqrt{x}}$

(d) $\dfrac{d}{dx}\left(\dfrac{10}{\sqrt{x}} + \dfrac{3}{\sqrt[3]{x}}\right) = \dfrac{d}{dx}\left(10x^{-\frac{1}{2}} + 3x^{-\frac{1}{3}}\right)$

$= -5x^{-\frac{3}{2}} - x^{-\frac{4}{3}}$

$= -\dfrac{5}{x^{\frac{3}{2}}} - \dfrac{1}{x^{\frac{4}{3}}}$

$= -\dfrac{5}{\sqrt{x^3}} - \dfrac{1}{\sqrt[3]{x^4}}$

(e) $\dfrac{d}{dx}\left(x^2 - \dfrac{3}{x}\right)^2 = \dfrac{d}{dx}\left(x^4 - 6x + \dfrac{9}{x^2}\right)$

$= \dfrac{d}{dx}(x^4 - 6x + 9x^{-2})$

$= 4x^3 - 6 - \dfrac{18}{x^3}$

(f) $\dfrac{d}{dx}\left(\dfrac{8x^2 + x}{\sqrt{x}}\right) = \dfrac{d}{dx}\left(8x^{\frac{3}{2}} + x^{\frac{1}{2}}\right)$

$= 12x^{\frac{1}{2}} + \dfrac{1}{2}x^{-\frac{1}{2}}$

$= 12\sqrt{x} + \dfrac{1}{2\sqrt{x}}$

(g) $\dfrac{d}{dx}\left(\dfrac{4}{9x^3} - \pi x + 6\right) = \dfrac{d}{dx}\left(\dfrac{4}{9}x^{-3} - \pi x + 6\right)$

$= -\dfrac{4}{3}x^{-4} - \pi$

$= -\dfrac{4}{3x^4} - \pi$

(h) $\dfrac{d}{dx}\left[\sqrt{x}\,(2 - x)\right] = \dfrac{d}{dx}(2x^{\frac{1}{2}} - x^{\frac{3}{2}})$

$= x^{-\frac{1}{2}} - \dfrac{3}{2}x^{\frac{1}{2}}$

$= \dfrac{1}{\sqrt{x}} - \dfrac{3}{2}\sqrt{x}$

2. $f(x) = 3x^{\frac{2}{3}} + 6x^{-\frac{1}{3}}$

$f'(x) = 2x^{-\frac{1}{3}} - 2x^{-\frac{4}{3}}$

$= \frac{2}{\sqrt[3]{x}} - \frac{2}{\sqrt[3]{x^4}}$

$f'(8) = \frac{2}{\sqrt[3]{8}} - \frac{2}{\sqrt[3]{8^4}}$

$= 1 - \frac{2}{16}$

$= \frac{7}{8}$

3. (a) $f(t) = \frac{6t^3}{\sqrt[3]{t}}$

$= 6t^{3-\frac{1}{3}}$

$= 6t^{\frac{8}{3}}$

(b) $f'(t) = 6\left(\frac{8}{3}t^{\frac{8}{3}-1}\right)$

$= 16t^{\frac{5}{3}}$

(c) $f'(\frac{1}{8}) = 16\left(\frac{1}{8}\right)^{\frac{5}{3}}$

$= 16\left(\frac{1}{2^3}\right)^{\frac{5}{3}}$

$= \frac{2^4}{2^5}$

$= \frac{1}{2}$

4. $s = 3t^2 + 5t - 7$

$\frac{ds}{dt} = 6t + 5$

Apabila $\frac{ds}{dt} < 0$

$6t + 5 < 0$

$6t < -5$

$t < -\frac{5}{6}$

5. $y = ax^3 + bx^2 + 3$

Pada titik $(1, 4)$, $4 = a(1)^3 + b(1)^2 + 3$

$4 = a + b + 3$

$a + b = 1$ …❶

dan $\frac{dy}{dx} = 7$

$3ax^2 + 2bx = 7$

Pada titik $(1, 4)$, $3a(1)^2 + 2b(1) = 7$

$3a + 2b = 7$ …❷

$2 \times$ ❶: $2a + 2b = 2$ …❸

❷ − ❸: $a = 5$

Gantikan $a = 5$ dalam ❶: $5 + b = 1$

$b = -4$

$\therefore a = 5$ dan $b = -4$

6. $\dfrac{dy}{dx} = 3$

$3x^2 - 6x + 6 = 3$
$3x^2 - 6x + 3 = 0$
$x^2 - 2x + 1 = 0$
$(x - 1)(x - 1) = 0$
$x = 1$

Apabila $x = 1$, $y = 1^3 - 3(1)^2 + 6(1) + 2$
$= 1 - 3 + 6 + 2$
$= 6$

$\therefore (1, 6)$

7. (a) $h(x) = kx^3 - 4x^2 - 5x$
$h'(x) = 3kx^2 - 8x - 5$

(b) $h'(1) = 8$
$3k(1)^2 - 8(1) - 5 = 8$
$3k - 13 = 8$
$3k = 21$
$k = 7$

8. (a) $y = \dfrac{3}{4}\left(\dfrac{x}{6} - 1\right)^4$

$\dfrac{dy}{dx} = \left[\dfrac{3}{4}(4)\left(\dfrac{x}{6} - 1\right)^{4-1}\right]\dfrac{d}{dx}\left(\dfrac{x}{6} - 1\right)$
$= 3\left(\dfrac{x}{6} - 1\right)^3\left(\dfrac{1}{6}\right)$
$= \dfrac{1}{2}\left(\dfrac{x}{6} - 1\right)^3$

(b) $y = \dfrac{1}{12}(10x - 3)^6$

$\dfrac{dy}{dx} = \left[\dfrac{1}{12}(6)(10x - 3)^{6-1}\right]\dfrac{d}{dx}(10x - 3)$
$= \dfrac{1}{2}(10x - 3)^5(10)$
$= 5(10x - 3)^5$

(c) $y = \dfrac{8}{2 - 5x}$

$y = 8(2 - 5x)^{-1}$

$\dfrac{dy}{dx} = [8(-1)(2 - 5x)^{-1-1}]\dfrac{d}{dx}(2 - 5x)$
$= -8(2 - 5x)^{-2}(-5)$
$= \dfrac{40}{(2 - 5x)^2}$

(d) $y = \left(x - \dfrac{1}{x}\right)^3$
$= (x - x^{-1})^3$

$\dfrac{dy}{dx} = 3(x - x^{-1})^{3-1}\dfrac{d}{dx}(x - x^{-1})$
$= 3(x - x^{-1})^2(1 + x^{-2})$
$= 3\left(1 + \dfrac{1}{x^2}\right)\left(x - \dfrac{1}{x}\right)^2$

(e) $y = \dfrac{1}{\sqrt[3]{3 - 9x}}$
$= (3 - 9x)^{-\frac{1}{3}}$

$$\frac{dy}{dx} = -\frac{1}{3}(3-9x)^{-\frac{1}{3}-1}\frac{d}{dx}(3-9x)$$
$$= -\frac{1}{3}(3-9x)^{-\frac{4}{3}}(-9)$$
$$= \frac{3}{(3-9x)^{\frac{4}{3}}}$$
$$= \frac{3}{\sqrt[3]{(3-9x)^4}}$$

(f) $y = \sqrt{x^2+6x+6}$
$= (x^2+6x+6)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(x^2+6x+6)^{\frac{1}{2}-1}\frac{d}{dx}(x^2+6x+6)$$
$$= \left[\frac{1}{2}(x^2+6x+6)^{-\frac{1}{2}}\right](2x+6)$$
$$= \frac{x+3}{\sqrt{x^2+6x+6}}$$

9. $y = \dfrac{24}{(3x-5)^2}$
$= 24(3x-5)^{-2}$

$$\frac{dy}{dx} = -48(3x-5)^{-2-1}\frac{d}{dx}(3x-5)$$
$$= -48(3x-5)^{-3}(3)$$
$$= -\frac{144}{(3x-5)^3}$$

Apabila $x = 2$, $\dfrac{dy}{dx} = -\dfrac{144}{(3(2)-5)^3}$
$= -144$

10. $\dfrac{d}{dx}\left(\dfrac{1}{(3x-2)^3}\right) = \dfrac{d}{dx}(3x-2)^{-3}$
$= -3(3x-2)^{-4}(3)$
$= -\dfrac{9}{(3x-2)^4}$

Secara perbandingan, $a = 9$ dan $b = 4$

11. (a) $\dfrac{d}{dx}[4x(2x-1)^5] = 4x[5(2x-1)^4(2)] + (2x-1)^5(4)$
$= 40x(2x-1)^4 + 4(2x-1)^5$
$= 4(2x-1)^4[10x + (2x-1)]$
$= 4(12x-1)(2x-1)^4$

(b) $\dfrac{d}{dx}[x^4(3x+1)^7] = x^4[7(3x+1)^6(3)] + (3x+1)^7(4x^3)$
$= 21x^4(3x+1)^6 + 4x^3(3x+1)^7$
$= x^3(3x+1)^6[21x + 4(3x+1)]$
$= x^3(33x+4)(3x+1)^6$

(c) $\dfrac{d}{dx}\left(x\sqrt{x+3}\right) = x\left[\dfrac{1}{2}(x+3)^{-\frac{1}{2}}(1)\right] + \sqrt{x+3}(1)$
$= \dfrac{x}{2\sqrt{x+3}} + \sqrt{x+3}$
$= \dfrac{x+2(x+3)}{2\sqrt{x+3}}$
$= \dfrac{3x+6}{2\sqrt{x+3}}$
$= \dfrac{3(x+2)}{2\sqrt{x+3}}$

(d) $\frac{d}{dx}[(x+7)^5(x-5)^3] = (x+7)^5[3(x-5)^2(1) + (x-5)^3[5(x+7)^4(1)]$

$= 3(x+7)^5(x-5)^2 + 5\ (x+7)^4(x-5)^3$

$= (x+7)^4(x-5)^2[3(x+7) + 5(x-5)]$

$= (x+7)^4(x-5)^2(3x + 21 + 5x - 25)$

$= (x+7)^4(x-5)^2(8x-4)$

$= 4(2x-1)\ (x+7)^4(x-5)^2$

(e) $\frac{d}{dx}\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) = \frac{\left(1+\sqrt{x}\right)\left(-\frac{1}{2}x^{-\frac{1}{2}}\right) - \left(1-\sqrt{x}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{\left(1+\sqrt{x}\right)^2}$

$= \frac{-\frac{1+\sqrt{x}}{2\sqrt{x}} - \frac{1-\sqrt{x}}{2\sqrt{x}}}{\left(1+\sqrt{x}\right)^2}$

$= \frac{-1-\sqrt{x}-1+\sqrt{x}}{2\sqrt{x}\left(1+\sqrt{x}\right)^2}$

$= -\frac{2}{2\sqrt{x}\left(1+\sqrt{x}\right)^2}$

$= -\frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}$

(f) $\frac{d}{dx}\left(\frac{x}{\sqrt{4x+1}}\right) = \frac{\left(\sqrt{4x+1}\right)(1) - x\left[\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)\right]}{\left(\sqrt{4x+1}\right)^2}$

$= \frac{\sqrt{4x+1} - \frac{2x}{\sqrt{4x+1}}}{4x+1}$

$= \frac{\frac{4x+1-2x}{\sqrt{4x+1}}}{4x+1}$

$= \frac{2x+1}{(4x+1)\sqrt{4x+1}}$

$= \frac{2x+1}{\sqrt{(4x+1)^3}}$

(g) $\frac{d}{dx}\left(\frac{1}{x^2+2x+7}\right) = \frac{(x^2+2x+7)(0) - 1(2x+2)}{(x^2+2x+7)^2}$

$= -\frac{2(x+1)}{(x^2+2x+7)^2}$

(h) $\frac{d}{dx}\left(\frac{1-2x^3}{x-1}\right) = \frac{(x-1)(-6x^2) - (1-2x^3)(1)}{(x-1)^2}$

$= \frac{-6x^3 + 6x^2 - 1 + 2x^3}{(x-1)^2}$

$= \frac{6x^2 - 4x^3 - 1}{(x-1)^2}$

12. $f(x) = x\sqrt{x^2+3}$

$= x(x^2+3)^{\frac{1}{2}}$

$f'(x) = x\left(\frac{1}{2}\right)(2x)(x^2+3)^{-\frac{1}{2}} + (1)(x^2+3)^{\frac{1}{2}}$

$= \frac{x^2}{\sqrt{x^2+3}} + \sqrt{x^2+3}$

$= \frac{x^2 + x^2 + 3}{\sqrt{x^2+3}}$

$= \frac{2x^2+3}{\sqrt{x^2+3}}$ (Tertunjuk)

13. $y = \dfrac{4x - 3}{x^2 + 1}$

$$\frac{dy}{dx} = \frac{(x^2 + 1)(4) - (4x - 3)(2x)}{(x^2 + 1)^2}$$

$$= \frac{4x^2 + 4 - 8x^2 + 6x}{(x^2 + 1)^2}$$

$$= \frac{4 + 6x - 4x^2}{(x^2 + 1)^2}$$

Untuk $\dfrac{dy}{dx}$ adalah positif, $4 + 6x - 4x^2 > 0$

$$2x^2 - 3x - 2 < 0$$

$$(2x + 1)(x - 2) < 0$$

$$-\frac{1}{2} < x < 2 \quad \ldots ❶$$

Untuk y adalah positif, $4x - 3 > 0$

$$x > \frac{3}{4} \quad \ldots ❷$$

Gabungkan ❶ dan ❷, diperoleh $\dfrac{3}{4} < x < 2$.

