Jawapan

BAB 3 PENGAMIRAN

Aktiviti Penerokaan 1 (Halaman 82)

3. (a) Graf fungsi $g(x) = \int f'(x)\,dx$ adalah sama dengan graf fungsi $f(x)$.

(b) Graf fungsi $k(x) = \int h'(x)\,dx$ adalah sama dengan graf fungsi $h(x)$.

(c) Graf fungsi $n(x) = \int m'(x)\,dx$ adalah sama dengan graf fungsi $m(x)$.

Kuiz Pantas (Halaman 83)

1. Mencari isi padu air dalam baldi daripada kadar pengaliran air sebuah pili.

2. Mencari jarak yang dilalui oleh suatu objek daripada kadar perubahan jarak atau laju.

3. Mencari luas pembiakan kulat daripada kadar pertambahan luas kulat pada roti.

Latihan Kendiri 3.1

1. $\int (15x^2 + 4)\,dx = 5x^3 + 4x$

2. $\int 24x^2\,dx = 8x^3$

3. (a) $\dfrac{dJ}{dt} = 300t^2 + 60t$

(b) $\dfrac{dJ}{dt} = 1\,500t^2 + 300t$

$= 5(300t^2 + 60t)$

$J = 5(100t^3 + 30t^2)$

$= 500t^3 + 150t^2$

Apabila $t = 2$, $J = 500(2)^3 + 150(2)^2$

$= 4\,600$ liter

Latihan Formatif 3.1

1. $\dfrac{dy}{dx} = 3(3)(2)(2x + 2)^2$

$= 18(2x + 2)^2$

$\int 18(2x + 2)^2\,dx = 3(2x + 2)^3$

2. $f'(x) = \dfrac{(2 - 3x)(5) - (-3)(5x + 2)}{(2 - 3x)^2}$

$= \dfrac{16}{(2 - 3x)^2}$

$\int f'(x)\,dx = \int \dfrac{16}{(2 - 3x)^2}\,dx$

$= \dfrac{5x + 2}{2 - 3x}$

3. $\dfrac{dy}{dx} = 15(x + 2)^2$

Bandingkan dengan $\dfrac{dy}{dx}$,

$h = 15$ dan $k = 2$

$h + k = 15 + 2$

$= 17$

$$\frac{1}{10}\int\left(\frac{dy}{dx}\right)dx = \frac{1}{10}\int 15(x+2)^2\,dx$$
$$= \frac{1}{10}[5(x+2)^3]$$
$$= \frac{(x+2)^3}{2}$$

Apabila $x = 2$, nilai bagi $\frac{1}{10}\int\left(\frac{dy}{dx}\right)dx = \frac{(2+2)^3}{2}$
$= 32$

4. $f(x) = 3x(2x+1)^2$
$f'(x) = 3(2x+1)^2 + 2(2)(2x+1)(3x)$
$= (2x+1)[3(2x+1) + 12x]$
$= (2x+1)(6x+3+12x)$
$= (2x+1)(18x+3)$
$= 36x^2 + 6x + 18x + 3$
$= 36x^2 + 24x + 3$
$= 3(12x^2 + 8x + 1)$

$$\int(12x^2 + 3x + 1)\,dx = \int\frac{1}{3}f'(x)\,dx$$
$$= \frac{1}{3}\int f'(x)\,dx$$
$$af(x) = \frac{1}{3}f(x)$$
$$a = \frac{1}{3}$$

5. (a) $\frac{dA}{dt} = 200t + 150t^2$

Apabila $t = 5$, $\frac{dA}{dt} = 200(5) + 150(5)^2$
$= 4\,750$

Kadar keuntungan harian hasil jualan tiket bas bagi syarikat *K* ialah RM4 750 sehari.

(b) Fungsi keuntungan harian hasil jualan tiket bas bagi syarikat *H*

$= \int 30t^2 + 40t\,dt$
$= \frac{1}{5}\int 200t + 150t^2\,dt$
$= \frac{1}{5}[100t^2 + 50t^3]$
$= 20t^2 + 10t^3$

Maka, syarikat *K* mendapat keuntungan 5 kali ganda lebih daripada syarikat *H*.

Aktiviti Penerokaan 2 (Halaman 85)
Kes 1

Fungsi	$f'(x)$	$\int f'(x)\,dx$
$f(x) = 2x + c$	2	$2x + c$
$f(x) = 3x + c$	3	$3x + c$
$f(x) = 0.5x + c$	0.5	$0.5x + c$
$f(x) = -7x + c$	-7	$-7x + c$
$f(x) = -4x + c$	-4	$-4x + c$

Maka, $\int a\,dx = ax + c$.

Kes 2

Fungsi	$f'(x)$	$\int f'(x)\,dx$	Pola
$f(x) = 2x^2 + c$	$4x$	$\frac{4x^2}{2} + c = 2x^2 + c$	$\frac{4x^{1+1}}{1+1} + c$
$f(x) = 2x^3 + c$	$6x^2$	$\frac{6x^3}{3} + c = 2x^3 + c$	$\frac{6x^{2+1}}{2+1} + c$
$f(x) = 2x^4 + c$	$8x^3$	$\frac{8x^4}{4} + c = 2x^4 + c$	$\frac{8x^{3+1}}{3+1} + c$
$f(x) = 2x^5 + c$	$10x^4$	$\frac{10x^5}{5} + c = 2x^5 + c$	$\frac{10x^{4+1}}{4+1} + c$
$f(x) = 2x^6 + c$	$12x^5$	$\frac{12x^6}{6} + c = 2x^6 + c$	$\frac{12x^{5+1}}{5+1} + c$

Maka, $\int ax^n\,dx = \frac{ax^{n+1}}{n+1}$.

Kuiz Pantas (Halaman 86)

(a) $\int dx = \int 1\,dx$
$= x + c$

(b) $\int 0\,dx = 0 + c$
$= c$

(c) $\int |x|\,dx = \begin{cases} \frac{1}{2}x^2 + c \text{ jika } x \geqslant 0 \\ -\frac{1}{2}x^2 + c \text{ jika } x < 0 \end{cases}$

Perbincangan (Halaman 87)

Walaupun kamiran bagi fungsi yang melibatkan penambahan atau penolakan akan melibatkan beberapa pemalar pengamiran, namun hasil tambah semua pemalar ini masih lagi merupakan suatu pemalar. Oleh itu, kamiran bagi suatu fungsi yang melibatkan penambahan dan penolakan sebutan-sebutan algebra boleh diwakilkan dengan satu pemalar pengamiran sahaja.

Latihan Kendiri 3.2

1. (a) $\int 2\,dx = 2x + c$

(b) $\int \frac{5}{6}\,dx = \frac{5}{6}x + c$

(c) $\int -2\,dx = -2x + c$

(d) $\int \frac{\pi}{3}\,dx = \frac{\pi}{3}x + c$

2. (a) $\int 3x^2\,dx = \frac{3x^3}{3} + c$
$= x^3 + c$

(b) $\int \frac{4}{3}x^3\,dx = \frac{4}{3}\left(\frac{x^4}{4}\right) + c$
$= \frac{x^4}{3} + c$

(c) $\int -x\,dx = -\frac{x^2}{2} + c$

(d) $\int -\frac{2}{x^2}\,dx = \int -2x^{-2}\,dx$
$= \frac{-2x^{-1}}{-1} + c$
$= \frac{2}{x} + c$

(e) $\int \frac{3}{x^3}\,dx = \int 3x^{-3}\,dx$

$= \frac{3x^{-2}}{-2} + c$

$= -\frac{3}{2x^2} + c$

(f) $\int 3\sqrt{x}\,dx = \int 3x^{\frac{1}{2}}\,dx$

$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= 2x^{\frac{3}{2}} + c$

$= 2\sqrt{x^3} + c$

(g) $\int \frac{2}{\sqrt[3]{x}}\,dx = \int 2x^{-\frac{1}{3}}\,dx$

$= \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} + c$

$= 3x^{\frac{2}{3}} + c$

$= 3\sqrt[3]{x^2} + c$

(h) $\int \left(-\frac{3}{\sqrt{x}}\right)^3 dx = \int -27x^{-\frac{3}{2}}\,dx$

$= \frac{-27x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$= \frac{54}{\sqrt{x}} + c$

3. (a) $\int 2x + 3\,dx = \frac{2x^2}{2} + 3x + c$

$= x^2 + 3x + c$

(b) $\int 4x^2 + 5x\,dx = \frac{4x^3}{3} + \frac{5x^2}{2} + c$

$= \frac{4}{3}x^3 + \frac{5}{2}x^2 + c$

(c) $\int \frac{1}{2}x^3 + 5x - 2\,dx = \frac{1}{2}\left(\frac{x^4}{4}\right) + \frac{5x^2}{2} - 2x + c$