14. $y = \dfrac{x - 2}{x^2 + 5}$

$$\frac{dy}{dx} = \frac{x^2 + 5 - 2x\,(x - 2)}{(x^2 + 5)^2}$$

$$= \frac{x^2 + 5 - 2x^2 + 4x}{(x^2 + 5)^2}$$

$$= \frac{5 + 4x - x^2}{(x^2 + 5)^2}$$

Untuk $\dfrac{dy}{dx} < 0$,

$$5 + 4x - x^2 < 0$$

$$x^2 - 4x - 5 > 0$$

$$(x + 1)(x - 5) > 0$$

$$x < -1 \text{ atau } x > 5 \quad \ldots ❶$$

Untuk $y < 0$,

$$x - 2 < 0$$

$$x < 2 \quad \ldots ❷$$

Gabungkan ❶ dan ❷, diperoleh $x < -1$.

Kuiz Pantas (Halaman 50)

Diberi $y = 5x - 3$.

$$\frac{dy}{dx} = 5$$

(a) $\left(\dfrac{dy}{dx}\right)^2 = 5^2$

$$= 25$$

(b) $\dfrac{d^2y}{dx^2} = 0$

Jadi, $\left(\dfrac{dy}{dx}\right)^2 \neq \dfrac{d^2y}{dx^2}$.

Latihan Kendiri 2.6

1. (a) $y = 3x^4 - 5x^2 + 2x - 1$

$$\frac{dy}{dx} = 12x^3 - 10x + 2$$

$$\frac{d^2y}{dx^2} = 36x^2 - 10$$

(b) $y = 4x^2 - \frac{2}{x}$

$= 4x^2 - 2x^{-1}$

$\frac{dy}{dx} = 8x + 2x^{-2}$

$= 8x + \frac{2}{x^2}$

$\frac{d^2y}{dx^2} = 8 - 4x^{-3}$

$= 8 - \frac{4}{x^3}$

(c) $y = (3x + 2)^8$

$\frac{dy}{dx} = 8(3x + 2)^7(3)$

$= 24(3x + 2)^7$

$\frac{d^2y}{dx^2} = 24(7)(3x + 2)^6(3)$

$= 504(3x + 2)^6$

2. (a) $f(x) = \sqrt{x} + \frac{1}{x^2}$

$= x^{\frac{1}{2}} + x^{-2}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-3}$

$= \frac{1}{2\sqrt{x}} - \frac{2}{x^3}$

$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} + 6x^{-4}$

$= -\frac{1}{4x^{\frac{3}{2}}} + \frac{6}{x^4}$

(b) $f(x) = \frac{x^4 + 2}{x^2}$

$= x^2 + 2x^{-2}$

$f'(x) = 2x - 4x^{-3}$

$= 2x - \frac{4}{x^3}$

$f''(x) = 2 + 12x^{-4}$

$= 2 + \frac{12}{x^4}$

(c) $f(x) = \frac{2x + 5}{x - 1}$

$f'(x) = \frac{(x - 1)(2) - (2x + 5)(1)}{(x - 1)^2}$

$= \frac{2x - 2 - 2x - 5}{(x - 1)^2}$

$= -\frac{7}{(x - 1)^2}$

$f''(x) = \frac{(x - 1)^2(0) - [(-7)(2)(x - 1)(1)]}{[(x - 1)^2]^2}$

$= \frac{14(x - 1)}{(x - 1)^4}$

$= \frac{14}{(x - 1)^3}$

3. $y = x^3 + 3x^2 - 9x + 2$

$$\frac{dy}{dx} = 0$$

$$3x^2 + 6x - 9 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x = -3 \text{ atau } x = 1$$

Apabila $x = -3$, $y = (-3)^3 + 3(-3)^2 - 9(-3) + 2$
$= -27 + 27 + 27 + 2$
$= 29$

Apabila $x = 1$, $y = 1^3 + 3(1)^2 - 9(1) + 2$
$= 1 + 3 - 9 + 2$
$= -3$

Jadi, koordinat titik A yang mungkin ialah $(-3, 29)$ dan $(1, -3)$.

$$\frac{d^2y}{dx^2} = 6x + 6$$

Apabila $x = -3$, $\frac{d^2y}{dx^2} = 6(-3) + 6$
$= -18 + 6$
$= -12$

Apabila $x = 1$, $\frac{d^2y}{dx^2} = 6(1) + 6$
$= 6 + 6$
$= 12$

Latihan Formatif 2.3

1. $xy - 2x^2 = 3$

$$xy = 3 + 2x^2$$

$$y = \frac{3 + 2x^2}{x}$$

$$= 2x + \frac{3}{x}$$

$$= 2x + 3x^{-1}$$

$$\frac{dy}{dx} = 2 - 3x^{-2}$$

$$= 2 - \frac{3}{x^2}$$

$$\frac{d^2y}{dx^2} = 6x^{-3}$$

$$= \frac{6}{x^3}$$

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = x^2\left(\frac{6}{x^3}\right) + x\left(2 - \frac{3}{x^2}\right)$$

$$= \frac{6}{x} + 2x - \frac{3}{x}$$

$$= 2x + \frac{3}{x} \text{ (Tertunjuk)}$$

2. (a) $f(x) = 3x - 2x^3$

$$f'(x) = 3 - 6x^2$$

$$f''(x) = -12x$$

$$f'(1) = 3 - 6(1)^2$$

$$= -3$$

$$f''(1) = -12(1)$$

$$= -12$$

(b) $f(x) = x^2(5x - 3)$

$$= 5x^3 - 3x^2$$

$$f'(x) = 15x^2 - 6x$$

$$f''(x) = 30x - 6$$

$$f'(1) = 15(1)^2 - 6(1)$$

$$= 9$$

$$f''(1) = 30(1) - 6$$
$$= 24$$

(c) $f(x) = \dfrac{x^3 + x}{x^2}$

$$= x + \frac{1}{x}$$
$$= x + x^{-1}$$
$$f'(x) = 1 - x^{-2}$$
$$= 1 - \frac{1}{x^2}$$
$$f''(x) = 2x^{-3}$$
$$= \frac{2}{x^3}$$
$$f'(1) = 1 - \frac{1}{1^2}$$
$$= 0$$
$$f''(1) = \frac{2}{1^3}$$
$$= 2$$

3. $f(x) = \sqrt{x^2 - 5}$

$$f'(x) = \left(\frac{1}{2}\right)(x^2 - 5)^{-\frac{1}{2}}(2x)$$
$$= \frac{x}{\sqrt{x^2 - 5}}$$
$$f''(x) = \frac{\sqrt{x^2 - 5}\,(1) - x\left[\frac{1}{2}(x^2 - 5)^{-\frac{1}{2}}(2x)\right]}{\left(\sqrt{x^2 - 5}\right)^2}$$
$$= \frac{\sqrt{x^2 - 5} - \dfrac{x^2}{\sqrt{x^2 - 5}}}{x^2 - 5}$$
$$= \frac{x^2 - 5 - x^2}{(x^2 - 5)\sqrt{x^2 - 5}}$$
$$= -\frac{5}{(x^2 - 5)\sqrt{x^2 - 5}}$$
$$= -\frac{5}{\sqrt{(x^2 - 5)^3}}$$
$$f'(3) = \frac{3}{\sqrt{3^2 - 5}}$$
$$= \frac{3}{\sqrt{4}}$$
$$= \frac{3}{2}$$
$$f''(-3) = -\frac{5}{\sqrt{[(-3)^2 - 5]^3}}$$
$$= -\frac{5}{\sqrt{4^3}}$$
$$= -\frac{5}{8}$$

4. $a = t^3 + 2t^2 + 3t + 4$

$$\frac{da}{dt} = 3t^2 + 4t + 3$$
$$\frac{d^2a}{dt^2} = 6t + 4$$

$$\frac{da}{dt} = \frac{d^2a}{dt^2}$$

$$3t^2 + 4t + 3 = 6t + 4$$

$$3t^2 - 2t - 1 = 0$$

$$(3t + 1)(t - 1) = 0$$

$$t = -\frac{1}{3} \text{ atau } t = 1$$

5. $g(x) = hx^3 - 4x^2 + 5x$

$g'(x) = 3hx^2 - 8x + 5$

$g''(x) = 6hx - 8$

Diberi $g''(1) = 4$

$$6h(1) - 8 = 4$$

$$6h = 12$$

$$h = 2$$

6. $f(x) = x^3 - x^2 - 8x + 9$

(a) $f'(x) = 0$

$$3x^2 - 2x - 8 = 0$$

$$(3x + 4)(x - 2) = 0$$

$$x = -\frac{4}{3} \text{ atau } x = 2$$

(b) $f''(x) = 6x - 2$

(c) $f''(x) = 0$

$$6x - 2 = 0$$

$$6x = 2$$

$$x = \frac{1}{3}$$

(d) $f''(x) < 0$

$$6x - 2 < 0$$

$$6x < 2$$

$$x < \frac{1}{3}$$

Latihan Kendiri 2.7

1. (a) (i) $y = 9x + \frac{1}{x}$

$$= 9x + x^{-1}$$

$$\frac{dy}{dx} = 9 - x^{-2}$$

$$= 9 - \frac{1}{x^2}$$

Apabila $x = \frac{1}{4}$, $\frac{dy}{dx} = 9 - \frac{1}{\left(\frac{1}{4}\right)^2}$

$$= 9 - 16$$

$$= -7$$

Apabila $x = 1$, $\frac{dy}{dx} = 9 - \frac{1}{1^2}$

$$= 9 - 1$$

$$= 8$$

(ii) Pada $x = \frac{1}{4}$, kecerunan tangennya adalah negatif, iaitu -7 (< 0). Jadi, garis tangen condong ke kiri.

Pada $x = 1$ pula, kecerunan tangennya adalah positif, iaitu 8 (> 0). Jadi, garis tangen condong ke kanan.

(b) $\frac{dy}{dx} = 0$

$9 - \frac{1}{x^2} = 0$

$9 = \frac{1}{x^2}$

$9x^2 = 1$

$x^2 = \frac{1}{9}$

$x = \pm\frac{1}{3}$

Apabila $x = \frac{1}{3}$, $y = 9\left(\frac{1}{3}\right) + \frac{1}{\frac{1}{3}}$, Apabila $x = -\frac{1}{3}$, $y = 9\left(-\frac{1}{3}\right) + \frac{1}{\left(-\frac{1}{3}\right)}$

$= 3 + 3$ $= -3 - 3$

$= 6$ $= -6$

$\therefore \left(\frac{1}{3}, 6\right)$ dan $\left(-\frac{1}{3}, -6\right)$

2. (a) $y = ax^2 + \frac{b}{x}$

$= ax^2 + bx^{-1}$

$\frac{dy}{dx} = 2ax - bx^{-2}$

$= 2ax - \frac{b}{x^2}$

Apabila $x = \frac{1}{2}$, $\frac{dy}{dx} = -14$, Apabila $x = 2$, $\frac{dy}{dx} = 7$

$2a\left(\frac{1}{2}\right) - \frac{b}{\left(\frac{1}{2}\right)^2} = -14$, $2a(2) - \frac{b}{2^2} = 7$

$a - 4b = -14$ …❶ , $4a - \frac{1}{4}b = 7$

$16a - b = 28$ … ❷

❷ × 4: $64a - 4b = 112$ …❸

❸ − ❶: $63a = 126$

$a = \frac{126}{63}$

$= 2$

Gantikan $a = 2$ ke dalam ❶: $2 - 4b = -14$

$4b = 16$

$b = 4$

$\therefore a = 2, b = 4$

(b) $\frac{dy}{dx} = 0$

$2(2)(x) - \frac{4}{x^2} = 0$

$4x - \frac{4}{x^2} = 0$

$4x = \frac{4}{x^2}$

$x^3 = 1$

$x = 1$

Apabila $x = 1$, $y = 2(1)^2 + \frac{4}{1}$

$= 2 + 4$

$= 6$

$\therefore (1, 6)$

Latihan Kendiri 2.8

1. (a) $f(x) = 5x^2 - 7x - 1$

$f'(x) = 10x - 7$

Apabila $x = 1, f'(1) = 10(1) - 7$

$= 3$

Persamaan tangen: $y - (-3) = 3(x - 1)$

$y + 3 = 3x - 3$

$y = 3x - 6$

Persamaan normal: $y - (-3) = -\frac{1}{3}(x - 1)$

$3y + 9 = -x + 1$

$3y + x + 8 = 0$

(b) $f(x) = x^3 - 5x + 6$

$f'(x) = 3x^2 - 5$

Apabila $x = 2, f'(2) = 3(2)^2 - 5$

$= 12 - 5$

$= 7$

Persamaan tangen: $y - 4 = 7(x - 2)$

$y - 4 = 7x - 14$

$y = 7x - 10$

Persamaan normal: $y - 4 = -\frac{1}{7}(x - 2)$

$7y - 28 = -x + 2$

$7y + x = 30$

(c) $f(x) = \sqrt{2x + 1}$

$f'(x) = \frac{1}{2}(2x + 1)^{-\frac{1}{2}}(2)$

$= \frac{1}{\sqrt{2x + 1}}$

Apabila $x = 4, f'(4) = \frac{1}{\sqrt{2(4) + 1}}$

$= \frac{1}{\sqrt{9}}$

$= \frac{1}{3}$

Persamaan tangen: $y - 3 = \frac{1}{3}(x - 4)$

$3y - 9 = x - 4$

$3y - x = 5$

Persamaan normal: $y - 3 = -3(x - 4)$

$y - 3 = -3x + 12$

$y = -3x + 15$

(d) $f(x) = \frac{x + 1}{x - 1}$

$f'(x) = \frac{(x - 1)(1) - (x + 1)(1)}{(x - 1)^2}$

$= \frac{x - 1 - x - 1}{(x - 1)^2}$

$= -\frac{2}{(x - 1)^2}$

Apabila $x = 3, f'(3) = -\frac{2}{(3 - 1)^2}$

$= -\frac{1}{2}$

Persamaan tangen: $y - 2 = -\frac{1}{2}(x - 3)$

$2y - 4 = -x + 3$

$2y = -x + 7$

Persamaan normal: $y - 2 = 2(x - 3)$

$y - 2 = 2x - 6$

$y = 2x - 4$

2. (a) $y = 2x^3 - 4x + 3$

$\frac{dy}{dx} = 6x^2 - 4$

Apabila $x = 1$, $y = 2(1)^3 - 4(1) + 3$

$= 2 - 4 + 3$

$= 1$

dan $\frac{dy}{dx} = 6(1)^2 - 4$

$= 6 - 4$

$= 2$

Persamaan tangen: $y - 1 = 2(x - 1)$

$y - 1 = 2x - 2$

$y = 2x - 1$

Persamaan normal: $y - 1 = -\frac{1}{2}(x - 1)$

$2y - 2 = -x + 1$

$2y + x = 3$

(b) $y = \sqrt{x} - \frac{1}{\sqrt{x}}$

$= x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x^3}}$

Apabila $x = 4$, $y = \sqrt{4} - \frac{1}{\sqrt{4}}$

$= 2 - \frac{1}{2}$

$= \frac{3}{2}$

dan $\frac{dy}{dx} = \frac{1}{2\sqrt{4}} + \frac{1}{2\sqrt{4^3}}$

$= \frac{1}{4} + \frac{1}{16}$

$= \frac{5}{16}$

Persamaan tangen: $y - \frac{3}{2} = \frac{5}{16}(x - 4)$

$16y - 24 = 5x - 20$

$16y - 5x = 4$

Persamaan normal: $y - \frac{3}{2} = -\frac{16}{5}(x - 4)$

$5y - \frac{15}{2} = -16x + 64$

$10y - 15 = -32x + 128$

$10y = -32x + 143$

(c) $y = \sqrt{x+1}$

$y = (x+1)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x+1}}$

Apabila $x = 3$, $y = \sqrt{3+1}$

$= \sqrt{4}$

$= 2$

dan $\frac{dy}{dx} = \frac{1}{2\sqrt{3+1}}$

$= \frac{1}{4}$

Persamaan tangen: $y - 2 = \frac{1}{4}(x-3)$

$y = \frac{1}{4}x - \frac{3}{4} + 2$

$y = \frac{1}{4}x + \frac{5}{4}$

Persamaan normal: $y - 2 = -4(x-3)$

$y = -4x + 12 + 2$

$y = -4x + 14$

(d) $y = \frac{5}{x^2+1}$

$\frac{dy}{dx} = \frac{(x^2+1)(0) - 5(2x)}{(x^2+1)^2}$

$= -\frac{10x}{(x^2+1)^2}$

Apabila $x = -2$, $y = \frac{5}{(-2)^2+1}$

$= 1$

dan $\frac{dy}{dx} = -\frac{10(-2)}{[(-2)^2+1]^2}$

$= \frac{4}{5}$

Persamaan tangen: $y - 1 = \frac{4}{5}(x+2)$

$5y - 5 = 4x + 8$

$5y - 4x = 13$

Persamaan normal: $y - 1 = -\frac{5}{4}(x+2)$

$4y - 4 = -5x - 10$

$4y + 5x + 6 = 0$

(e) $y = 2 + \frac{1}{x}$

$\frac{dy}{dx} = -\frac{1}{x^2}$

Apabila $x = -1$, $y = 2 + \frac{1}{-1}$

$= 1$

dan $\frac{dy}{dx} = -\frac{1}{(-1)^2}$

$= -1$

Persamaan tangen: $y - 1 = -[x - (-1)]$

$y - 1 = -x - 1$

$y = -x$

Persamaan normal: $y - 1 = x - (-1)$

$y = x + 1 + 1$

$y = x + 2$

(f) $y = \dfrac{x^2 + 3}{x + 1}$

$\dfrac{dy}{dx} = \dfrac{(x + 1)(2x) - (1)(x^2 + 3)}{(x + 1)^2}$

$= \dfrac{2x^2 + 2x - x^2 - 3}{(x + 1)^2}$

$= \dfrac{x^2 + 2x - 3}{(x + 1)^2}$

Apabila $x = 3$, $y = \dfrac{3^2 + 3}{3 + 1}$

$= 3$

dan $\dfrac{dy}{dx} = \dfrac{3^2 + 2(3) - 3}{(3 + 1)^2}$

$= \dfrac{3}{4}$

Persamaan tangen: $y - 3 = \dfrac{3}{4}(x - 3)$

$y = \dfrac{3}{4}x - \dfrac{9}{4} + 3$

$y = \dfrac{3}{4}x + \dfrac{3}{4}$

Persamaan normal: $y - 3 = -\dfrac{4}{3}(x - 3)$

$y = -\dfrac{4}{3}x + 4 + 3$

$y = -\dfrac{4}{3}x + 7$

3. (a) $y = x\sqrt{1 - 2x}$

$\dfrac{dy}{dx} = x\left[\dfrac{1}{2}(1 - 2x)^{-\frac{1}{2}}(-2)\right] + \sqrt{1 - 2x}\,(1)$

$= -\dfrac{x}{\sqrt{1 - 2x}} + \sqrt{1 - 2x}$

$= \dfrac{-x + 1 - 2x}{\sqrt{1 - 2x}}$

$= \dfrac{1 - 3x}{\sqrt{1 - 2x}}$

Apabila $x = -4$, $\dfrac{dy}{dx} = \dfrac{1 - 3(-4)}{\sqrt{1 - 2(-4)}}$

$= \dfrac{13}{\sqrt{9}}$

$= \dfrac{13}{3}$

(b) Apabila $x = -4$, $y = -4\sqrt{1 - 2(-4)}$

$= -4(3)$

$= -12$

Persamaan tangen: $y + 12 = \dfrac{13}{3}(x + 4)$

$3y + 36 = 13x + 52$

$3y - 13x = 16$

(c) Persamaan normal: $y + 12 = -\dfrac{3}{13}(x + 4)$

$13y + 156 = -3x - 12$

$13y + 3x + 168 = 0$

4. (a) $y = (x - 2)^2$

$= x^2 - 4x + 4$

$\frac{dy}{dx} = 2x - 4$

Apabila $x = 3$, $\frac{dy}{dx} = 2(3) - 4$

$= 2$

Persamaan tangen: $y - 1 = 2(x - 3)$

$y - 1 = 2x - 6$

$y = 2x - 5$

Pada titik $(k, 7)$: $7 = 2k - 5$

$2k = 12$

$k = 6$

(b) $y = 7x - \frac{6}{x}$

$= 7x - 6x^{-1}$

Apabila $x = 1$, $y = 7(1) - \frac{6}{1}$

$= 1$

dan $\frac{dy}{dx} = 7 + 6x^{-2}$

$= 7 + \frac{6}{x^2}$

$= 7 + \frac{6}{1^2}$

$= 13$

Persamaan normal: $y - 1 = -\frac{1}{13}(x - 1)$

$13y - 13 = -x + 1$

$13y + x = 14$

Pada paksi-x, $y = 0$

$13(0) + x = 14$

$x = 14$

$\therefore A(14, 0)$

Latihan Kendiri 2.9

1. (a) $y = x^2 - 3x + 4$

$\frac{dy}{dx} = 2x - 3$

Apabila $x = 1$, $\frac{dy}{dx} = 2(1) - 3$

$= 2 - 3$

$= -1$

Persamaan tangen di titik $A(1, 2)$ ialah

$y - 2 = -1(x - 1)$

$y - 2 = -x + 1$

$y + x = 3$

(b) Apabila $x = 3$, $\frac{dy}{dx} = 2(3) - 3$

$= 6 - 3$

$= 3$

Persamaan normal di titik $B(3, 4)$ dengan kecerunan $-\frac{1}{3}$ ialah

$y - 4 = -\frac{1}{3}(x - 3)$

$3y - 12 = -x + 3$

$3y + x = 15$

(c) $y + x = 3$ …❶

$3y + x = 15$ …❷

❷ – ❶: $2y = 12$

$y = 6$

Gantikan $y = 6$ ke dalam ❶:

$6 + x = 3$

$x = -3$

$\therefore C(-3, 6)$

2. (a) $y = 2x^2 - 5x - 2$

$\frac{dy}{dx} = 4x - 5$

$= 4(1) - 5$

$= -1$

Persamaan normal: $y + 5 = x - 1$

$y = x - 6$

(b) $x - 6 = 2x^2 - 5x - 2$

$2x^2 - 6x + 4 = 0$

$2(x^2 - 3x + 2) = 0$

$2(x - 2)(x - 1) = 0$

$x = 2$ atau $x = 1$

Apabila $x = 2$, $y = 2 - 6$

$= -4$

$\therefore B(2, -4)$

(c) Titik tengah AB, $M = \left(\frac{1 + 2}{2}, \frac{-5 + (-4)}{2}\right)$

$= \left(\frac{3}{2}, -\frac{9}{2}\right)$

3. (a) $m_{PQ} = \frac{1 - 0}{2 - 1\frac{1}{2}}$

$= 2$

$y = ax^3 - 4x + b$

$\frac{dy}{dx} = 3ax^2 - 4$

Pada titik $P(2, 1)$, $\frac{dy}{dx} = 2$

$3a(2)^2 - 4 = 2$

$12a = 6$

$a = \frac{1}{2}$

dan $1 = \frac{1}{2}(2)^3 - 4(2) + b$

$1 = 4 - 8 + b$

$1 = -4 + b$

$b = 5$

Maka, $a = \frac{1}{2}$ dan $b = 5$.

(b) $P(2, 1)$ dan $m = -\frac{1}{2}$

Persamaan normal: $y - 1 = -\frac{1}{2}(x - 2)$

$2y - 2 = -x + 2$

$2y + x = 4$

(c) Pada paksi-x, $2(0) + x = 4$

$x = 4$

Jadi, koordinat R ialah $(4, 0)$.