$= \frac{1}{8}x^4 + \frac{5}{2}x^2 - 2x + c$

(d) $\int \frac{3}{x^2} + 4x - 2\,dx = \int 3x^{-2} + 4x - 2\,dx$

$= \frac{3x^{-1}}{-1} + \frac{4x^2}{2} - 2x + c$

$= -\frac{3}{x} + 2x^2 - 2x + c$

4. (a) $\int (x + 2)(x - 4)\,dx = \int x^2 + 2x - 4x - 8\,dx$

$= \int x^2 - 2x - 8\,dx$

$= \frac{x^3}{3} - \frac{2x^2}{2} - 8x + c$

$= \frac{x^3}{3} - x^2 - 8x + c$

(b) $\int x^2(3x^2 + 5x)\,dx = \int 3x^4 + 5x^3\,dx$

$= \frac{3x^5}{5} + \frac{5x^4}{4} + c$

$= \frac{3}{5}x^5 + \frac{5}{4}x^4 + c$

(c) $\int (5x^2 - 3\sqrt{x})\, dx = \int \left(5x^2 - 3x^{\frac{1}{2}}\right) dx$

$$= \frac{5x^3}{3} - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{5}{3}x^3 - 2x^{\frac{3}{2}} + c$$

$$= \frac{5}{3}x^3 - 2\sqrt{x^3} + c$$

(d) $\int (5x - 3)^2\, dx = \int 25x^2 - 30x + 9\, dx$

$$= \frac{25x^3}{3} - \frac{30x^2}{2} + 9x + c$$

$$= \frac{25}{3}x^3 - 15x^2 + 9x + c$$

(e) $\int \left(\frac{5x^2 - 3x}{x}\right) dx = \int 5x - 3\, dx$

$$= \frac{5x^2}{2} - 3x + c$$

$$= \frac{5}{2}x^2 - 3x + c$$

(f) $\int \left(x + \sqrt{x}\right)^2 dx = \int (x^2 + 2x^{\frac{3}{2}} + x)\, dx$

$$= \frac{x^3}{3} + \frac{2x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^2}{2} + c$$

$$= \frac{1}{3}x^3 + \frac{4}{5}x^{\frac{5}{2}} + \frac{1}{2}x^2 + c$$

Perbincangan (Halaman 88)

Tidak boleh. Kaedah penggantian hanya boleh digunakan untuk fungsi dalam kurungan yang berbentuk linear sahaja. Kita perlu mengembangkan fungsi dalam kurungan yang bukan linear bagi mendapatkan kamiran.

Latihan Kendiri 3.3

1. (a) $\int (x - 3)^2\, dx$

Katakan $u = x - 3$ dan $\frac{du}{dx} = 1$

$$\int (x - 3)^2\, dx = \int u^2\, du$$

$$= \frac{u^3}{3} + c$$

$$= \frac{(x - 3)^3}{3} + c$$

(b) Katakan $u = 3x - 5$ dan $\frac{du}{dx} = 3$

$$\int (3x - 5)^9\, dx = \int \frac{u^9}{3}\, du$$

$$= \frac{u^{10}}{30} + c$$

$$= \frac{(3x - 5)^{10}}{30} + c$$

(c) Katakan $u = 5x - 2$ dan $\frac{du}{dx} = 5$

$$\int 4(5x - 2)^5\, dx = \int \frac{4u^5}{5}\, du$$

$$= \frac{4u^6}{30} + c$$

$$= \frac{2}{15}(5x - 2)^6 + c$$

(d) Katakan $u = 7x - 3$ dan $\frac{du}{dx} = 7$

$$\int \frac{(7x-3)^4}{3}\,dx = \int \frac{u^4}{21}\,du$$
$$= \frac{u^5}{105} + c$$
$$= \frac{(7x-3)^5}{105} + c$$

(e) Katakan $u = 2x - 6$ dan $\frac{du}{dx} = 2$

$$\int \frac{12}{(2x-6)^3}\,dx = \int \frac{12u^{-3}}{2}\,du$$
$$= \frac{6u^{-2}}{-2} + c$$
$$= -\frac{3}{(2x-6)^2} + c$$

(f) Katakan $u = 3x - 2$ dan $\frac{du}{dx} = 3$

$$\int \frac{2}{3(3x-2)^2}\,dx = \int \frac{2u^{-2}}{9}\,du$$
$$= \frac{2u^{-1}}{-9} + c$$
$$= -\frac{2}{9(3x-2)} + c$$

2. (a) $\int (4x+5)^4\,dx = \frac{(4x+5)^5}{5(4)} + c$

$$= \frac{(4x+5)^5}{20} + c$$

(b) $\int 2(3x-2)^3\,dx = \frac{2(3x-2)^4}{4(3)} + c$

$$= \frac{(3x-2)^4}{6} + c$$

(c) $\int (5x-11)^4\,dx = \frac{(5x-11)^5}{5(5)} + c$

$$= \frac{(5x-11)^5}{25} + c$$

(d) $\int \frac{(3x-2)^5}{5}\,dx = \frac{(3x-5)^6}{5(3)(6)} + c$

$$= \frac{(3x-5)^6}{90} + c$$

(e) $\int \frac{5}{(6x-3)^6}\,dx = \int 5(6x-3)^{-6}\,dx$

$$= \frac{5(6x-3)^{-5}}{-5(6)} + c$$
$$= -\frac{1}{6(6x-3)^5} + c$$

(f) $\int \frac{12}{(3x-5)^8}\,dx = \int 12(3x-5)^{-8}\,dx$

$$= \frac{12(3x-5)^{-7}}{-7(3)} + c$$
$$= -\frac{4}{7(3x-5)^7} + c$$

Latihan Kendiri 3.4

1. (a) $y = \int 4x - 2\,dx$

$y = \frac{4x^2}{2} - 2x + c$

$y = 2x^2 - 2x + c$ …❶

Gantikan $x = -1$ dan $y = 7$ ke dalam persamaan ❶:

$7 = 2(-1)^2 - 2(-1) + c$

$c = 3$

(b) $y = \int -6x - \frac{6}{x^3}\, dx$

$y = \int -6x - 6x^{-3}\, dx$

$y = -\frac{6x^2}{2} - \frac{6x^{-2}}{-2} + c$

$y = -3x^2 + \frac{3}{x^2} + c \ldots$❶

Gantikan $x = -1$ dan $y = 6$ ke dalam persamaan ❶:

$6 = -3(-1)^2 + \frac{3}{(-1)^2} + c$

$c = 6$

2. $y = \int 20x^3 - 6x^2 - 6\, dx$

$y = \frac{20x^4}{4} - \frac{6x^3}{3} - 6x + c$

$y = 5x^4 - 2x^3 - 6x + c \ldots$❶

Gantikan $x = 1$ dan $y = 2$ ke dalam persamaan ❶:

$2 = 5(1)^4 - 2(1)^3 - 6(1) + c$

$c = 5$

$y = 5x^4 - 2x^3 - 6x + 5 \ldots$❷

Gantikan $x = \frac{1}{2}$ ke dalam persamaan ❷:

$y = 5\left(\frac{1}{2}\right)^4 - 2\left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right) + 5$

$y = \frac{33}{16}$

3. (a) $y = \int 9x^2 - 2\, dx$

$y = \frac{9x^3}{3} - 2x + c$

$y = 3x^3 - 2x + c \ldots$❶

Gantikan titik (1, 6) ke dalam persamaan ❶:

$6 = 3(1)^3 - 2(1) + c$

$c = 5$

Persamaan lengkung ialah $y = 3x^3 - 2x + 5$

(b) $y = \int 10x - 2\, dx$

$y = \frac{10x^2}{2} - 2x + c$

$y = 5x^2 - 2x + c \ldots$❶

Gantikan titik (2, 13) ke dalam persamaan ❶:

$13 = 5(2)^2 - 2(2) + c$

$c = -3$

Persamaan lengkung ialah $y = 5x^2 - 2x - 3$

(c) $y = \int 24x^2 - 5\, dx$

$y = \frac{24x^3}{3} - 5x + c$

$y = 8x^3 - 5x + c \ldots$❶

Gantikan titik (1, 1) ke dalam persamaan ❶:

$1 = 8(1)^3 - 5(1) + c$

$c = -2$

Persamaan lengkung ialah $y = 8x^3 - 5x - 2$

(d) $y = \int 18x^2 + 10x\, dx$

$y = \frac{18x^3}{3} + \frac{10x^2}{2} + c$

$y = 6x^3 + 5x^2 + c$ …❶

Gantikan titik (–2, –10) ke dalam persamaan ❶:

$-10 = 6(-2)^3 + 5(-2)^2 + c$

$c = 18$

Persamaan lengkung ialah $y = 6x^3 + 5x^2 + 18$

Latihan Formatif 3.2

1. (a) $\int \frac{1}{2}\, dx = \frac{1}{2}x + c$

(b) $\int \frac{5}{3x^3}\, dx = \int \frac{5x^{-3}}{3}\, dx$

$= \frac{5x^{-2}}{-6} + c$

$= -\frac{5}{6x^2} + c$

(c) $\int \frac{1}{\sqrt{x}}\, dx = \int x^{-\frac{1}{2}}\, dx$

$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2x^{\frac{1}{2}} + c$

(d) $\int \left(\frac{2}{x^3} - \frac{3}{x^4}\right) dx = \int 2x^{-3} - 3x^{-4}\, dx$

$= \frac{2x^{-2}}{-2} - \frac{3x^{-3}}{-3} + c$

$= -\frac{1}{x^2} + \frac{1}{x^3} + c$

2. (a) $\int \frac{5x^2 - 3x^3}{x}\, dx = \int 5x - 3x^2\, dx$

$= \frac{5x^2}{2} - \frac{3x^3}{3} + c$

$= \frac{5}{2}x^2 - x^3 + c$

(b) $\int \frac{6x^3 + 2x^2}{2x^2}\, dx = \int 3x + 1\, dx$

$= \frac{3x^2}{2} + x + c$

$= \frac{3}{2}x^2 + x + c$

(c) $\int (5 - 6x)^3\, dx = \frac{(5 - 6x)^4}{-24} + c$

$= -\frac{(5 - 6x)^4}{24} + c$

(d) $\int \frac{1}{\sqrt[4]{5 - 2x}}\, dx = \int (5 - 2x)^{-\frac{1}{4}}\, dx$

$= \frac{(5 - 2x)^{\frac{3}{4}}}{\left(\frac{3}{4}\right)(-2)} + c$

$= -\frac{2(5 - 2x)^{\frac{3}{4}}}{3} + c$

3. $\frac{dy}{dx} = 10x + \frac{p}{x^2}$ …❶

Gantikan $\frac{dy}{dx} = 20\frac{1}{2}$ dan $x = 2$ ke dalam persamaan ❶:

$\frac{41}{2} = 10(2) + \frac{p}{(2)^2}$

$p = 2$

$y = \int 10x + \frac{2}{x^2}\, dx$

$y = \int 10x + 2x^{-2}\, dx$

$y = \frac{10x^2}{2} + \frac{2x^{-1}}{-1} + c$

$y = 5x^2 - \frac{2}{x} + c$ …❷

Gantikan $y = 19$ dan $x = 2$ ke dalam persamaan ❷:

$19 = 5(2)^2 - \frac{2}{2} + c$

$c = 0$

$y = 5x^2 - \frac{2}{x}$ …❸

Gantikan $x = -2$ ke dalam persamaan ❸:

$y = 5(-2)^2 - \frac{2}{-2}$

$= 21$

4. (a) $y = \int 4x^3 - 15x^2 + 6\, dx$

$y = \frac{4x^4}{4} - \frac{15x^3}{3} + 6x + c$

$y = x^4 - 5x^3 + 6x + c$ …❶

Gantikan $x = 3$ dan $y = -20$ ke dalam persamaan ❶:

$-20 = (3)^4 - 5(3)^3 + 6(3) + c$

$c = 16$

$y = x^4 - 5x^3 + 6x + 16$ …❷

Gantikan $x = -2$ ke dalam persamaan ❷:

$y = (-2)^4 - 5(-2)^3 + 6(-2) + 16$

$y = 60$

(b) $y = \int 2x + 2\, dx$

$y = \frac{2x^2}{2} + 2x + c$

$y = x^2 + 2x + c$ …❶

Gantikan $x = 2$ dan $y = 2$ ke dalam persamaan ❶:

$2 = (2)^2 + 2(2) + c$

$c = -6$

$y = x^2 + 2x - 6$ …❷

Gantikan $y = -6$ ke dalam persamaan ❷:

$-6 = x^2 + 2x - 6$

$x^2 + 2x = 0$

$x(x + 2) = 0$

$x = 0$ atau $x = -2$

5. $y = \int 3x^2 - 8x\, dx$

$y = \frac{3x^3}{3} - \frac{8x^2}{2} + c$

$y = x^3 - 4x^2 + c$ …❶

Gantikan titik $(1, -1)$ ke dalam persamaan ❶:

$-1 = (1)^3 - 4(1)^2 + c$

$c = 2$

Persamaan lengkung tersebut ialah $y = x^3 - 4x^2 + 2$.

6. $\frac{dy}{dx} = -(6x - 2)$

$y = \int 2 - 6x \, dx$

$y = 2x - \frac{6x^2}{2} + c$

$y = 2x - 3x^2 + c$ …❶

Gantikan titik $(2, 2)$ ke dalam ❶:

$2 = 2(2) - 3(2)^2 + c$

$c = 10$

Maka, $y = 2x - 3x^2 + 10$

7. $\frac{dy}{dx} = ax + b$

Pada titik $(-2, 8)$, $\frac{dy}{dx} = -7$

$a(-2) + b = -7$

$-2a + b = -7$ …❶

Pada titik $(0, 6)$, $\frac{dy}{dx} = 5$

$a(0) + b = 5$

$b = 5$

Gantikan $b = 5$ ke dalam ❶,

$-2a + 5 = -7$

$-2a = -12$

$a = 6$

$y = \int 6x + 5 \, dx$

$= \frac{6x^2}{2} + 5x + c$

$= 3x^2 + 5x + c$ …❷

Gantikan titik $(-2, 8)$ ke dalam ❷:

$8 = 3(-2)^2 + 5(-2) + c$

$c = 8 - 12 + 10$

$= 6$

Maka, persamaan lengkung ialah $y = 3x^2 + 5x + 6$.

8. $s = \int 10t - 2 \, dt$

$s = \frac{10t^2}{2} - 2t + c$

$s = 5t^2 - 2t + c$ …❶

Gantikan $s = 8$ dan $t = 1$ ke dalam ❶:

$8 = 5(1)^2 - 2(1) + c$

$c = 5$

$s = 5t^2 - 2t + 5$ …❷

Gantikan $t = 3$ ke dalam ❷:

$s = 5(3)^2 - 2(3) + 5$

$= 44$ m

Kuiz Pantas (Halaman 92)

(a) $\int_1^2 1 \, dx = [x]_1^2$

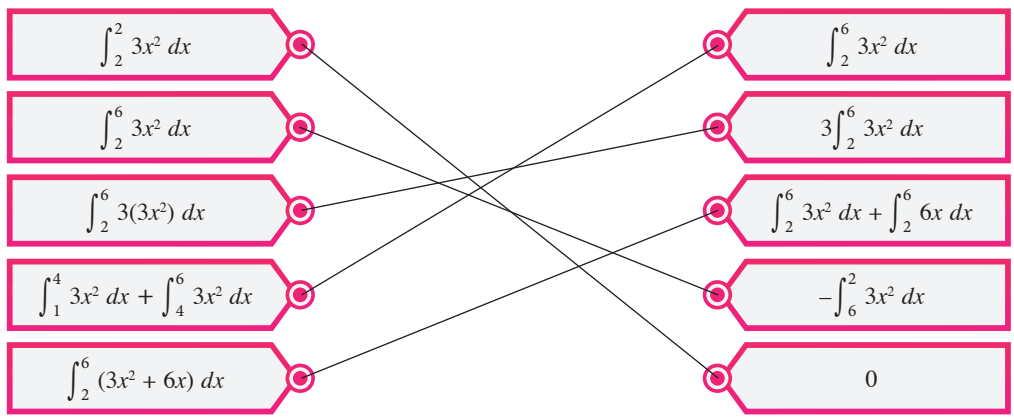
$= 2 - 1$

$= 1$

(b) $\int_1^2 0\ dx = 0$

Aktiviti Penerokaan 3 (Halaman 93)

4.



Latihan Kendiri 3.5

1. (a) $\int_2^4 x^3\ dx = \left[\frac{x^4}{4}\right]_2^4$

$= \frac{4^4}{4} - \frac{2^4}{4}$

$= 60$

(b) $\int_1^4 \frac{2}{x^2}\ dx = \int_1^4 2x^{-2}\ dx$

$= \left[\frac{2x^{-1}}{-1}\right]_1^4$

$= \left(\frac{-2}{4}\right) - \left(-\frac{2}{1}\right)$

$= -\frac{1}{2} + 2$

$= \frac{3}{2}$

(c) $\int_1^5 (2x^2 + 3x)\ dx = \left[\frac{2x^3}{3} + \frac{3x^2}{2}\right]_1^5$

$= \left[\frac{2(5)^3}{3} + \frac{3(5)^2}{2}\right] - \left[\frac{2(1)^3}{3} + \frac{3(1)^2}{2}\right]$

$= \frac{356}{3}$

(d) $\int_2^6 \left(\frac{1}{x^3} - 2x\right) dx = \int_2^6 (x^{-3} - 2x)\ dx$

$= \left[\frac{x^{-2}}{-2} - \frac{2x^2}{2}\right]_2^6$

$= \left[-\frac{1}{2(6)^2} - 6^2\right] - \left[-\frac{1}{2(2)^2} - 2^2\right]$

$= -\frac{287}{9}$

(e) $\int_1^3 \left(3x - \sqrt{x}\right) dx = \int_1^3 \left(3x - x^{\frac{1}{2}}\right) dx$

$= \left[\frac{3x^2}{2} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^3$

$= \left[\frac{3(3)^2}{2} - \frac{3^{\frac{3}{2}}}{\frac{3}{2}}\right] - \left[\frac{3(1)^2}{2} - \frac{1^{\frac{3}{2}}}{\frac{3}{2}}\right]$

$= 9.203$

(f) $\int_3^5 \left(x - \frac{1}{\sqrt{x}}\right) dx = \int_3^5 \left(x - x^{-\frac{1}{2}}\right) dx$

$= \left[\frac{x^2}{2} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_3^5$

$= \left[\frac{5^2}{2} - 2\sqrt{5}\right] - \left[\frac{3^2}{2} - 2\sqrt{3}\right]$

$= 6.992$

2. (a) $\int_2^4 \left(\frac{x^3 + x^2}{x}\right) dx = \int_2^4 x^2 + x \, dx$

$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_2^4$

$= \left[\frac{4^3}{3} + \frac{4^2}{2}\right] - \left[\frac{2^3}{3} + \frac{2^2}{2}\right]$

$= \frac{74}{3}$

(b) $\int_1^3 \left(\frac{5 + x^2}{x^2}\right) dx = \int_1^3 5x^{-2} + 1 \, dx$

$= \left[\frac{5x^{-1}}{-1} + x\right]_1^3$

$= \left[-\frac{5}{3} + 3\right] - \left[-\frac{5}{1} + 1\right]$

$= \frac{16}{3}$

(c) $\int_1^5 \left(\frac{(2x + 3)(x - 2)}{x^4}\right) dx = \int_1^5 (2x^{-2} - x^{-3} - 6x^{-4}) \, dx$