(d) Luas $\Delta PQR = \frac{1}{2}\begin{vmatrix} 2 & \frac{3}{2} & 4 & 2 \\ 1 & 0 & 0 & 1 \end{vmatrix}$

$= \frac{1}{2}\left|4 - \frac{3}{2}\right|$

$= \frac{1}{2}\left(\frac{5}{2}\right)$

$= 1\frac{1}{4}$ unit^2

4. (a) $3y - x = 14$

$3y = x + 14$

$y = \frac{1}{3}x + \frac{14}{3}$

Jadi, kecerunan tangen ialah -3.

$y = ax + \frac{b}{x}$

$= ax + bx^{-1}$

$\frac{dy}{dx} = a - \frac{b}{x^2}$

Pada titik $P(1, 5)$, $\frac{dy}{dx} = -3$

$a - \frac{b}{1^2} = -3$

$a - b = -3$ ……❶

dan $5 = a(1) + \frac{b}{1}$

$a + b = 5$ ……❷

❶ + ❷: $2a = 2$

$a = 1$

Gantikan $a = 1$ ke dalam ❶:

$1 - b = -3$

$b = 4$

Maka, $a = 1$ dan $b = 4$.

(b) $P(1, 5)$ dan $m = -3$

Persamaan tangen: $y - 5 = -3(x - 1)$

$y - 5 = -3x + 3$

$y + 3x = 8$

(c) $3y - x = 14$

$y = \frac{x + 14}{3}$ ……❶

$y = x + \frac{4}{x}$ ……❷

Gantikan ❶ ke dalam ❷:

$\frac{x + 14}{3} = x + \frac{4}{x}$

$x(x + 14) = 3x^2 + 12$

$x^2 + 14x = 3x^2 + 12$

$2x^2 - 14x + 12 = 0$

$x^2 - 7x + 6 = 0$

$(x - 1)(x - 6) = 0$

$x = 1$ atau $x = 6$

Gantikan $x = 6$ ke dalam ❶: $y = \frac{6 + 14}{3}$

$= 6\frac{2}{3}$

Maka, koordinat Q ialah $\left(6, 6\frac{2}{3}\right)$.

(d) $M_{PQ} = \left(\frac{1 + 6}{2}, \frac{5 + 6\frac{2}{3}}{2}\right)$

$= \left(3\frac{1}{2}, 5\frac{5}{6}\right)$

5. (a) $y = \sqrt{2x + 1}$

$$\frac{dy}{dx} = \frac{1}{2}(2x + 1)^{-\frac{1}{2}}(2)$$

$$= \frac{1}{\sqrt{2x + 1}}$$

Pada titik $A(4, 3)$, $\frac{dy}{dx} = \frac{1}{\sqrt{2(4) + 1}}$

$$= \frac{1}{3}$$

Persamaan tangen: $y - 3 = \frac{1}{3}(x - 4)$

$$3y - 9 = x - 4$$

$$3y = x + 5$$

Pada paksi-x, $3(0) = x + 5$

$$x = -5$$

Jadi, koordinat B ialah $(-5, 0)$.

Maka, $d_{AB} = \sqrt{(-5 - 4)^2 + (0 - 3)^2}$

$$= \sqrt{81 + 9}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10} \text{ unit}$$

(b) $y = x^2 + 6x + 4$

$$\frac{dy}{dx} = 2x + 6$$

Pada titik $(-2, -4)$, $\frac{dy}{dx} = 2(-2) + 6$

$$= 2$$

Pada titik $(1, \frac{1}{2})$, $\frac{dy}{dx} = -\frac{1}{2}$

$$3h(1)^2 + k = -\frac{1}{2}$$

$$6h + 2k = -1 \quad \ldots\ldots ❶$$

dan $\frac{1}{2} = h(1)^3 + k(1) + 2$

$$\frac{1}{2} = h + k + 2$$

$$h + k = -\frac{3}{2}$$

$$2h + 2k = -3 \quad \ldots\ldots ❷$$

❶ − ❷: $4h = 2$

$$h = \frac{1}{2}$$

Gantikan $h = \frac{1}{2}$ ke dalam ❶: $6\left(\frac{1}{2}\right) + 2k = -1$

$$3 + 2k = -1$$

$$2k = -4$$

$$k = -2$$

Maka, $h = \frac{1}{2}$ dan $k = -2$.

Aktiviti Penerokaan 7 (Halaman 57)

4.

Koordinat-x bagi titik P	–1	0	1	2	3
Kecerunan lengkung pada titik P, $\frac{dy}{dx}$	4	2	0	–2	–4
Tanda bagi $\frac{dy}{dx}$	+	+	0	–	–
Lakaran tangen	/	/	–	\	\
Lakaran graf					

5.

Koordinat-x bagi titik P	–3	–2	–1	0	1
Kecerunan lengkung pada titik P, $\frac{dy}{dx}$	–4	–2	0	2	4
Tanda bagi $\frac{dy}{dx}$	–	–	0	+	+
Lakaran tangen	\	\	–	/	/
Lakaran graf					

6.

Koordinat-x bagi titik P	–2	–1	0	1	2
Kecerunan lengkung pada titik P, $\frac{dy}{dx}$	12	3	0	3	12
Tanda bagi $\frac{dy}{dx}$	+	+	0	+	+
Lakaran tangen	/	/	–	/	/
Lakaran graf					

7. (a) (i) $(1, 4)$

(ii) Nilai $\frac{dy}{dx}$ berubah daripada 4 kepada 0 dan kemudian kepada -4 apabila x menokok melalui titik pegun $(1, 4)$

(iii) Tandanya berubah daripada positif kepada negatif apabila x menokok melalui titik pegun $(1, 4)$

(iv) Titik pusingan maksimum

(b) (i) $(-1, -4)$

(ii) Nilai $\frac{dy}{dx}$ berubah daripada -4 kepada 0 dan kemudian kepada 4 apabila x menokok melalui titik pegun $(-1, -4)$

(iii) Tandanya berubah daripada negatif kepada positif apabila x menokok melalui titik pegun $(-1, -4)$

(iv) Titik pusingan minimum

(c) (i) $(0, 4)$

(ii) Nilai $\frac{dy}{dx}$ berubah daripada 12 kepada 0 dan kemudian kepada 12 sekali lagi apabila x menokok melalui titik pegun $(0, 4)$

(iii) Tandanya tidak berubah apabila x menokok melalui titik pegun $(0, 4)$

(iv) Titik lengkok balas

Perbincangan (Halaman 62)

$y = x^3 + 6x^2 + 12x + 7$, $y = x^3 - 6x^2 + 12x - 5$ dan lain-lain lagi.

Latihan Kendiri 2.10

1. (a) $y = x^3 - 12x$

$\frac{dy}{dx} = 3x^2 - 12$

Untuk titik pusingan, $\frac{dy}{dx} = 0$

$3x^2 - 12 = 0$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

Apabila $x = -2$, $y = (-2)^3 - 12(-2)$

$= -8 + 24$

$= 16$

Apabila $x = 2$, $y = 2^3 - 12(2)$

$= 8 - 24$

$= -16$

Maka, titik pusingan ialah $(-2, 16)$ dan $(2, -16)$.

Apabila $x = -2$, $\frac{d^2y}{dx^2} = 6x$

$= 6(-2)$

$= -12 (< 0)$

Maka, $(-2, 16)$ ialah titik maksimum.

Apabila $x = 2$, $\frac{d^2y}{dx^2} = 6x$

$= 6(2)$

$= 12 (> 0)$

Maka, $(2, -16)$ ialah titik minimum.

(b) $y = x(x - 6)^2$

$= x(x^2 - 12x + 36)$

$= x^3 - 12x^2 + 36x$

$\frac{dy}{dx} = 3x^2 - 24x + 36$

Untuk titik pusingan, $\frac{dy}{dx} = 0$

$3x^2 - 24x + 36 = 0$

$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

$x = 2$ atau $x = 6$

Apabila $x = 2$, $y = 2^3 - 12(2)^2 + 36(2)$

$= 8 - 48 + 72$

$= 32$

Apabila $x = 6$, $y = 6^3 - 12(6)^2 + 36(6)$

$= 216 - 432 + 216$

$= 0$

Maka, titik pusingan ialah $(2, 32)$ dan $(6, 0)$.

Apabila $x = 2$, $\frac{d^2y}{dx^2} = 6x - 24$

$= 6(2) - 24$

$= -12 (< 0)$

Maka, $(2, 32)$ ialah titik maksimum.

Apabila $x = 6$, $\frac{d^2y}{dx^2} = 6x - 24$

$= 6(6) - 24$

$= 12 (> 0)$

Maka, $(6, 0)$ ialah titik minimum.

(c) $y = x\sqrt{18 - x^2}$

$$\frac{dy}{dx} = x\left[\frac{1}{2}(18 - x^2)^{-\frac{1}{2}}(-2x)\right] + \sqrt{18 - x^2}(1)$$

$$= -\frac{x^2}{\sqrt{18 - x^2}} + \sqrt{18 - x^2}$$

$$= \frac{-x^2 + 18 - x^2}{\sqrt{18 - x^2}}$$

$$= \frac{18 - 2x^2}{\sqrt{18 - x^2}}$$

$$= \frac{2(9 - x^2)}{\sqrt{18 - x^2}}$$

$$= \frac{2(3 + x)(3 - x)}{\sqrt{18 - x^2}}$$

Untuk titik pusingan, $\frac{dy}{dx} = 0$

$$\frac{2(3 + x)(3 - x)}{\sqrt{18 - x^2}} = 0$$

$$2(3 + x)(3 - x) = 0$$

$$x = -3 \text{ atau } x = 3$$

Apabila $x = -3$, $y = -3\sqrt{18 - (-3)^2}$

$$= -3\sqrt{9}$$

$$= -9$$

Apabila $x = 3$, $y = 3\sqrt{18 - 3^2}$

$$= 3\sqrt{9}$$

$$= 9$$

Maka, titik pusingan ialah $(-3, -9)$ dan $(3, 9)$.

$$\frac{d^2y}{dx^2} = \frac{\sqrt{18 - x^2}(-4x) - (18 - 2x^2)\left[\frac{1}{2}(18 - x^2)^{-\frac{1}{2}}(-2x)\right]}{\left(\sqrt{18 - x^2}\right)^2}$$

$$= \frac{-4x\sqrt{18 - x^2} + \frac{18x - 2x^3}{\sqrt{18 - x^2}}}{18 - x^2}$$

$$= \frac{-4x(18 - x^2) + 18x - 2x^3}{(18 - x^2)^{\frac{3}{2}}}$$

$$= \frac{-72x + 4x^3 + 18x - 2x^3}{\sqrt{(18 - x^2)^3}}$$

$$= \frac{2x^3 - 54x}{\sqrt{(18 - x^2)^3}}$$

$$= \frac{2x(x^2 - 27)}{\sqrt{(18 - x^2)^3}}$$

Apabila $x = -3$, $\frac{d^2y}{dx^2} = \frac{2(-3)[(-3)^2 - 27]}{\sqrt{[18 - (-3)^2]^3}}$

$$= \frac{108}{27}$$

$$= 4 \ (> 0)$$

Maka, $(-3, -9)$ ialah titik minimum.

Apabila $x = 3$, $\frac{d^2y}{dx^2} = \frac{2(3)[(3)^2 - 27]}{\sqrt{[18 - (3)^2]^3}}$

$= -\frac{108}{27}$

$= -4 \ (< 0)$

Maka, (3, 9) ialah titik maksimum.

(d) $y = (x - 6)(4 - 2x)$

$= 4x - 2x^2 - 24 + 12x$

$= -2x^2 + 16x - 24$

$\frac{dy}{dx} = -4x + 16$

Untuk titik pusingan, $\frac{dy}{dx} = 0$

$-4x + 16 = 0$

$x = 4$

Apabila $x = 4$, $y = (4 - 6)[4 - 2(4)]$

$= (-2)(-4)$

$= 8$

Maka, titik pusingan ialah (4, 8).

Apabila $x = 4$, $\frac{d^2y}{dx^2} = -4 \ (< 0)$

Maka, (4, 8) ialah titik maksimum.

(e) $y = x + \frac{4}{x}$

$= x + 4x^{-1}$

$\frac{dy}{dx} = 1 - 4x^{-2}$

$= 1 - \frac{4}{x^2}$

Untuk titik pusingan, $\frac{dy}{dx} = 0$

$1 - \frac{4}{x^2} = 0$

$1 = \frac{4}{x^2}$

$x^2 = 4$

$x = \pm 2$

Apabila $x = -2$, $y = -2 + \frac{4}{(-2)}$

$= -4$

Apabila $x = 2$, $y = 2 + \frac{4}{2}$

$= 4$

Maka, titik pusingan ialah $(-2, -4)$ dan (2, 4).