$= \left[\frac{2x^{-1}}{-1} - \frac{x^{-2}}{-2} - \frac{6x^{-3}}{-3}\right]_1^5$

$= \left[-\frac{2}{5} + \frac{1}{2(5)^2} + \frac{2}{5^3}\right] - \left[-\frac{2}{1} + \frac{1}{2(1)^2} + \frac{2}{1^3}\right]$

$= -\frac{108}{125}$

(d) $\int_3^4 (3x - 4)^2 \, dx = \left[\frac{(3x - 4)^3}{9}\right]_3^4$

$= \left[\frac{[3(4) - 4]^3}{9}\right] - \left[\frac{[3(3) - 4]^3}{9}\right]$

$= 43$

(e) $\int_{-3}^{-1} \frac{3}{(5 - 3x)^3} dx = \int_{-3}^{-1} 3(5 - 3x)^{-3} \, dx$

$= \left[\frac{3(5 - 3x)^{-2}}{6}\right]_{-3}^{-1}$

$= \left[\frac{1}{2[5 - 3(-1)]^2}\right] - \left[\frac{1}{2[5 - 3(-3)]^2}\right]$

$= \frac{33}{6\,272}$

(f) $\int_{-2}^0 \frac{2}{\sqrt{3 - 2x}} dx = \int_{-2}^0 2(3 - 2x)^{-\frac{1}{2}} \, dx$

$= \left[-2(3 - 2x)^{\frac{1}{2}}\right]_{-2}^0$

$= \left[-2[3 - 2(0)]^{\frac{1}{2}}\right] - \left[-2[3 - 2(-2)]^{\frac{1}{2}}\right]$

$= 1.827$

3. (a) $\int_5^2 f(x) \, dx = -\int_2^5 f(x) \, dx$

$= -3$

(b) $\int_2^5 \frac{1}{2} f(x)\,dx = \frac{1}{2}\int_2^5 f(x)\,dx$

$= \frac{1}{2}(3)$

$= \frac{3}{2}$

(c) $\int_2^5 [3f(x) - 2]\,dx = 3\int_2^5 f(x)\,dx - \int_2^5 2\,dx$

$= 3(3) - \left[2x\right]_2^5$

$= 9 - [2(5) - 2(2)]$

$= 3$

4. (a) $\int_3^7 [f(x) + k(x)]\,dx = \int_3^7 f(x)\,dx + \int_3^7 k(x)\,dx$

$= 5 + 7$

$= 12$

(b) $\int_3^5 f(x)\,dx - \int_7^5 f(x)\,dx = \int_3^5 f(x)\,dx + \int_5^7 f(x)\,dx$

$= \int_3^7 f(x)\,dx$

$= 5$

(c) $\int_3^7 [f(x) + 2x]\,dx = \int_3^7 f(x)\,dx + \int_3^7 2x\,dx$

$= 5 + \left[\frac{2x^2}{2}\right]_3^7$

$= 5 + [7^2 - 3^2]$

$= 45$

Aktiviti Penerokaan 4 (Halaman 95)

4.

Bilangan segi empat tepat, *n*	**Hasil tambah luas segi empat tepat di bawah lengkung**	**Luas rantau di bawah lengkung yang sebenar**
1	54	36
2	54	
3	50	
4	47	
5	45	
6	44	
7	43	
8	42	
9	42	
10	41	
11	41	
12	40	
13	40	
14	40	
15	39	
16	39	
17	39	
18	39	
19	39	
20	39	

Perbincangan (Halaman 96)

Kita bahagikan luas di bawah suatu lengkung dengan n trapezium.

Oleh itu, terdapat $(n + 1)$ garis menegak yang menjadi sempadan bagi setiap trapezium.

Lebar setiap trapezium $= \frac{b-a}{n}$

Luas trapezium pertama $= \left(\frac{b-a}{n}\right)\left(\frac{y_1 + y_2}{2}\right)$

Maka,

$\int_a^b f(x)\,dx$ = Jumlah luas di bawah lengkung

$$= \left(\frac{b-a}{n}\right)\left[\frac{y_1 + y_2}{2} + \frac{y_2 + y_3}{2} + \frac{y_3 + y_4}{2} + \ldots + \frac{y_n + y_{n+1}}{2}\right]$$

Aktiviti Penerokaan 5 (Halaman 97)

6.

Kamiran	Nilai kamiran	Kedudukan rantau
$\int_0^5 \frac{1}{3}x^3\,dx$	$\frac{625}{12}$	Di atas paksi-x
$\int_{-5}^0 \frac{1}{3}x^3\,dx$	$-\frac{625}{12}$	Di bawah paksi-x

Aktiviti Penerokaan 6 (Halaman 99)

6.

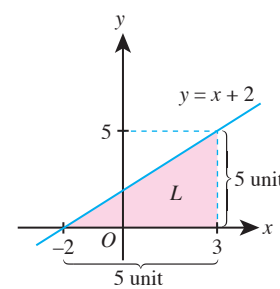
Kamiran	Nilai kamiran	Kedudukan rantau
$\int_0^5 y^{\frac{1}{3}}\,dy$	6.412	Di sebelah kanan paksi-y
$\int_{-5}^0 y^{\frac{1}{3}}\,dy$	–6.412	Di sebelah kiri paksi-y

Perbincangan (Halaman 103)

Luas di bawah garis lurus $y = x + 2$ antara batasan $x = –2$ dengan $x = 3$ dapat dicari menggunakan rumus luas segi tiga.

Luas segi tiga $= \frac{1}{2} \times 5 \times 5$

$= \frac{25}{2}$ unit^2



Maka, luas rantau $L = \int_{-2}^{3} (-x^2 + 2x + 8)\,dx - \frac{25}{2}$

$$= \left[-\frac{x^3}{3} + x^2 + 8x\right]_{-2}^{3} - \frac{25}{2}$$

$$= \left(\left[-\frac{3^3}{3} + 3^2 + 8(3)\right] - \left[-\frac{(-2)^3}{3} + (-2)^3 + 8(-2)\right]\right) - \frac{25}{2}$$

$$= \frac{100}{3} - \frac{25}{2}$$

$$= \frac{125}{6} \text{ unit}^2$$

Latihan Kendiri 3.6

1. (a) Luas rantau berlorek $= \int_0^3 3x - x^2 + 2\,dx$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{3} + 2x\right]_0^3$$

$$= \left[\frac{3(3)^2}{2} - \frac{3^3}{3} + 2(3)\right] - \left[\frac{3(0)^2}{2} - \frac{0^3}{3} + 2(0)\right]$$

$$= \frac{21}{2} \text{ unit}^2$$

(b) Luas rantau berlorek $= \int_{-3}^{2} \frac{1}{2}x^2\,dx$

$= \left[\frac{x^3}{6}\right]_{-3}^{2}$

$= \left[\frac{2^3}{6}\right] - \left[\frac{(-3)^3}{6}\right]$

$= \frac{35}{6}$ unit^2

(c) Luas rantau berlorek $= \int_{-2}^{1} y^2 + y - 6\,dy$

$= \left[\frac{y^3}{3} + \frac{y^2}{2} - 6y\right]_{-2}^{1}$

$= \left[\frac{1^3}{3} + \frac{1^2}{2} - 6(1)\right] - \left[\frac{(-2)^3}{3} + \frac{(-2)^2}{2} - 6(-2)\right]$

$= -\frac{33}{2}$

$= \frac{33}{2}$ unit^2

2. (a) Pada paksi-x, $y = 0$

$-x(x + 3)(x - 4) = 0$

$x = 0$ atau $x = -3$ atau $x = 4$

Luas rantau berlorek $= \left|\int_{-2}^{0} -x(x + 3)(x - 4)\,dx\right| + \int_{0}^{4} -x(x + 3)(x - 4)\,dx$

$= \left|\int_{-2}^{0} -x^3 + x^2 + 12x\,dx\right| + \int_{0}^{4} -x^3 + x^2 + 12x\,dx$

$= \left|\left[-\frac{x^4}{4} + \frac{x^3}{3} + \frac{12x^2}{2}\right]_{-2}^{0}\right| + \left[-\frac{x^4}{4} + \frac{x^3}{3} + \frac{12x^2}{2}\right]_{0}^{4}$

$= \left|\left[-\frac{0^4}{4} + \frac{0^3}{3} + 6(0)^2\right] - \left[-\frac{(-2)^4}{4} + \frac{(-2)^3}{3} + 6(-2)^2\right]\right|$

$- \left\{\left[-\frac{4^4}{4} + \frac{4^3}{3} + 6(4)^2\right] - \left[-\frac{0^4}{4} + \frac{0^3}{3} + 6(0)^2\right]\right\}$

$= \left|-\frac{52}{3}\right| + \frac{160}{3}$

$= \frac{212}{3}$ unit^2

(b) $y = x^2 - 4x + 5$ …❶

$y = -2x + 5$ …❷

Gantikan ❶ ke dalam ❷:

$x^2 - 4x + 5 = -2x + 5$

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ atau $x = 2$

Luas rantau berlorek $= \int_{0}^{2} (-2x + 5)\,dx - \int_{0}^{2} (x^2 - 4x + 5)\,dx$

$= \left[-\frac{2x^2}{2} + 5x\right]_{0}^{2} - \left[\frac{x^3}{3} - \frac{4x^2}{2} + 5x\right]_{0}^{2}$

$= \{[-(2)^2 + 5(2)] - [-(0)^2 + 5(0)]\} - \left\{\left[\frac{2^3}{3} - 2(2)^2 + 5(2)\right] - \left[\frac{0^3}{3} - 2(0)^2 + 5(0)\right]\right\}$

$= \frac{4}{3}$ unit^2

(c) $y^2 = 5x$ …❶

$x = -2y$ …❷

Gantikan ❷ ke dalam ❶:

$y^2 = 5(-2y)$

$y^2 = -10y$

$y^2 + 10y = 0$

$y(y + 10) = 0$

$y = 0$ atau $y = -10$

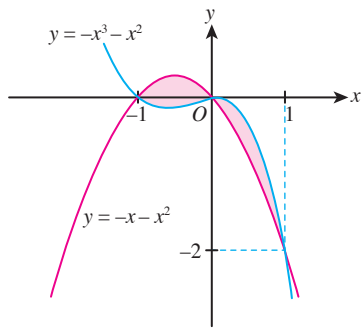
Luas rantau berlorek $= \int_{-10}^{0} (-2y)\, dy - \int_{-10}^{0} \left(\frac{y^2}{5}\right) dy$

$= \left[-\frac{2y^2}{2}\right]_{-10}^{0} - \left[\frac{y^3}{15}\right]_{-10}^{0}$

$= \{[-(0)^2] - [-(-10)^2]\} - \left\{\left[\frac{0^3}{15}\right] - \left[\frac{(-10)^3}{15}\right]\right\}$

$= \frac{100}{3}$ unit^2

3. (a)



Luas rantau di antara dua lengkung

$= \int_{-1}^{0} -x - x^2\, dx + \left|\int_{-1}^{0} -x^3 - x^2\, dx\right| + \left|\int_{0}^{1} -x - x^2\, dx\right| - \left|\int_{0}^{1} -x^3 - x^2\, dx\right|$

$= \left[-\frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{0} + \left|\left[-\frac{x^4}{4} - \frac{x^3}{3}\right]_{-1}^{0}\right| + \left|\left[-\frac{x^2}{2} - \frac{x^3}{3}\right]_{0}^{1}\right| + \left|\left[-\frac{x^4}{4} - \frac{x^3}{3}\right]_{0}^{1}\right|$

$= \left\{\left[-\frac{0^2}{2} - \frac{0^3}{3}\right] - \left[-\frac{(-1)^2}{2} - \frac{(-1)^3}{3}\right]\right\} + \left|\left\{\left[-\frac{0^4}{4} - \frac{0^3}{3}\right] - \left[-\frac{(-1)^4}{4} - \frac{(-1)^3}{3}\right]\right\}\right|$

$+ \left|\left[-\frac{1^2}{2} - \frac{1^3}{3}\right] - \left[-\frac{0^2}{2} - \frac{0^3}{3}\right]\right| - \left|\left[-\frac{1^4}{4} - \frac{1^3}{3}\right] - \left[-\frac{0^4}{4} - \frac{0^3}{3}\right]\right|$

$= \frac{1}{6} + \frac{1}{12} + \frac{5}{6} + \frac{7}{12}$

$= \frac{5}{3}$ unit^2

(b) $y = x^2 - 4x$ …❶

$y = 2x - x^2$ …❷

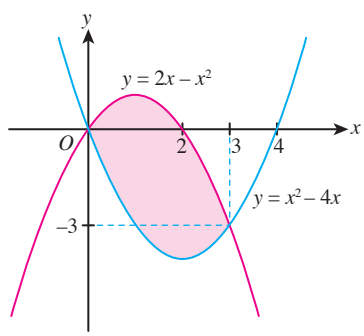
Gantikan ❶ ke dalam ❷:

$x^2 - 4x = 2x - x^2$

$2x^2 - 6x = 0$

$2(x)(x - 3) = 0$

$x = 0$ atau $x = 3$



Luas rantau di antara dua lengkung

$$= \int_0^2 (2x - x^2)\,dx + \left|\int_0^3 x^2 - 4x\,dx\right| - \left|\int_2^3 (2x - x^2)\,dx\right|$$

$$= \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 + \left|\left[\frac{x^3}{3} - \frac{4x^2}{2}\right]_0^3\right| - \left|\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_2^3\right|$$

$$= \left\{\left[2^2 - \frac{2^3}{3}\right] - \left[0^2 - \frac{0^3}{3}\right]\right\} + \left|\left\{\left[\frac{3^3}{3} - 2(3)^2\right] - \left[\frac{0^3}{3} - 2(0)^2\right]\right\}\right| - \left|\left\{\left[3^2 - \frac{3^3}{3}\right] - \left[2^2 - \frac{2^3}{3}\right]\right\}\right|$$

$$= \frac{4}{3} + 9 - \frac{4}{3}$$

$$= 9 \text{ unit}^2$$

Perbincangan (Halaman 108)

(a) Kon

(b) Silinder

Latihan Kendiri 3.7

1. (a) Isi padu janaan $= \int_0^2 \pi(-x^2 + 3x)^2\,dx$

$$= \pi \int_0^2 (x^4 - 6x^3 + 9x^2)\,dx$$

$$= \pi\left[\frac{x^5}{5} - \frac{6x^4}{4} + \frac{9x^3}{3}\right]_0^2$$

$$= \pi\left\{\left[\frac{2^5}{5} - \frac{3(2)^4}{2} + 3(2)^3\right] - \left[\frac{0^5}{5} - \frac{3(0)^4}{2} + 3(0)^3\right]\right\}$$

$$= \frac{32}{5}\pi \text{ unit}^3$$

(b) Isi padu janaan $= \int_0^6 \pi\left(\frac{6 - y}{2}\right) dy$

$$= \pi \int_0^6 \left(3 - \frac{y}{2}\right) dy$$

$$= \pi\left[3y - \frac{y^2}{4}\right]_0^6$$

$$= \pi\left\{\left[3(6) - \frac{6^2}{4}\right] - \left[3(0) - \frac{0^2}{4}\right]\right\}$$

$$= 9\pi \text{ unit}^3$$

2. Isi padu janaan $= \int_0^2 \pi\left(-\frac{y^2}{4}\right)^2 dy$

$$= \pi \int_0^2 \left(\frac{y^4}{16}\right) dy$$

$$= \pi\left[\frac{y^5}{80}\right]_0^2$$

$$= \pi\left[\frac{2^5}{80} - \frac{0^5}{80}\right]$$

$$= \frac{2}{5}\pi \text{ unit}^3$$

3. Isi padu janaan $= \int_0^5 \pi(5 - x)^2\,dx - \int_0^2 \pi(-x^2 + 4)^2\,dx$

$$= \pi\int_0^5 (25 - 10x + x^2)\,dx - \pi\int_0^2 (x^4 - 8x^2 + 16)\,dx$$

$$= \pi\left[25x - \frac{10x^2}{2} + \frac{x^3}{3}\right]_0^5 - \pi\left[\frac{x^5}{5} - \frac{8x^3}{3} + 16x\right]_0^2$$

$$= \pi\left\{\left[25(5) - 5(5)^2 + \frac{5^3}{3}\right] - \left[25(0) - 5(0)^2 + \frac{0^3}{3}\right]\right\}$$

$$- \pi\left\{\left[\frac{2^5}{5} - \frac{8(2)^3}{3} + 16(2)\right] - \left[\frac{0^5}{5} - \frac{8(0)^3}{3} + 16(0)\right]\right\}$$

$$= \frac{125}{3}\pi - \frac{256}{15}\pi$$

$$= \frac{123}{5}\pi \text{ unit}^3$$

4. (a) $y^2 = 4 - x$ …❶

$y = x - 2$ …❷

Gantikan ❶ ke dalam ❷:

$(x - 2)^2 = 4 - x$

$x^2 - 4x + 4 = 4 - x$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0$ atau $x = 3$

$x = 0, y = 0 - 2$

$= -2$

Oleh itu, $A(0, -2)$

(b) $x = 3, y = 3 - 2$

$= 1$

Oleh itu, $B(3, 1)$

(c) Isi padu janaan $= \int_{-2}^{1} \pi(4 - y^2)^2 \, dy - \int_{-2}^{1} \pi(y + 2)^2 \, dy$

$= \pi\int_{-2}^{1} (16 - 8y^2 + y^4) \, dy - \pi\int_{-2}^{1} (y^2 + 4y + 4) \, dy$

$= \pi\left[16y - \frac{8y^3}{3} + \frac{y^5}{5}\right]_{-2}^{1} - \pi\left[\frac{y^3}{3} - \frac{4y^2}{2} + 4y\right]_{-2}^{1}$

$= \pi\left\{\left[16(1) - \frac{8(1)^3}{3} + \frac{1^5}{5}\right] - \left[16(-2) - \frac{8(-2)^3}{3} + \frac{(-2)^5}{5}\right]\right\}$

$- \pi\left\{\left[\frac{1^3}{3} + \frac{4(1)^2}{2} + 4(1)\right] - \left[\frac{(-2)^3}{3} - \frac{4(-2)^2}{2} + 4(-2)\right]\right\}$

$= \frac{108}{5}\pi \text{ unit}^3$

Latihan Formatif 3.3

1. (a) $\int_{-1}^{3} (2 - x)^5 \, dx = \left[\frac{(2 - x)^6}{-6}\right]_{-1}^{3}$

$= \left\{-\frac{[2 - 3]^6}{6}\right\} - \left\{-\frac{[2 - (-1)]^6}{6}\right\}$

$= \frac{364}{3}$

(b) $\int_{-3}^{2} \frac{8x - 6x^2 + 8}{2 - x} \, dx = \int_{-3}^{2} \frac{(6x + 4)(2 - x)}{2 - x} \, dx$