Apabila $x = -2$, $\frac{d^2y}{dx^2} = 8x^{-3}$

$= \frac{8}{x^3}$

$= \frac{8}{(-2)^3}$

$= -1 \ (< 0)$

Maka, $(-2, -4)$ ialah titik maksimum.

Apabila $x = 2$, $\frac{d^2y}{dx^2} = \frac{8}{2^3}$

$= 1 \ (> 0)$

Maka, (2, 4) ialah titik minimum.

(f) $y = x^2 + \frac{1}{x^2}$

$= x^2 + x^{-2}$

$\frac{dy}{dx} = 2x - 2x^{-3}$

$= 2x - \frac{2}{x^3}$

Untuk titik pusingan, $\frac{dy}{dx} = 0$

$2x - \frac{2}{x^3} = 0$

$2x = \frac{2}{x^3}$

$x^4 = 1$

$x = 1$

Apabila $x = 1$, $y = 1^2 + \frac{1}{1^2}$

$= 2$

Maka, titik pusingan ialah (1, 2).

Apabila $x = 1$, $\frac{d^2y}{dx^2} = 2 + 6x^{-4}$

$= 2 + \frac{6}{x^4}$

$= 2 + \frac{6}{1^4}$

$= 8 \ (> 0)$

Maka, (1, 2) ialah titik minimum.

(g) $y = x + \frac{1}{x - 1}$

$= x + (x - 1)^{-1}$

$\frac{dy}{dx} = 1 - 1(x - 1)^{-2}(1)$

$= 1 - \frac{1}{(x - 1)^2}$

Untuk titik pusingan, $\frac{dy}{dx} = 0$

$1 - \frac{1}{(x - 1)^2} = 0$

$1 = \frac{1}{(x - 1)^2}$

$(x - 1)^2 = 1$

$x - 1 = \pm 1$

$x = \pm 1 + 1$

$x = -1 + 1$ atau $x = 1 + 1$

$= 0$ $= 2$

Apabila $x = 0$, $y = 0 + \frac{1}{0 - 1}$

$= -1$

Apabila $x = 2$, $y = 2 + \frac{1}{2 - 1}$

$= 3$

Maka, titik pusingan ialah (0, –1) dan (2, 3).

Apabila $x = 0$, $\frac{d^2y}{dx^2} = \frac{2}{(0 - 1)^3}$

$= -2 \ (< 0)$

Maka, (0, –1) ialah titik maksimum.

Apabila $x = 2$, $\frac{d^2y}{dx^2} = \frac{2}{(2 - 1)^3}$

$= 2 \ (> 0)$

Maka, (2, 3) ialah titik minimum.

(h) $y = \dfrac{(x-3)^2}{x}$

$= \dfrac{x^2 - 6x + 9}{x}$

$= x - 6 + 9x^{-1}$

$\dfrac{dy}{dx} = 1 - 9x^{-2}$

$= 1 - \dfrac{9}{x^2}$

Untuk titik pusingan, $\dfrac{dy}{dx} = 0$

$1 - \dfrac{9}{x^2} = 0$

$1 = \dfrac{9}{x^2}$

$x^2 = 9$

$x = \pm 3$

Apabila $x = -3$, $y = \dfrac{(-3-3)^2}{-3}$

$= -12$

Apabila $x = 3$, $y = \dfrac{(3-3)^2}{3}$

$= 0$

Maka, titik pusingan ialah $(-3, -12)$ dan $(3, 0)$.

Apabila $x = -3$, $\dfrac{d^2y}{dx^2} = \dfrac{18}{x^3}$

$= \dfrac{18}{(-3)^3}$

$= -\dfrac{2}{3}\ (< 0)$

Maka, $(-3, -12)$ ialah titik maksimum.

Apabila $x = 3$, $\dfrac{d^2y}{dx^2} = \dfrac{18}{3^3}$

$= \dfrac{2}{3}\ (> 0)$

Maka, $(3, 0)$ ialah titik minimum.

2. (a) $y = x(x-2)^3$

$\dfrac{dy}{dx} = x[3(x-2)^2(1)] + (x-2)^3(1)$

$= 3x(x-2)^2 + (x-2)^3$

$= (x-2)^2[3x + (x-2)]$

$= (4x-2)(x-2)^2$

$= 2(2x-1)(x-2)^2$

(b) Untuk titik pegun, $\dfrac{dy}{dx} = 0$

$2(2x-1)(x-2)^2 = 0$

$2x - 1 = 0$ atau $(x-2)^2 = 0$

$x = \dfrac{1}{2}$ $x = 2$

Apabila $x = \dfrac{1}{2}$, $y = \dfrac{1}{2}\left(\dfrac{1}{2} - 2\right)^3$

$= \dfrac{1}{2}\left(-\dfrac{27}{8}\right)$

$= -\dfrac{27}{16}$

Apabila $x = 2$, $y = 2(2-2)^3$

$= 0$

$\therefore P\left(\dfrac{1}{2}, -\dfrac{27}{16}\right)$ dan $Q(2, 0)$

(c) Q ialah titik lengkok balas.

Kuiz Pantas (Halaman 64)

Boleh.

Gantikan $j = \sqrt{\frac{512}{\pi t}}$ ke dalam $L = 8j^2 + 2\pi jt$. Kemudian, tentukan nilai t apabila $\frac{dL}{dt} = 0$.

Seterusnya, gantikan nilai $t = 10.186$ yang diperoleh ke dalam $j = \sqrt{\frac{512}{\pi t}}$ untuk mencari nilai j.

Latihan Kendiri 2.11

1. (a) $s_{PQ} = 80 - 2j$

$$j\theta = 80 - 2j$$

$$\theta = \frac{80 - 2j}{j}$$

Luas sektor $POQ, A = \frac{1}{2}j^2\theta$

$$= \frac{1}{2}j^2\left(\frac{80 - 2j}{j}\right)$$

$$= \frac{1}{2}j(80 - 2j) \text{ (Tertunjuk)}$$

(b) $A = \frac{1}{2}j(80 - 2j)$

$$= 40j - j^2$$

Untuk luas maksimum, $\frac{dA}{dj} = 0$

$$40 - 2j = 0$$

$$2j = 40$$

$$j = 20$$

Apabila $j = 20$, $A = \frac{1}{2}(20)[80 - 2(20)]$

$$= 400$$

$\frac{d^2A}{dj^2} = -2 \ (> 0)$

Jadi, A adalah maksimum.

Maka, luas maksimum bagi sektor POQ ialah 400 cm^2.

2. (a) $2y + 2(13x) + 24x = 240$

$$2y + 50x = 240$$

$$y + 25x = 120$$

$$y = 120 - 25x$$

(b) $L = 24xy + \frac{1}{2}(24x)(5x)$

$$= 24x(120 - 25x) + 60x^2$$

$$= 2\,880x - 600x^2 + 60x^2$$

$$= 2\,880x - 540x^2 \text{ (Tertunjuk)}$$

(c) (i) Untuk nilai maksimum, $\frac{dL}{dx} = 0$

$$2\,880 - 1\,080x = 0$$

$$1\,080x = 2\,880$$

$$x = 2\frac{2}{3}$$

Apabila $x = 2\frac{2}{3}$, $y = 120 - 25\left(2\frac{2}{3}\right)$

$$= 53\frac{1}{3}$$

dan $\frac{d^2L}{dx^2} = -1\,080 \ (< 0)$

Maka, L mempunyai nilai maksimum apabila $x = 2\frac{2}{3}$ cm dan $y = 53\frac{1}{3}$ cm.

(ii) $L_{\text{Maks}} = 2\,880\left(2\frac{2}{3}\right) - 540\left(2\frac{2}{3}\right)^2$

$= 7\,680 - 3\,840$

$= 3\,840$

Maka, luas maksimum rantau ialah 3 840 cm^2.

3. (a) Katakan jejari dan tinggi silinder masing-masing ialah j cm dan t cm.

$\pi j^2 t = 32\pi$

$t = \frac{32\pi}{\pi j^2}$

$= \frac{32}{j^2}$ …❶

Fungsi kos, $C = 2\pi j^2(2) + 2\pi jt(1)$

$= 4\pi j^2 + 2\pi j\left(\frac{32}{j^2}\right)$

$= 4\pi j^2 + \frac{64\pi}{j}$ (Tertunjuk)

(b) Untuk C minimum, $\frac{dC}{dj} = 0$

$8\pi j - 64\pi j^{-2} = 0$

$8\pi j = \frac{64\pi}{j^2}$

$j^3 = 8$

$j = \sqrt[3]{8}$

$= 2$ cm

dan $t = \frac{32}{j^2} = \frac{32}{2^2} = 8$ cm

$\frac{d^2C}{dj^2} = 8\pi + \frac{128\pi}{j^3}$

Apabila $j = 2$, $\frac{d^2C}{dj^2} = 8\pi + \frac{128\pi}{2^3}$

$= 8\pi + 16\pi$

$= 24\pi\ (> 0)$

Jadi, C adalah minimum apabila $j = 2$. Maka, kilang itu mesti mengeluarkan tin berbentuk silinder dengan jejari 2 cm dan tinggi 8 cm untuk memperoleh kos minimum.

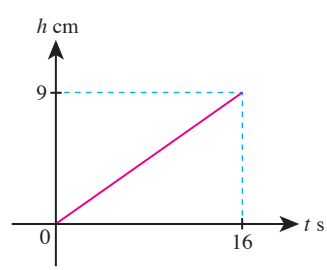
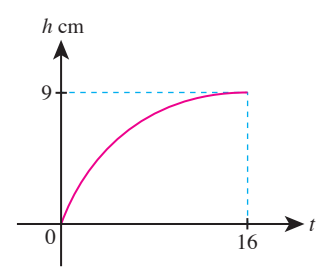
Aktiviti Penerokaan 8 (Halaman 65)

2. 3π cm$^3 \rightarrow 1$ saat

48π cm$^3 \rightarrow \frac{48\pi}{3\pi} = 16$ saat

Jadi, masa yang diambil untuk memenuhkan air di dalam setiap bekas ialah 16 saat.

3.

Bekas	Silinder	Kon
Graf kedalaman-masa		

4. (b) Luas permukaan air di dalam bekas berbentuk silinder sentiasa seragam apabila air diisikan ke dalamnya. Jadi, aras air meningkat secara seragam terhadap masa dan kedalaman air dikatakan meningkat pada kadar malar. Luas permukaan air di dalam bekas berbentuk kon pula bertambah apabila aras air meningkat. Jadi, kadar perubahan kedalaman aras air berubah-ubah apabila air diisikan, iaitu kadar perubahan kedalaman air menyusut terhadap masa.

Latihan Kendiri 2.12

1. (a) $y = 3x^2 - 4$

$\frac{dy}{dx} = 6x$

Apabila $x = \frac{1}{2}$, $\frac{dy}{dx} = 6\left(\frac{1}{2}\right)$

$= 3$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dt} = 3 \times 2 = 6$ unit s^{-1}

(b) $y = 2x^2 + \frac{1}{x} = 2x^2 + x^{-1}$

$\frac{dy}{dx} = 4x - \frac{1}{x^2}$

Apabila $x = 1$, $\frac{dy}{dx} = 4(1) - \frac{1}{1^2}$

$= 4 - 1$

$= 3$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dt} = 3 \times 2 = 6$ unit s^{-1}

(c) $y = \frac{2}{(3x-5)^3} = 2(3x-5)^{-3}$

$\frac{dy}{dx} = -6(3x-5)^{-4}(3) = -\frac{18}{(3x-5)^4}$

Apabila $x = 2$, $\frac{dy}{dx} = -\frac{18}{[3(2)-5]^4}$

$= -18$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dt} = -18 \times 2 = -36$ unit s^{-1}

(d) $y = (4x-3)^5$

$\frac{dy}{dx} = 5(4x-3)^4(4) = 20(4x-3)^4$

Apabila $x = \frac{1}{2}$, $\frac{dy}{dx} = 20\left[4\left(\frac{1}{2}\right) - 3\right]^4$

$= 20$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dt} = 20 \times 2 = 40$ unit s^{-1}

(e) $y = \frac{x}{x+1}$

$\frac{dy}{dx} = \frac{(x+1)(1) - x(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$

Apabila $y = 2$, $2 = \frac{x}{x+1}$

$2x + 2 = x$

$x = -2$

dan $\frac{dy}{dx} = \frac{1}{(-2+1)^2}$

$= 1$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dt} = 1 \times 2 = 2$ unit s^{-1}

(f) $y = x^3 + 2$

$\frac{dy}{dx} = 3x^2$

Apabila $y = 10$, $10 = x^3 + 2$

$x^3 = 8$

$x = 2$

dan $\frac{dy}{dx} = 3(2)^2$

$= 12$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dt} = 12 \times 2 = 24$ unit s^{-1}

2. (a) $y = x^3 - 2x^2$

$\frac{dy}{dx} = 3x^2 - 4x$

Apabila $x = 1$, $\frac{dy}{dx} = 3(1)^2 - 4(1)$

$= 3 - 4$

$= -1$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$6 = -1 \times \frac{dx}{dt}$

$\frac{dx}{dt} = -6$ unit s^{-1}

(b) $y = x^2 + \frac{4}{x}$

$= x^2 + 4x^{-1}$

$\frac{dy}{dx} = 2x - \frac{4}{x^2}$

Apabila $x = 2$, $\frac{dy}{dx} = 2(2) - \frac{4}{2^2}$

$= 4 - 1$

$= 3$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$6 = 3 \times \frac{dx}{dt}$

$\frac{dx}{dt} = 2$ unit s^{-1}

(c) $y = \frac{2x^2}{x - 1}$

$\frac{dy}{dx} = \frac{(x - 1)(4x) - 2x^2(1)}{(x - 1)^2}$

$= \frac{4x^2 - 4x - 2x^2}{(x - 1)^2}$

$= \frac{2x^2 - 4x}{(x - 1)^2}$

$= \frac{2x(x - 2)}{(x - 1)^2}$

Apabila $x = 3$, $\frac{dy}{dx} = \frac{2(3)(3 - 2)}{(3 - 1)^2}$

$= \frac{6}{4}$

$= \frac{3}{2}$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$6 = \frac{3}{2} \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = 6\left(\frac{2}{3}\right) = 4 \text{ unit s}^{-1}$$

(d) $y = (x - 6)\sqrt{x - 1}$

$$\frac{dy}{dx} = (x - 6)\left[\frac{1}{2}(x - 1)^{-\frac{1}{2}}(1)\right] + \sqrt{x - 1}$$

$$= \frac{x - 6}{2\sqrt{x - 1}} + \sqrt{x - 1}$$

$$= \frac{x - 6 + 2(x - 1)}{2\sqrt{x - 1}}$$

$$= \frac{x - 6 + 2x - 2}{2\sqrt{x - 1}}$$

$$= \frac{3x - 8}{2\sqrt{x - 1}}$$

Apabila $x = 2$, $\frac{dy}{dx} = \frac{3(2) - 8}{2\sqrt{2 - 1}}$

$$= -\frac{2}{2}$$

$$= -1$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$6 = -1 \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = -6 \text{ unit s}^{-1}$$

(e) $y = \frac{2x - 1}{x + 1}$

$$\frac{dy}{dx} = \frac{(x + 1)(2) - (2x - 1)(1)}{(x + 1)^2}$$

$$= \frac{2x + 2 - 2x + 1}{(x + 1)^2}$$

$$= \frac{3}{(x + 1)^2}$$

Apabila $y = 3$, $3 = \frac{2x - 1}{x + 1}$

$$3x + 3 = 2x - 1$$

$$x = -4$$

dan $\frac{dy}{dx} = \frac{3}{(-4 + 1)^2}$

$$= \frac{3}{9}$$

$$= \frac{1}{3}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$6 = \frac{1}{3} \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = 18 \text{ unit s}^{-1}$$

(f) $y = \sqrt{2x + 7}$

$$\frac{dy}{dx} = \frac{1}{2}(2x + 7)^{-\frac{1}{2}}(2)$$

$$= \frac{1}{\sqrt{2x + 7}}$$

Apabila $y = 3$, $3 = \sqrt{2x + 7}$

$9 = 2x + 7$

$2x = 2$

$x = 1$

dan $\frac{dy}{dx} = \frac{1}{\sqrt{2(1) + 7}}$

$= \frac{1}{\sqrt{9}}$

$= \frac{1}{3}$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$6 = \frac{1}{3} \times \frac{dx}{dt}$

$\frac{dx}{dt} = 18$ unit s^{-1}

3. (a) $y = (x - 8)\sqrt{x + 4}$

$\frac{dy}{dx} = (x - 8)\left[\frac{1}{2}(x + 4)^{-\frac{1}{2}}(1)\right] + \sqrt{x + 4}(1)$

$= \frac{x - 8}{2\sqrt{x + 4}} + \sqrt{x + 4}$

$= \frac{x - 8 + 2(x + 4)}{2\sqrt{x + 4}}$

$= \frac{x - 8 + 2x + 8}{2\sqrt{x + 4}}$

$= \frac{3x}{2\sqrt{x + 4}}$

(b) Apabila $x = 5$, $\frac{dy}{dx} = \frac{3(5)}{2\sqrt{5 + 4}}$

$= \frac{15}{2(3)}$

$= \frac{15}{6}$

$= \frac{5}{2}$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{5}{2} \times 6$

$= 15$ unit s^{-1}

Perbincangan (Halaman 68)

(a) $V = \frac{1}{3}\pi r^2 h$ …❶

$\frac{r}{8} = \frac{h}{16}$

$r = \frac{h}{16} \times 8$

$= \frac{h}{2}$ …❷

Gantikan ❷ ke dalam ❶:

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$= \frac{1}{12}\pi h^3$

Kadar perubahan V diberi oleh:

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \text{ (petua rantai)}$$
$$= \frac{d}{dh}\left(\frac{1}{12}\pi h^3\right) \times \frac{dh}{dt}$$
$$= \frac{1}{4}\pi h^2 \times \frac{dh}{dt}$$

Apabila $h = 8$ dan $\frac{dV}{dt} = 64\pi$, kita peroleh

$$64\pi = \frac{1}{4}\pi(8)^2 \times \frac{dh}{dt}$$
$$64\pi = 16\pi \times \frac{dh}{dt}$$
$$\frac{dh}{dt} = 4$$

Jadi, kadar perubahan kedalaman air dalam bekas itu ialah 4 cms^{-1}.

(b) $L = \pi r^2$ …❸

Gantikan ❷ ke dalam ❸:

$$L = \pi\left(\frac{h}{2}\right)^2$$
$$= \frac{1}{4}\pi h^2$$

Kadar perubahan L diberi oleh:

$$\frac{dL}{dt} = \frac{dL}{dh} \times \frac{dh}{dt} \text{ (petua rantai)}$$
$$= \frac{d}{dh}\left(\frac{1}{4}\pi h^2\right) \times \frac{dh}{dt}$$
$$= \frac{1}{2}\pi h \times 4$$

Apabila $h = 8$ dan $\frac{dh}{dt} = 4$, kita peroleh

$$\frac{dL}{dt} = \frac{1}{2}\pi(8) \times 4$$
$$= 16\pi$$

Jadi, kadar perubahan luas permukaan mengufuk ialah 16π cm^2s^{-1}.

Latihan Kendiri 2.13

1. $y = \frac{1}{8}x^2$

$\frac{dy}{dx} = \frac{1}{4}x$

Apabila $x = 4$, $\frac{dy}{dx} = \frac{1}{4}(4) = 1$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$
$$\frac{dy}{dt} = 1 \times 3$$
$$= 3 \text{ unit } s^{-1}$$

2. $L = x^2$

$\frac{dL}{dx} = 2x$

Apabila $L = 4$,

$x^2 = 4$

$x^2 = \sqrt{4}$

$= 2 \ (> 0)$

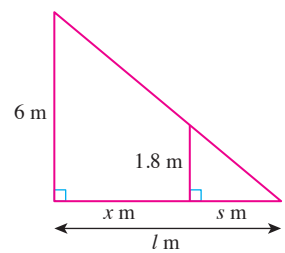
dan $\frac{dL}{dx} = 2(2) = 4$

$$\frac{dL}{dt} = \frac{dL}{dx} \times \frac{dx}{dt}$$
$$8 = 4 \times \frac{dx}{dt}$$
$$\frac{dx}{dt} = \frac{8}{4}$$
$$= 2 \text{ cms}^{-1}$$

3. $I = x^3$
$$\frac{dI}{dx} = 3x^2$$
Apabila $x = 10$, $\frac{dI}{dx} = 3(10)^2$
$$= 300$$
$$\frac{dI}{dt} = \frac{dI}{dx} \times \frac{dx}{dt}$$
$$-10.5 = 300 \times \frac{dx}{dt}$$
$$\frac{dx}{dt} = -\frac{10.5}{300} = -\frac{7}{200} \text{ cmmin}^{-1}$$

4. (a) $V = \pi r^2 h$
$$= \pi(3)^2 h$$
$$= 9\pi h$$
(b) $\frac{dV}{dt} = 9\pi \times -0.6$
$$= -5.4\pi \text{ cm}^3\text{min}^{-1}$$

5. Katakan panjang bayang-bayang dan panjang hujung bayang-bayang dari kaki tiang lampu masing-masing ialah s m dan l m.



$$\frac{6}{x+s} = \frac{1.8}{s}$$
$$6s = 1.8(x + s)$$
$$6s = 1.8x + 1.8s$$
$$4.2s = 1.8x$$
$$s = \frac{3}{7}x$$

(a) $s = \frac{3}{7}x$
$$\frac{ds}{dx} = \frac{3}{7}$$
Jadi, $\frac{ds}{dt} = \frac{ds}{dx} \times \frac{dx}{dt}$
$$= \frac{3}{7} \times 3.5$$
$$= 1.5$$
Maka, kadar perubahan panjang bayang-bayang ialah 1.5 ms^{-1}.

(b) $l = x + s$
$$= x + \frac{3}{7}x$$
$$= \frac{10}{7}x$$
$$\frac{dl}{dx} = \frac{10}{7}$$
Jadi, $\frac{dl}{dt} = \frac{dl}{dx} \times \frac{dx}{dt}$
$$= \frac{10}{7} \times 3.5$$
$$= 5$$
Maka, kadar perubahan hujung bayang-bayang yang bergerak ialah 5 ms^{-1}.

Perbincangan (Halaman 71)
Boleh digunakan tetapi jawapannya tidak tepat dan bukan penghampiran yang terbaik.

Latihan Kendiri 2.14

1. (a) $y = 4x^3 - 3x^2$

$\frac{dy}{dx} = 12x^2 - 6x$

Apabila $x = 1$, $\delta x = 1.05 - 1$
$= 0.05$

dan $\frac{dy}{dx} = 12(1)^2 - 6(1)$
$= 12 - 6$
$= 6$

Jadi, $\delta y \approx \frac{dy}{dx} \times \delta x$
$= 6 \times 0.05$
$= 0.3$ unit

(b) $y = 4\sqrt{x} + 3x^2$

$\frac{dy}{dx} = 4\left(\frac{1}{2}x^{-\frac{1}{2}}\right) + 6x$
$= \frac{2}{\sqrt{x}} + 6x$

Apabila $x = 4$, $\delta x = 3.98 - 4$
$= -0.02$

dan $\frac{dy}{dx} = \frac{2}{\sqrt{4}} + 6(4)$
$= 1 + 24$
$= 25$

Jadi, $\delta y \approx \frac{dy}{dx} \times \delta x$
$= 25 \times -0.02$
$= -0.5$ unit

2. (a) $y = 2x^{\frac{3}{2}}$

$\frac{dy}{dx} = 3x^{\frac{1}{2}}$
$= 3\sqrt{x}$

Apabila $y = 16$, $16 = 2x^{\frac{3}{2}}$
$x^{\frac{3}{2}} = 8$
$x = \left(2^3\right)^{\frac{2}{3}}$
$= 4$

$\delta y = 15.7 - 16$
$= -0.3$

dan $\frac{dy}{dx} = 3\sqrt{4}$
$= 3(2)$
$= 6$

Jadi, $\delta y \approx \frac{dy}{dx} \times \delta x$
$-0.3 = 6 \times \delta x$
$\delta x = -\frac{0.3}{6}$
$= -0.05$ unit