$= \int_{-3}^{2} 6x + 4 \, dx$

$= \left[\frac{6x^2}{2} + 4x\right]_{-3}^{2}$

$= [3(2)^2 + 4(2)] - [3(-3)^2 + 4(-3)]$

$= 5$

(c) $\int_{-2}^{3} 2x^2(x^2 - x) \, dx = \int_{-2}^{3} 2x^4 - 2x^3 \, dx$

$= \left[\frac{2x^5}{5} - \frac{2x^4}{4}\right]_{-2}^{3}$

$= \left[\frac{2(3)^5}{5} - \frac{2(3)^4}{4}\right] - \left[\frac{2(-2)^5}{5} - \frac{2(-2)^4}{4}\right]$

$= \frac{155}{2}$

2. (a) $\int_{3}^{0} \frac{1}{2}f(x) \, dx + \int_{2}^{5} 3g(x) \, dx = -\frac{1}{2}\int_{0}^{3} f(x) \, dx + 3\int_{2}^{5} g(x) \, dx$

$= -\frac{1}{2}(2) + 3(7)$

$= -1 + 21$

$= 20$

(b) $\int_1^3 [k(x) - 3]\,dx + \int_3^7 k(x)\,dx = \int_1^3 k(x)\,dx - \int_1^3 3\,dx + \int_3^7 k(x)\,dx$

$= \int_1^7 k(x)\,dx - \int_1^3 3\,dx$

$= 10 - \left[3x\right]_1^3$

$= 10 - [3(3) - 3(1)]$

$= 10 - 6$

$= 4$

3. $\int_1^4 (x^2 + hx - 5)\,dx = 28\frac{1}{2}$

$\left[\frac{x^3}{3} + \frac{hx^2}{2} - 5x\right]_1^4 = \frac{57}{2}$

$\left[\frac{4^3}{3} + \frac{h(4)^2}{2} - 5(4)\right] - \left[\frac{1^3}{3} + \frac{h(1)^2}{2} - 5(1)\right] = \frac{57}{2}$

$\frac{15}{2}h + 6 = \frac{57}{2}$

$h = 3$

4. (a) Persamaan HK ialah $y = -x + 2$

$y = -x + 2$ …❶

$y = x^2$ …❷

Gantikan ❷ ke dalam ❶:

$x^2 = -x + 2$

$x^2 + x - 2 = 0$

$(x - 1)(x + 2) = 0$

$x = 1$ atau $x = -2$

$x = 1,\ y = -1 + 2$

$= 1$

$\therefore$ Titik $K(1, 1)$

(b) Luas rantau $P = \int_1^4 (y)^{\frac{1}{2}}\,dy - \int_1^2 (-y + 2)\,dy$

$= \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 - \left[-\frac{y^2}{2} + 2y\right]_1^2$

$= \left\{\left[\frac{2(4)^{\frac{3}{2}}}{3}\right] - \left[\frac{2(1)^{\frac{3}{2}}}{3}\right]\right\} - \left\{\left[-\frac{2^2}{2} + 2(2)\right] - \left[-\frac{1^2}{2} + 2(1)\right]\right\}$

$= \frac{14}{3} - \frac{1}{2}$

$= \frac{25}{6}$ unit^2

Luas rantau $Q = \int_0^1 (y)^{\frac{1}{2}}\,dy + \int_1^2 (-y + 2)\,dy$

$= \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + \left[-\frac{y^2}{2} + 2y\right]_1^2$

$= \left\{\left[\frac{2(1)^{\frac{3}{2}}}{3}\right] - \left[\frac{2(0)^{\frac{3}{2}}}{3}\right]\right\} + \left\{\left[-\frac{2^2}{2} + 2(2)\right] - \left[-\frac{1^2}{2} + 2(1)\right]\right\}$

$= \frac{2}{3} + \frac{1}{2}$

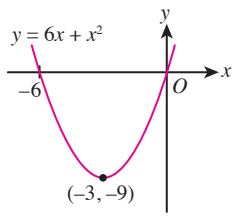
$= \frac{7}{6}$ unit^2

Nisbah luas rantau P terhadap luas rantau $Q = \dfrac{\text{Luas rantau } P}{\text{Luas rantau } Q}$

$$= \frac{\frac{25}{6}}{\frac{7}{6}}$$

$$= 25 : 7$$

5. (a)



(b) $\dfrac{dy}{dx} = 6 + 2x$

Pada asalan (0, 0), persamaan tangen ialah $y - 0 = [6 + 2(0)](x - 0)$

$y = 6x$

Pada $x = 2$, $y = 6(2) + (2)^2$

$= 16$

Pada titik (2, 16), persamaan tangen ialah $y - 16 = [6 + 2(2)](x - 2)$

$y = 10x - 20 + 16$

$y = 10x - 4$

(c) $6x = 10x - 4$

$-4x = -4$

$x = 1$

$x = 1$, $y = 6(1)$

$= 6$

Titik $A(1, 6)$

$6x + x^2 = 10x - 4$

$x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

$x = 2$

Pada $x = 2$, $y = 10(2) - 4$

$= 16$

Titik $B(2, 16)$

Luas rantau antara garis-garis tangen dan lengkung $= \int_0^2 6x + x^2\,dx - \int_0^1 6x\,dx - \int_1^2 10x - 4\,dx$

$$= \left[\frac{6x^2}{2} + \frac{x^3}{3}\right]_0^2 - \left[\frac{6x^2}{2}\right]_0^1 - \left[\frac{10x^2}{2} - 4x\right]_1^2$$

$$= \left\{\left[3(2)^2 + \frac{2^3}{3}\right] - \left[3(0)^2 + \frac{0^3}{3}\right]\right\}$$

$$- [3(1)^2 - 3(0)^2] - \{[5(2)^2 - 4(2)] - [5(1)^2 - 4(1)]\}$$

$$= \frac{2}{3} \text{ unit}^2$$

6. Pada $x = 1$, $y = 1^2 + 2$

$= 3$

Pada $x = 2$, $y = 2^2 + 2$

$= 6$

Isi padu janaan $= \int_3^6 \pi(y-2)\,dy$

$= \pi\left[\frac{y^2}{2} - 2y\right]_3^6$

$= \pi\left\{\left[\frac{6^2}{2} - 2(6)\right] - \left[\frac{3^2}{2} - 2(3)\right]\right\}$

$= \frac{15}{2}\pi$ unit^3

7. (a) $\frac{dy}{dx} = 2x$

$x = 1, \frac{dy}{dx} = 2(1)$

$= 2$

$y - 5 = 2(x - 1)$

$y = 2x - 2 + 5$

$y = 2x + 3$

$\therefore$ Titik $Q(0, 3)$

(b) Luas rantau berlorek $= \int_0^1 x^2 + 4\,dx - \int_0^1 (2x + 3)\,dx$

$= \left[\frac{x^3}{3} + 4x\right]_0^1 - \left[\frac{2x^2}{2} + 3x\right]_0^1$

$= \left\{\left[\frac{1^3}{3} + 4(1)\right] - \left[\frac{0^3}{3} + 4(0)\right]\right\} - \{[1^2 + 3(1)] - [0^2 + 3(0)]\}$

$= \frac{1}{3}$ unit^2

(c) Isi padu janaan $= \int_4^8 \pi(y-4)\,dy$

$= \pi\int_4^8 (y-4)\,dy$

$= \pi\left[\frac{y^2}{2} - 4y\right]_4^8$

$= \pi\left\{\left[\frac{8^2}{2} - 4(8)\right] - \left[\frac{4^2}{2} - 4(4)\right]\right\}$

$= 8\pi$ unit^3

8. (a) $x = 6 - y^2$ …❶

$3y = 8 + 2x$ …❷

Gantikan ❶ ke dalam ❷:

$3y = 8 + 2(6 - y^2)$

$3y = 8 + 12 - 2y^2$

$2y^2 + 3y - 20 = 0$

$(2y - 5)(y + 4) = 0$

$y = \frac{5}{2}$ atau $y = -4$

Daripada rajah, $y = \frac{5}{2}, x = 6 - \left(\frac{5}{2}\right)^2$

$= -\frac{1}{4}$

Jadi, $A\left(-\frac{1}{4}, \frac{5}{2}\right)$

(b) $x = 0,\ 3y = 8$

$y = \frac{8}{3}$

$x = 0,\ 0 = 6 - y^2$

$y^2 = 6$

$y = \pm\sqrt{6}$

$y = \sqrt{6}$

Luas rantau $Q = \int_{-\frac{1}{4}}^{0} \left(\frac{2}{3}x + \frac{8}{3}\right) dx - \int_{-\frac{1}{4}}^{0} \sqrt{6 - x}\ dx$

$= \frac{31}{48} - 0.619$

$= 0.027 \text{ unit}^2$

(c) Isi padu janaan bagi rantau $P = \int_{-\frac{1}{4}}^{0} \pi(6 - x)\ dx$

$= \pi \int_{-\frac{1}{4}}^{0} (6 - x)\ dx$

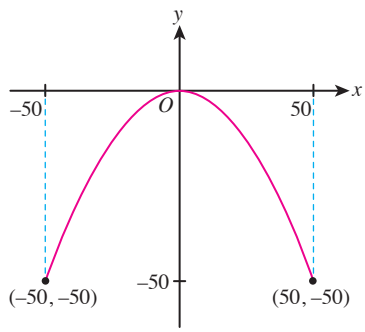
$= \pi \left[6x - \frac{x^2}{2}\right]_{-\frac{1}{4}}^{0}$

$= \pi \left\{\left[6(0) - \frac{0^2}{2}\right] - \left[6\left(-\frac{1}{4}\right) - \frac{\left(-\frac{1}{4}\right)^2}{2}\right]\right\}$

$= \frac{49}{32}\pi \text{ unit}^3$

Latihan Kendiri 3.8

1. (a) Lakarkan tudung saji pada satah Cartes:



Gantikan titik (50, –50):

$-50 = -k(50)^2$

$k = \frac{1}{50}$

(b) Isi padu bahagian dalaman $= \int_{-50}^{0} \pi(-50y)\ dy$

$= \pi \left[-\frac{50y^2}{2}\right]_{-50}^{0}$

$= \pi\{[-25(0)^2] - [-25(50)^2)]\}$

$= 62\ 500\pi \text{ cm}^3$

2. (a) $S'(t) = \frac{A}{1\ 000}(20 - t)$

Nilai susutan selepas 7 tahun $= \int_{0}^{7} \frac{48\ 000}{1\ 000}(20 - t)\ dt$

$= \int_{0}^{7} 960 - 48t\ dt$

$= \left[960t - \frac{48t^2}{2}\right]_{0}^{7}$

$= [960(7) - 24(7)^2] - [960(0) - 24(0)^2]$

$= \text{RM5 544}$

Harga kereta tersebut selepas 7 tahun = RM48 000 – RM5 544

= RM42 456

(b) Nilai susutan kereta selepas 5 tahun $= \int_0^5 \frac{88\,500}{1\,000}(20 - t)\,dt$

$= \int_0^5 1\,770 - \frac{177}{2}t\,dt$

$= \left[1\,770t - \frac{177t^2}{4}\right]_0^5$

$= \left[1\,770(5) - \frac{177(5)^2}{4}\right] - \left[1\,770(0) - \frac{177(0)^2}{4}\right]$

$=$ RM7 743.75

Peratus nilai susut $= \frac{7\,743.75}{88\,500} \times 100\%$

$= 8.75\%$

Latihan Formatif 3.4

1. Diberi $\frac{dh}{dt} = 5$ cm min^{-1} dan $\frac{dV}{dh} = \frac{3}{5}t - 6$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$= \left(\frac{3}{5}t - 6\right) \times 5$

$= 3t - 30$

Isi padu minyak yang mengalir keluar dari tangki $= \int_0^{30} 3t - 30\,dt$

$= \left[\frac{3t^2}{2} - 30t\right]_0^{30}$

$= \left[\frac{3}{2}(30)^2 - 30(30)\right] - \left[\frac{3}{2}(0)^2 - 30(0)\right]$

$= 450$ cm^3

2. Isi padu janaan bagi fungsi bahagian dalaman $= \int_0^{2.8} \pi(44.8 - 16y)\,dy$

$= 197.0407$

Isi padu janaan bagi fungsi bahagian luaran $= \int_0^3 \pi(60 - 20y)\,dy$

$= 282.7433$

Isi padu filamen plastik yang digunakan untuk menghasilkan 20 penutup mesin
$= (282.7433 - 197.0407) \times 20$
$= 1\,714.052$ cm^3

Anggaran kos bagi filamen plastik yang digunakan untuk menghasilkan 20 penutup mesin
$= 1\,714.052 \times$ RM0.07
$=$ RM119.98

3. (a) $\frac{dK}{dt} = 50\left[1 + \frac{300}{(t + 25)^2}\right]$

$\frac{dK}{dt} = 50 + \frac{15\,000}{(t + 25)^2}$

Bilangan mesin yang dihasilkan selepas 5 tahun $= \int_0^5 50 + \frac{15\,000}{(t + 25)^2}\,dt$

$= \int_0^5 50 + 15\,000(t + 25)^{-2}\,dt$

$= \left[50t + \frac{15\,000(t + 25)^{-1}}{-1}\right]_0^5$

$= \left[50(5) - \frac{15\,000}{(5 + 25)}\right] - \left[50(0) - \frac{15\,000}{(0 + 25)}\right]$

$= 350$

(b) Bilangan mesin yang dihasilkan selepas 6 tahun $= \int_0^6 50 + \frac{15\,000}{(t+25)^2}\,dt$

$= \int_0^6 50 + 15\,000(t+25)^{-2}\,dt$

$= \left[50t + \frac{15\,000(t+25)^{-1}}{-1}\right]_0^6$

$= \left[50(6) - \frac{15\,000}{(6+25)}\right] - \left[50(0) - \frac{15\,000}{(0+25)}\right]$

$= 416.129$

≈ 416

Bilangan mesin yang dihasilkan pada tahun ke-6 $= 416 - 350$

$= 66$

Latihan Sumatif

1. (a) $\int x(x-2)(x+3)\,dx = \int x^3 + x^2 - 6x\,dx$

$= \frac{x^4}{4} + \frac{x^3}{3} - \frac{6x^2}{2} + c$

$= \frac{1}{4}x^4 + \frac{1}{3}x^3 - 3x^2 + c$

(b) $\int \frac{2}{(2x-3)^3}\,dx = \int 2(2x-3)^{-3}\,dx$

$= \frac{2(2x-3)^{-2}}{2(-2)} + c$

$= -\frac{1}{2(2x-3)^2} + c$

2. (a) $\int \frac{2}{(3x-2)^n}\,dx = \int 2(3x-2)^{-n}\,dx$

$= \frac{2(3x-2)^{-n+1}}{3(-n+1)} + c$

$= \frac{2}{(3-3n)}(3x-2)^{-n+1} + c$

Bandingkan dengan $a(3x-2)^{-2} + c$:

$-n + 1 = -2$

$n = 3$

$a = \frac{2}{3 - 3(3)}$

$a = -\frac{1}{3}$

(b) $\int_1^3 \frac{8}{(3x-2)^3}\,dx = 4\left[-\frac{1}{3(3x-2)^2}\right]_1^3$

$= 4\left[\left\{-\frac{1}{3[3(3)-2]^2}\right\} - \left\{-\frac{1}{3[3(1)-2]^2}\right\}\right]$

$= \frac{64}{49}$

3. $\frac{dy}{dx} = \frac{3(2)(2)(2x+1)(5x-1) - (5)[3(2x+1)^2]}{(5x-1)^2}$

$= \frac{60x^2 - 24x - 27}{(5x-1)^2}$

$= \frac{3(20x^2 - 8x - 9)}{(5x-1)^2}$

$\int_1^4 \frac{3(20x^2 - 8x - 9)}{(5x-1)^2}\,dx = \left[\frac{3(2x+1)^2}{5x-1}\right]_1^4$

$= \left[\frac{3[2(4)+1]^2}{5(4)-1}\right] - \left[\frac{3[2(1)+1]^2}{5(1)-1}\right]$

$= \frac{459}{76}$

4. $f(x) = \int 2x^2 + 5x - r\ dx$

$f(x) = \frac{2x^3}{3} + \frac{5x^2}{2} - rx + c$

$f(1) = 14$

$\frac{2(1)^3}{3} + \frac{5(1)^2}{2} - r(1) + c = 14$

$\frac{2}{3} + \frac{5}{2} - r + c = 14$

$r = \frac{2}{3} + \frac{5}{2} - 14 + c$

$r = c - \frac{65}{6}$ …❶

$f(-2) = -16$

$\frac{2(-2)^3}{3} + \frac{5(-2)^2}{2} - r(-2) + c = -16$

$-\frac{16}{3} + 10 + 2r + c = -16$

$2r + c = -16 + \frac{16}{3} - 10$

$2r + c = -\frac{62}{3}$ …❷

Gantikan ❶ ke dalam ❷:

$2\left(c - \frac{65}{6}\right) + c = -\frac{62}{3}$

$c = \frac{1}{3}$

$r = \frac{1}{3} - \frac{65}{6}$

$= -\frac{21}{2}$

5. (a) $\int_0^2 f(x)\ dx - \int_4^2 f(x)\ dx = \int_0^2 f(x)\ dx + \int_2^4 f(x)\ dx$

$= \int_0^4 f(x)\ dx$

$= 4$

(b) $\int_0^4 f(x)\ dx + \int_1^v [g(x) + x]\ dx = 19$

$\int_0^4 f(x)\ dx + \int_1^v g(x)\ dx + \int_1^v x\ dx = 19$

$4 + 3 + \left[\frac{x^2}{2}\right]_1^v = 19$

$7 + \frac{v^2}{2} - \frac{1}{2} = 19$

$\frac{v^2}{2} = \frac{25}{2}$

$v^2 = 25$

$v = 5$ atau $v = -5$

Oleh itu, $v = 5$ (> 1)

6. $\frac{dV}{dt} = 10t + 3$

$V = \int 10t + 3\ dt$

$V = \frac{10t^2}{2} + 3t + c$

$V = 5t^2 + 3t + c$

Gantikan $t = 2$ dan $V = 24$ ke dalam fungsi:

$24 = 5(2)^2 + 3(2) + c$

$c = -2$

$V = 5t^2 + 3t - 2$

Apabila $t = 5$, $V = 5(5)^2 + 3(5) - 2$

$= 138\ \text{cm}^3$

7. (a) $3y = 4x - 13$ …❶
$x = 2y^2 + 2$ …❷

Gantikan ❶ ke dalam ❷:
$3y = 4(2y^2 + 2) - 13$
$8y^2 - 3y - 5 = 0$
$(8y + 5)(y - 1) = 0$
$y = -\frac{5}{8}$ atau $y = 1$
$y = 1, x = 2(1)^2 + 2$
$= 4$
Maka, $K(4, 1)$

(b) Luas rantau berlorek $= \int_0^1 (2y^2 + 2)\, dy$
$= \left[\frac{2y^3}{3} + 2y\right]_0^1$
$= \left[\frac{2(1)^3}{3} + 2(1)\right] - \left[\frac{2(0)^3}{3} + 2(0)\right]$
$= \frac{8}{3}$ unit^2