(b) $y = \frac{x + 2}{2}$

$= \frac{1}{2}x + 1$

$\frac{dy}{dx} = \frac{1}{2}$

Apabila $y = 2$, $2 = \frac{x+2}{2}$

$4 = x + 2$

$x = 2$

$\delta y = 2 + p - 2$

$= p$

Jadi, $\delta y \approx \frac{dy}{dx} \times \delta x$

$p = \frac{1}{2} \times \delta x$

$\delta x = 2p$ unit

3. $y = \frac{16}{x^2} = 16x^{-2}$

$\frac{dy}{dx} = -32x^{-3} = -\frac{32}{x^3}$

Apabila $x = 2$, $y = \frac{16}{2^2}$

$y = \frac{16}{4}$

$= 4$

$\delta x = 2.02 - 2$

$= 0.02$

dan $\frac{dy}{dx} = -\frac{32}{2^3}$

$= -4$

$f(x + \delta x) \approx y + \frac{dy}{dx}\delta x$

$\frac{16}{(2 + 0.02)^2} = 4 + (-4)(0.02)$

$= 4 - 0.08$

$= 3.92$

4. $y = x^{\frac{5}{4}}$

$\frac{dy}{dx} = \frac{5}{4}x^{\frac{1}{4}}$

$\delta y = \frac{4}{100}y$

$= 0.04y$

$= 0.04x^{\frac{5}{4}}$

$\delta y \approx \frac{dy}{dx} \times \delta x$

$0.04x^{\frac{5}{4}} = \frac{5}{4}x^{\frac{1}{4}} \times \delta x$

$\delta x = \frac{0.04x^{\frac{5}{4}}}{\frac{5}{4}x^{\frac{1}{4}}}$

$= 0.032x$

Peratus perubahan hampir dalam $x = \frac{\delta x}{x} \times 100\%$

$= \frac{0.032x}{x} \times 100\%$

$= 3.2\%$

Latihan Kendiri 2.15

1. $T = 2\pi\sqrt{\frac{l}{10}}$

$\frac{dT}{dl} = 2\pi\left[\frac{1}{2}\left(\frac{l}{10}\right)^{-\frac{1}{2}}\left(\frac{1}{10}\right)\right]$

$= \frac{\pi}{10\sqrt{\frac{l}{10}}}$

Apabila $l = 9$, $\frac{dT}{dl} = \frac{\pi}{10\sqrt{\frac{9}{10}}}$

$= \frac{\pi\sqrt{10}}{30}$

dan $\delta l = 9.05 - 9 = 0.05$

Maka, $\delta T \approx \frac{dT}{dl} \times \delta l$

$= \frac{\pi\sqrt{10}}{30} \times 0.05$

$= \frac{\pi\sqrt{10}}{600}$ saat

2. $L = \pi j^2$

$\frac{dL}{dj} = 2\pi j$

$L = 4\pi$

$\pi j^2 = 4\pi$

$j^2 = 4$

$j = 2\ (> 0)$

Apabila $j = 2$, $\frac{dL}{dj} = 2\pi(2)$

$= 4\pi$

$\delta L = 4.01\pi - 4\pi$

$= 0.01\pi$

dan $\delta L \approx \frac{dL}{dj} \times \delta j$

$0.01\pi = 4\pi \times \delta j$

$\delta j = \frac{0.01\pi}{4\pi}$

$= 0.0025$ cm

3. $V = x^3$

$\frac{dV}{dx} = 3x^2$

Apabila $x = 2$, $\delta x = 1.99 - 2$

$= -0.01$

dan $\frac{dV}{dx} = 3(2)^2$

$= 12$

Maka, $\delta V = \frac{dV}{dx} \times \delta x$

$= 12 \times -0.01$

$= -0.12$ cm^3

4. $I = \frac{4}{3}\pi j^3$

$\frac{dI}{dj} = 4\pi j^2$

Apabila $j = 5$, $\delta j = 4.98 - 5$

$= -0.02$

dan $\frac{dI}{dj} = 4\pi(5)^2$
$= 100\pi$

Maka, $\delta I \approx \frac{dI}{dj} \times \delta j$
$= 100\pi \times -0.02$
$= -2\pi \text{ cm}^3$

Latihan Formatif 2.4

1. (a) $y = \sqrt{x+1}$

$\frac{dy}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}}(1)$

$= \frac{1}{2\sqrt{x+1}}$

Apabila $x = 0$, $\frac{dy}{dx} = \frac{1}{2\sqrt{0+1}}$

$= \frac{1}{2}$

Persamaan tangen: $y - 1 = \frac{1}{2}(x - 0)$

$2y - 2 = x$

$2y - x = 2$

Pada paksi-x, $y = 0$

$2(0) - x = 2$

$x = -2$

$\therefore Q(-2, 0)$

(b) Persamaan normal: $y - 1 = -2(x - 0)$

$y - 1 = -2x$

$y = -2x + 1$

Pada paksi-x, $y = 0$

$0 = -2x + 1$

$2x = 1$

$x = \frac{1}{2}$

$\therefore R\left(\frac{1}{2}, 0\right)$

(c) Luas $\Delta PQR = \frac{1}{2}\begin{vmatrix} 0 & -2 & \frac{1}{2} & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix}$

$= \frac{1}{2}\left|\frac{1}{2} - (-2)\right|$

$= \frac{1}{2}\left|\frac{1}{2} + 2\right|$

$= \frac{5}{4}$

$= 1\frac{1}{4} \text{ unit}^2$

2. (a) $2y = 4 - x$

$y = 2 - \frac{1}{2}x$

$y = x^2 - 4x + 1$

$\frac{dy}{dx} = 2x - 4$

Jadi, $2x - 4 = 2$

$2x = 2 + 4$

$2x = 6$

$x = 3$

Apabila $x = 3$, $y = 3^2 - 4(3) + 1$

$= 9 - 12 + 1$

$= -2$

$\therefore a = 3, b = -2$

(b) Persamaan tangen: $y + 2 = 2(x - 3)$

$y + 2 = 2x - 6$

$y = 2x - 8$

Pada paksi-x, $y = 0$

$0 = 2x - 8$

$2x = 8$

$x = 4$

$\therefore B(4, 0)$

(c) Persamaan normal: $y + 2 = -\frac{1}{2}(x - 3)$

$2y + 4 = -x + 3$

$2y + x + 1 = 0$

Pada paksi-x, $y = 0$

$2(0) + x + 1 = 0$

$x = -1$

$\therefore C(-1, 0)$

(d) Luas $\Delta BPC = \frac{1}{2}\begin{vmatrix} 4 & -1 & 3 & 4 \\ 0 & 0 & -2 & 0 \end{vmatrix}$

$= \frac{1}{2}|2 - (-8)|$

$= \frac{1}{2}|10|$

$= 5 \text{ unit}^2$

3. (a) Luas = 75

$x^2 + 4hx = 75$

$4hx = 75 - x^2$

$h = \frac{75 - x^2}{4x}$

Isipadu, $V = x^2h$

$= x^2\left(\frac{75 - x^2}{4x}\right)$

$= \frac{1}{4}x(75 - x^2)$

$= \frac{1}{4}(75x - x^3)$ (Tertunjuk)

(b) Untuk V maksimum, $\frac{dV}{dx} = 0$

$\frac{75}{4} - \frac{3}{4}x^2 = 0$

$\frac{75}{4} = \frac{3}{4}x^2$

$3x^2 = 75$

$x^2 = 25$

$x = 5 \ (> 0)$

$V = \frac{1}{4}[75(5) - 5^3]$

$= \frac{1}{4}(375 - 125)$

$= \frac{1}{4}(250)$

$= 62.5 \text{ cm}^3$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x$

Apabila $x = 5$, $\frac{d^2V}{dx^2} = -\frac{3}{2}(5)$

$= -7.5\ (< 0) \Rightarrow V$ adalah maksimum

Maka, V mempunyai nilai maksimum apabila $x = 5$ cm dan isi padu maksimum kotak ialah 62.5 cm^3.

4. (a) $x^2 + y^2 = 10^2$

$y^2 = 100 - x^2$

$y = \sqrt{100 - x^2}$

$\frac{dy}{dx} = \frac{1}{2}(100 - x^2)^{-\frac{1}{2}}(-2x)$

$= -\frac{x}{\sqrt{100 - x^2}}$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= -\frac{x}{\sqrt{100 - x^2}} \times 3$

$= -\frac{3x}{\sqrt{100 - x^2}}$

Apabila $x = 8$, $\frac{dy}{dt} = -\frac{3(8)}{\sqrt{100 - 8^2}}$

$= -4$

Jadi, kadar perubahan hujung kayu A ialah -4 ms^{-1}.

(b) $x^2 + y^2 = 10^2$

Apabila $y = 6$, $x^2 + 6^2 = 10^2$

$x^2 = 100 - 36$

$x = \sqrt{64}$

$= 8$

Apabila $x = 8$, $\frac{dy}{dx} = -\frac{8}{\sqrt{100 - 8^2}}$

$= -\frac{4}{3}$

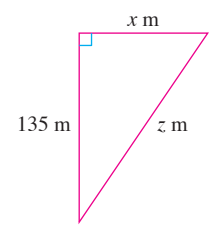
dan $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$-2 = -\frac{4}{3} \times \frac{dx}{dt}$

$\frac{dx}{dt} = 1.5$

Jadi, kadar perubahan hujung kayu B ialah 1.5 ms^{-1}.

5. Katakan x m ialah jarak mengufuk antara helikopter dengan budak lelaki dan z m ialah jarak antara helikopter dengan budak lelaki pada masa t.



$z^2 = x^2 + 135^2$

$z = \sqrt{x^2 + 18\ 225}$

$\frac{dz}{dx} = \frac{1}{2}(x^2 + 18\ 225)^{-\frac{1}{2}}(2x)$

$= \frac{x}{\sqrt{x^2 + 18\ 225}}$

Jadi, $\frac{dz}{dt} = \frac{dz}{dx} \times \frac{dx}{dt}$

Apabila $x = 72$ dan $\frac{dx}{dt} = -17$, $\frac{dz}{dt} = \frac{72}{\sqrt{72^2 + 18\ 225}} \times -17$

$= \frac{72}{153} \times -17$

$= -8$

$\therefore -8$ ms^{-1}

Latihan Sumatif

1. (a) $\underset{x \to -2}{\text{had}} \left(\frac{8 + 2x - x^2}{8 - 2x^2}\right) = \underset{x \to -2}{\text{had}} \left[\frac{(4 - x)(2 + x)}{2(4 - x^2)}\right]$

$= \underset{x \to -2}{\text{had}} \left[\frac{(4 - x)(2 + x)}{2(2 + x)(2 - x)}\right]$

$= \underset{x \to -2}{\text{had}} \left[\frac{(4 - x)}{2(2 - x)}\right]$

$= \frac{4 - (-2)}{2[2 - (-2)]}$

$= \frac{6}{8}$

$= \frac{3}{4}$

(b) $\underset{x \to 0}{\text{had}} \left(\frac{\sqrt{1 + x + x^2} - 1}{x}\right) = \underset{x \to 0}{\text{had}} \left(\frac{\sqrt{1 + x + x^2} - 1}{x}\right)\left(\frac{\sqrt{1 + x + x^2} + 1}{\sqrt{1 + x + x^2} + 1}\right)$

$= \underset{x \to 0}{\text{had}} \left[\frac{1 + x + x^2 - 1}{x\left(\sqrt{1 + x + x^2} + 1\right)}\right]$

$= \underset{x \to 0}{\text{had}} \left[\frac{x + x^2}{x\left(\sqrt{1 + x + x^2} + 1\right)}\right]$

$= \underset{x \to 0}{\text{had}} \left[\frac{x(1 + x)}{x\left(\sqrt{1 + x + x^2} + 1\right)}\right]$

$= \underset{x \to 0}{\text{had}} \left(\frac{1 + x}{\sqrt{1 + x + x^2} + 1}\right)$

$= \frac{1}{\sqrt{1} + 1}$

$= \frac{1}{2}$

(c) $\underset{x \to k}{\text{had}} \frac{9 - x^2}{4 - \sqrt{x^2 + 7}} = 8$

$\frac{9 - k^2}{4 - \sqrt{k^2 + 7}} = 8$

$\left(\frac{9 - k^2}{4 - \sqrt{k^2 + 7}}\right)\left(\frac{4 + \sqrt{k^2 + 7}}{4 + \sqrt{k^2 + 7}}\right) = 8$