8. (a) $3x^2 + 2x + 4 = (x - 4)^2$
$2x^2 + 10x - 12 = 0$
$2(x - 1)(x + 6) = 0$
$x = 1$ atau $x = -6$
$x = 1, y = (1 - 4)^2$
$= 9$
Titik keseimbangan P ialah $(1, 9)$

(b) Lebihan pengguna
$= \int_0^1 (x - 4)^2\, dx - 1 \times 9$
$= \int_0^1 x^2 - 8x + 16\, dx - 1 \times 9$
$= \left[\frac{x^3}{3} - \frac{8x^2}{2} + 16x\right]_0^1 - 9$
$= \left[\frac{1^3}{3} - 4(1)^2 + 16(1)\right] - \left[\frac{0^3}{3} - 4(0)^2 + 16(0)\right] - 9$
$= \frac{10}{3}$ unit^2

(c) Lebihan pengeluar
$= 1 \times 9 - \int_0^1 (3x^2 + 2x + 4)\, dx$
$= 9 - \left[\frac{3x^3}{3} + \frac{2x^2}{2} + 4x\right]_0^1$
$= 9 - \{[1^3 + 1^2 + 4(1)] - [0^3 + 0^2 + 4(0)]\}$
$= 3$ unit^2

9. (a) $x = \frac{3y - 18}{2}$ …❶
$4x = 4 - y^2$ …❷
Gantikan ❶ ke dalam ❷:
$4\left(\frac{3y - 18}{2}\right) = 4 - y^2$
$-y^2 - 6y + 40 = 0$
$-(y - 4)(y + 10) = 0$
$y = 4$ atau $y = -10$
Pada $y = 4, x = \frac{3(4) - 18}{2}$
$= -3$
$\therefore P(-3, 4)$

(b) Pada $x = 0$,

$3y = 18 + 2(0)$

$y = \frac{18}{3}$

$y = 6$

$4(0) = 4 - y^2$

$y^2 = 4$

$y = 2$ atau $y = -2$

$y = 2$

Luas rantau berlorek A

$= \left|\int_2^4 \frac{4 - y^2}{4}\, dy\right| + \left|\int_4^6 \frac{3y - 18}{2}\, dy\right|$

$= \left|\left[y - \frac{y^3}{12}\right]_2^4\right| + \left|\left[\frac{3y^2}{4} - 9y\right]_4^6\right|$

$= \left|\left[4 - \frac{4^3}{12}\right] - \left[2 - \frac{2^3}{12}\right]\right| + \left|\left[\frac{3(6)^2}{4} - 9(6)\right] - \left[\frac{3(4)^2}{4} - 9(4)\right]\right|$

$= \frac{17}{3}$ unit^2

(c) Isi padu janaan rantau berlorek B

$= \int_{-3}^{0} \pi(4 - 4x)\, dx$

$= \pi\left[4x - \frac{4x^2}{2}\right]_{-3}^{0}$

$= \pi\{[4(0) - 2(0)^2] - [4(-3) - 2(-3)^2]\}$

$= 30\pi$ unit^3

10. (a) $y = -x^2 + 4$

$\frac{dy}{dx} = -2x$

$x = 1, \frac{dy}{dx} = -2(1)$

$= -2$

$y - 3 = -2(x - 1)$

$y = -2x + 2 + 3$

$y = -2x + 5$

Apabila $x = 0$, $y = 5$

Apabila $y = 0$, $-2x + 5 = 0$

$x = \frac{5}{2}$

Oleh itu, $P(0, 5)$, $R\left(\frac{5}{2}, 0\right)$ dan $S(0, 4)$

(b) Luas rantau berlorek

$= \int_0^1 (-2x + 5)\, dx - \int_0^1 (-x^2 + 4)\, dx$

$= \left[-\frac{2x^2}{2} + 5x\right]_0^1 - \left[-\frac{x^3}{3} + 4x\right]_0^1$

$= \{[-(1)^2 + 5(1)] - [-(0)^2 + 5(0)]\} - \left\{\left[-\frac{1^3}{3} + 4(1)\right] - \left[-\frac{0^3}{3} + 4(0)\right]\right\}$

$= \frac{1}{3}$ unit^2

(c) Isi padu janaan $= \int_3^4 \pi(4 - y)\, dy$

$= \pi\int_3^4 (4 - y)\, dy$

$= \pi\left[4y - \frac{y^2}{2}\right]_3^4$

$= \pi\left\{\left[4(4) - \frac{4^2}{2}\right] - \left[4(3) - \frac{3^2}{2}\right]\right\}$

$= \frac{1}{2}\pi$ unit^3

11. $p(2)^2 + 6(2) = 24$

$4p + 12 = 24$

$4p = 12$

$p = 3$

$q = 24(2) - 30$

$q = 18$

12. (a) Luas rantau P

$= \int_{-28}^{-3} \sqrt{(x + 28)}\ dx + \int_{-3}^{-2} x^2 - 4\ dx$

$= \left[\frac{(x + 28)^{\frac{3}{2}}}{\frac{3}{2}}\right]_{-28}^{-3} + \left[\frac{x^3}{3} - 4x\right]_{-3}^{-2}$

$= \left[\frac{2(-3 + 28)^{\frac{3}{2}}}{3} - \frac{2(-28 + 28)^{\frac{3}{2}}}{3}\right] + \left\{\left[\frac{(-2)^3}{3} - 4(-2)\right] - \left[\frac{(-3)^3}{3} - 4(-3)\right]\right\}$

$= \frac{250}{3} + \frac{7}{3}$

$= \frac{257}{3}$ unit^2

(b) Isi padu janaan $= \int_{-4}^{10} \pi(y + 4)\ dy$

$= \pi\int_{-4}^{10} y + 4\ dy$

$= \pi\left[\frac{y^2}{2} + 4y\right]_{-4}^{10}$

$= \pi\left\{\left[\frac{10^2}{2} + 4(10)\right] - \left[\frac{(-4)^2}{2} + 4(-4)\right]\right\}$

$= 98\pi$ unit^3

13. (a) Gantikan $(5, 33)$ ke dalam fungsi $y = 2x^2 - 3x + c$:

$33 = 2(5)^2 - 3(5) + c$

$c = -2$

Pada paksi-x, $2x^2 - 3x - 2 = 0$

$(x - 2)(2x + 1) = 0$

$x = 2$ atau $x = -\frac{1}{2}$

$A(2, 0)$

(b) Luas rantau berlorek

$= \left|\int_0^2 (2x^2 - 3x - 2)\,dx\right| + \int_2^5 (2x^2 - 3x - 2)\ dx$

$= \left|\left[\frac{2x^3}{3} - \frac{3x^2}{2} - 2x\right]_0^2\right| + \left[\frac{2x^3}{3} - \frac{3x^2}{2} - 2x\right]_2^5$

$= \left|\left[\frac{2(2)^3}{3} - \frac{3(2)^2}{2} - 2(2)\right] - \left[\frac{2(0)^3}{3} - \frac{3(0)^2}{2} - 2(0)\right]\right| + \left\{\left[\frac{2(5)^3}{3} - \frac{3(5)^2}{2} - 2(5)\right] - \left[\frac{2(2)^3}{3} - \frac{3(2)^2}{2} - 2(2)\right]\right\}$

$= \frac{14}{3} + \frac{81}{2}$

$= \frac{271}{6}$ unit^2

(c) Isi padu janaan $(180°) = \int_0^2 \frac{\pi}{2}(2x^2 - 3x - 2)^2\ dx$

$= \frac{\pi}{2}\int_0^2 (2x^2 - 3x - 2)^2\ dx$

$= \frac{\pi}{2}\int_0^2 4x^4 - 12x^3 + x^2 + 12x + 4\ dx$

$= \frac{\pi}{2}\left[\frac{4x^5}{5} - \frac{12x^4}{4} + \frac{x^3}{3} + \frac{12x^2}{2} + 4x\right]_0^2$

$= \frac{\pi}{2}\left\{\left[\frac{4(2)^5}{5} - 3(2)^4 + \frac{2^3}{3} + 6(2)^2 + 4(2)\right] - \left[\frac{4(0)^5}{5} - 3(0)^4 + \frac{0^3}{3} + 6(0)^2 + 4(0)\right]\right\}$

$= \frac{92}{15}\pi$ unit^3

14. $30 = a(30)^2$

$a = \frac{1}{30}$

Isi padu bekas

$= \int_0^{30} \pi(30y)\, dy$

$= \pi\left[\frac{30y^2}{2}\right]_0^{30}$

$= \pi\{15(30)^2 - 15(0)^2\}$

$= 42\ 411.5\ \text{cm}^3$

Jisim beras yang boleh disimpan $= 42\ 411.5 \times 1.182$
$= 50\ 130.393\ \text{g}$
$= 50.13\ \text{kg}$

15. (a) Isi padu kolam renang

$= \int_0^5 (3t^2 + 14t)\, dt$

$= \left[\frac{3t^3}{3} + \frac{14t^2}{2}\right]_0^5$

$= [5^3 + 7(5)^2] - [0^3 + 7(0)^2]$
$= 300\ \text{m}^3$

(b) Luas dasar kolam × kedalaman kolam = isi padu kolam

Luas dasar kolam $= \frac{300}{1.2}$
$= 250\ \text{m}^2$

Kos mengecat dasar kolam $= \text{RM}250 \times 5$
$= \text{RM}1\ 250$

∴ Encik Razak tidak dapat mengecat keseluruhan dasar kolam renang itu kerana wang yang diperuntukkan adalah kurang daripada kos sebenar.