$\frac{(9 - k^2)\left(4 + \sqrt{k^2 + 7}\right)}{16 - (k^2 + 7)} = 8$

$\frac{(9 - k^2)\left(4 + \sqrt{k^2 + 7}\right)}{9 - k^2} = 8$

$4 + \sqrt{k^2 + 7} = 8$

$\sqrt{k^2 + 7} = 4$

$k^2 + 7 = 16$

$k^2 = 9$

$k = \pm 3$

2. $\underset{x \to -1}{\text{had}} \left(\frac{a - 5}{x + 4}\right) = -3$

$\frac{a - 5}{-1 + 4} = -3$

$\frac{a - 5}{3} = -3$

$a - 5 = -9$

$a = -9 + 5$

$a = -4$

3. (a) $\frac{d}{dx}\left(\frac{1}{2x+1}\right) = \frac{d}{dx}[(2x+1)^{-1}]$

$= -1(2x+1)^{-2}(2)$

$= -\frac{2}{(2x+1)^2}$

(b) $\frac{d}{dx}[4x(2x-1)^5] = 4x[5(2x-1)^4(2)] + (2x-1)^5(4)$

$= 40x(2x-1)^4 + 4(2x-1)^5$

$= 4(2x-1)^4[10x + (2x-1)]$

$= 4(12x-1)(2x-1)^4$

(c) $\frac{d}{dx}\left[\frac{6}{(2-x)^2}\right] = \frac{d}{dx}[6(2-x)^{-2}]$

$= 6(-2)(2-x)^{-3}(-1)$

$= \frac{12}{(2-x)^3}$

(d) $\frac{d}{dx}(x\sqrt{x+3}) = x\left(\frac{1}{2}\right)(x+3)^{-\frac{1}{2}}(1) + \sqrt{x+3}(1)$

$= \frac{x}{2\sqrt{x+3}} + \sqrt{x+3}$

$= \frac{x + 2(x+3)}{2\sqrt{x+3}}$

$= \frac{3x+6}{2\sqrt{x+3}}$

$= \frac{3(x+2)}{2\sqrt{x+3}}$

4. (a) $y = x(3-x)$

$= 3x - x^2$

$\frac{dy}{dx} = 3 - 2x$

$\frac{d^2y}{dx^2} = -2$

$y\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 12 = (3x - x^2)(-2) + x(3-2x) + 12$

$= -6x + 2x^2 + 3x - 2x^2 + 12$

$= 12 - 3x$

(b) $12 - 3x = 0$

$3x = 12$

$x = \frac{12}{3}$

$x = 4$

5. $y = ax + \frac{b}{x^2}$

$= ax + bx^{-2}$

$\frac{dy}{dx} = a - 2bx^{-3}$

$= a - \frac{2b}{x^3}$

Pada titik $\left(-1, -\frac{7}{2}\right)$, $-\frac{7}{2} = a(-1) + \frac{b}{(-1)^2}$

$-\frac{7}{2} = -a + b$

$a - b = \frac{7}{2}$

$2a - 2b = 7 \quad \ldots$❶

dan $\dfrac{dy}{dx} = 2$

$$a - \frac{2b}{(-1)^3} = 2$$

$$a + 2b = 2 \qquad \ldots ❷$$

❶ + ❷: $3a = 9$

$a = 3$

Gantikan $a = 3$ ke dalam ❶: $2(3) - 2b = 7$

$6 - 2b = 7$

$2b = -1$

$b = -\dfrac{1}{2}$

$\therefore a = 3, b = -\dfrac{1}{2}$

6. $V = \dfrac{4}{3}\pi r^3$

$\dfrac{dV}{dr} = 4\pi r^2$

$\dfrac{dV}{dt} = \dfrac{dV}{dr} \times \dfrac{dr}{dt}$

$20\pi = 4\pi r^2 \times 0.2$

$20 = 0.8r^2$

$r^2 = 25$

$r = 5\ (> 0)$

$\therefore r = 5$ cm

7. (a) $y = \dfrac{14}{\sqrt{6x^3 + 1}} = 14(6x^3 + 1)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = 14\left(-\frac{1}{2}\right)(6x^3 + 1)^{-\frac{3}{2}}(18x^2)$$

$$= -\frac{126x^2}{(6x^3 + 1)^{\frac{3}{2}}}$$

Apabila $x = 2$, $\delta x = 2.05 - 2$

$= 0.05$

dan $\dfrac{dy}{dx} = -\dfrac{126(2)^2}{[6(2)^3 + 1]^{\frac{3}{2}}}$

$= -\dfrac{504}{343}$

$= -\dfrac{72}{49}$

Jadi, $\delta y \approx \dfrac{dy}{dx} \times \delta x$

$= -\dfrac{72}{49} \times 0.05$

$= -0.0735$ unit

(b) Apabila $x = 2$, $y = \dfrac{14}{\sqrt{6(2)^3 + 1}}$

$= \dfrac{14}{7}$

$= 2$

$\delta x = 2.05 - 2 = 0.05$

dan $\dfrac{dy}{dx} = -\dfrac{72}{49}$

$$\frac{14}{\sqrt{6(2.05)^3 + 1}} = 2 + \left(-\frac{72}{49}\right)(0.05)$$

$= 2 - 0.0735$

$= 1.927$

8. $y = \frac{1}{\sqrt{x}}$

$= x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$

$= -\frac{1}{2\sqrt{x^3}}$

Apabila $x = 4$, $y = \frac{1}{\sqrt{4}}$

$= \frac{1}{2}$

$\delta x = \frac{2}{100} \times 4 = 0.08$

dan $\frac{dy}{dx} = -\frac{1}{2\sqrt{4^3}} = -\frac{1}{16}$

Jadi, $\delta y \approx \frac{dy}{dx} \times \delta x$

$= -\frac{1}{16} \times 0.08$

$= -0.005$

Maka, peratus perubahan hampir dalam $y = \frac{\delta y}{y} \times 100\%$

$= -\frac{0.005}{\frac{1}{2}} \times 100\%$

$= -1\%$

9. $y = 3x^2 - 4x + 6$

$\frac{dy}{dx} = 6x - 4$

Apabila $x = 2$, $y = 3(2)^2 - 4(2) + 6$

$= 12 - 8 + 6$

$= 10$

$\frac{dy}{dx} = 6(2) - 4$

$= 8$

dan $\delta x = \frac{p}{100} \times 2$

$= 0.02p$

$\delta y = \frac{dy}{dx} \times \delta x$

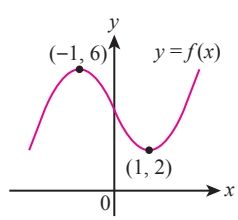
$= 8 \times 0.02p$

$= 0.16p$

Maka, peratus perubahan dalam y ialah $\frac{0.16p}{10} \times 100 = 1.6p\%$

10. (a) Titik maksimum ialah $(-1, 6)$ dan titik minimum ialah $(1, 2)$

(b)



11. (a) $y = 3x^3 - 4x + 2$

$\frac{dy}{dx} = 9x^2 - 4$

Pada titik $A(2, 1)$, $\frac{dy}{dx} = 9(2)^2 - 4$

$= 36 - 4$

$= 32$

Persamaan tangen di titik $A(2, 1)$ ialah:

$y - 1 = 32(x - 2)$

$y - 1 = 32x - 64$

$y = 32x - 63$

(b) $\frac{dy}{dx} = 32$

$9x^2 - 4 = 32$

$9x^2 = 36$

$x^2 = 4$

$x = \pm 2$

Apabila $x = -2$, $y = 3(-2)^3 - 4(-2) + 2$

$= -24 + 8 + 2$

$= -14$

$\therefore (-2, -14)$

12. (a) $j^2 = \left(6\sqrt{3}\right)^2 - t^2$

$j = \sqrt{108 - t^2}$ …❶

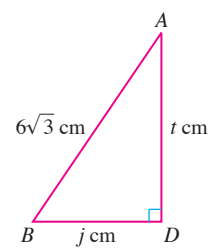
$I = \frac{1}{3}\pi j^2 t$ …❷

Gantikan ❶ ke dalam ❷:

$I = \frac{1}{3}\pi\left(\sqrt{108 - t^2}\right)^2 t$

$= \frac{1}{3}\pi(108 - t^2)t$

$= 36\pi t - \frac{1}{3}\pi t^3$



Untuk isi padu maksimum, $\frac{dI}{dt} = 0$

$36\pi - \pi t^2 = 0$

$36\pi = \pi t^2$

$36 = t^2$

$t = 6 \ (> 0)$

$\therefore t = 6$ cm

(b) $I = 36\pi t - \frac{1}{3}\pi t^3$

Apabila $t = 6$, $I = 36\pi(6) - \frac{1}{3}\pi(6)^3$

$= 216\pi - 72\pi$

$= 144\pi$

$\therefore$ Isi padu kon ialah 144π cm^3.

13. $AC = \sqrt{30^2 + x^2} = \sqrt{900 + x^2}$

Jumlah masa yang diambil dari A ke D ialah

$T = \frac{\sqrt{900 + x^2}}{40} + \frac{400 - x}{50}$

$T = \frac{1}{40}\sqrt{900 + x^2} + 8 - \frac{1}{50}x$

Untuk nilai pegun T, $\frac{dT}{dx} = 0$

$$\frac{1}{40}\left(\frac{1}{2}\right)(900 + x^2)^{-\frac{1}{2}}(2x) - \frac{1}{50} = 0$$

$$\frac{x}{40\sqrt{900 + x^2}} = \frac{1}{50}$$

$$50x = 40\sqrt{900 + x^2}$$

$$5x = 4\sqrt{900 + x^2}$$

$$25x^2 = 16(900 + x^2)$$

$$25x^2 = 14\ 400 + 16x^2$$

$$9x^2 = 14\ 400$$

$$x^2 = 1\ 600$$

$$x = \sqrt{1\ 600}$$

$$= 40$$

$\therefore$ Jarak dari B ke C ialah 40 m.

14. $I = 8$

$x^3 = 8$

$x = 2$

$L = 6x^2$

$\frac{dL}{dx} = 12x$

Apabila $x = 2$, $\frac{dL}{dx} = 12(2)$

$= 24$

$\frac{dL}{dt} = \frac{dL}{dx} \times \frac{dx}{dt}$

$\frac{dL}{dt} = 24 \times 2$

$= 48$

$\therefore$ Kadar perubahan jumlah luas permukaan kubus ialah 48 cm^2s^{-1}.

15. (a) Luas, $A = \frac{1}{2}xy$

$= \frac{1}{2}x(6x - x^2)$

$= \frac{1}{2}(6x^2 - x^3)$ (Tertunjuk)

(b) (i) $A = \frac{1}{2}(6x^2 - x^3) = 3x^2 - \frac{1}{2}x^3$

$\frac{dA}{dx} = 6x - \frac{3}{2}x^2$

$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$

$\frac{dA}{dt} = \left(6x - \frac{3}{2}x^2\right)(2)$

Apabila $x = 2$, $\frac{dA}{dt} = \left[6(2) - \frac{3}{2}(2)^2\right](2)$

$= (12 - 6)(2)$

$= 12$

Kadar tokokan A ialah 12 $unit^2\ s^{-1}$

(ii) Apabila $x = 5$, $\frac{dA}{dt} = \left[6(5) - \frac{3}{2}(5)^2\right](2)$

$= (30 - 37.5)(2)$

$= -15$

$\therefore$ Kadar susutan A ialah 15 $unit^2\ s^{-1}$

16. (a) $\frac{r}{12} = \frac{h}{20}$

$r = \frac{12}{20}h$

$r = \frac{3}{5}h \quad \ldots ❶$

$V = \frac{1}{3}\pi r^2 h \ldots ❷$

Gantikan ❶ ke dalam ❷: $V = \frac{1}{3}\pi\left(\frac{3}{5}h\right)^2 h$

$= \frac{1}{3}\pi\left(\frac{9}{25}h^2\right)h$

$= \frac{3}{25}\pi h^3$ (Tertunjuk)

(b) (i) $V = \frac{3}{25}\pi h^3$

$\frac{dV}{dh} = \frac{9}{25}\pi h^2$

Apabila $h = 5$, $\delta h = 4.99 - 5 = -0.01$

dan $\frac{dV}{dh} = \frac{9}{25}\pi(5)^2$

$= 9\pi$

$\delta V \approx \frac{dV}{dh} \times \delta h$

$\delta V = 9\pi \times -0.01$

$= -0.09\pi$

∴ Perubahan kecil dalam isi padu air ialah -0.09π cm^3.

(ii) Jika h menyusut sebanyak $p\%$,

$\delta h = -\frac{p}{100}(h) = -\frac{ph}{100}$

$\delta V \approx \frac{dV}{dh} \times \delta h$

$\delta V = \frac{9}{25}\pi h^2 \times -\frac{ph}{100}$

$= -\frac{9}{2\,500}\pi p h^3$

Jadi, $\frac{\delta V}{V} \times 100\% = \frac{-\frac{9}{2\,500}\pi p h^3}{\frac{3}{25}\pi h^3} \times 100\%$

$= -3p$

∴ Isi padu menyusut sebanyak $3p\%$.