Jawapan

BAB 6 FUNGSI TRIGONOMETRI

Kuiz Pantas (Halaman 190)

1. $0° = 0$ rad	**4.** $120° = 120 \times \dfrac{\pi}{180}$ rad $= \dfrac{2}{3}\pi$ rad $= 2.094$ rad	**7.** $270° = 270 \times \dfrac{\pi}{180}$ rad $= \dfrac{3}{2}\pi$ rad $= 4.712$ rad
2. $45° = 45 \times \dfrac{\pi}{180}$ rad $= \dfrac{\pi}{4}$ rad $= 0.785$ rad	**5.** $180° = 180 \times \dfrac{\pi}{180}$ rad $= \pi$ rad $= 3.142$ rad	**8.** $300° = 300 \times \dfrac{\pi}{180}$ rad $= \dfrac{5}{3}\pi$ rad $= 5.236$ rad
3. $90° = 90 \times \dfrac{\pi}{180}$ rad $= \dfrac{\pi}{2}$ rad $= 1.571$ rad	**6.** $225° = 225 \times \dfrac{\pi}{180}$ rad $= \dfrac{5}{4}\pi$ rad $= 3.927$ rad	**9.** $360° = 360 \times \dfrac{\pi}{180}$ rad $= 2\pi$ rad $= 6.283$ rad

Aktiviti Penerokaan 1 (Halaman 191)

Sudut	Sukuan	Sudut	Sukuan	Sudut	Sukuan
$140°$	II	$1\,000°$	IV	$-550°$	II
$\dfrac{7}{6}\pi$ rad	III	$\dfrac{13}{2}\pi$ rad	–	$-\dfrac{16}{3}\pi$ rad	II
$500°$	II	$-135°$	III	$-850°$	III
$\dfrac{11}{6}\pi$ rad	IV	$-\dfrac{5}{6}\pi$ rad	III	$-\dfrac{27}{8}\pi$ rad	I

Latihan Kendiri 6.1

1. (a) $290°\ 10' = 290.17 \times \dfrac{\pi}{180}$
$= 5.064$ rad

(b) $-359.4° = -359.4° \times \dfrac{\pi}{180}$
$= -6.273$ rad

(c) $620° = 620 \times \dfrac{\pi}{180}$
$= 10.82$ rad

(d) $-790° = -790° \times \dfrac{\pi}{180}$
$= -13.79$ rad

2. (a) 1.3 rad $= 1.3 \times \dfrac{180}{\pi}$
$= 74.48°$

(b) $\dfrac{13}{4}$ rad $= \dfrac{13}{4} \times \dfrac{180}{\pi}$
$= 186.21°$

(c) -2.7π rad $= -2.7\pi \times \dfrac{180}{\pi}$
$= -486°$

(d) $\dfrac{13}{4}\pi$ rad $= \dfrac{13}{4}\pi \times \dfrac{180}{\pi}$
$= 585°$

3. (a) $75°$ dalam sukuan I

(b) $-340.5°$ dalam sukuan 1
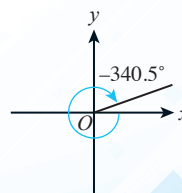
(c) $550° = 550° − 360°$
   $= 190°$
   $550°$ dalam sukuan III

(d) $−735° = −735° − 2(−360°)$
   $= −15°$
   $−735°$ dalam sukuan IV

(e) 0.36 rad $= 20.63°$
   0.36 rad dalam sukuan I
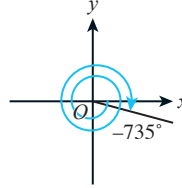
(f) $−4$ rad $= −229.18°$
   $−4$ rad dalam sukuan II

(g) $\frac{5}{3}\pi$ rad $= 300°$
   $\frac{5}{3}\pi$ rad dalam sukuan IV
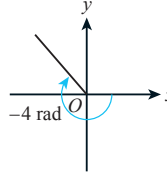
(h) $−1\,200° − 3(−360°) = −120°$
   $−\frac{20}{3}\pi$ rad dalam sukuan III
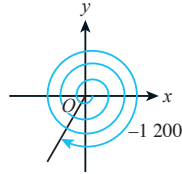
Latihan Formatif 6.1

1. $0° = 0$ rad

   $30° = 30 \times \frac{\pi}{180} = \frac{\pi}{6} = 0.5236$ rad

   $90° = 90 \times \frac{\pi}{180} = \frac{\pi}{2} = 1.571$ rad

   $150° = 150 \times \frac{\pi}{180} = \frac{5\pi}{6} = 2.618$ rad

   $210° = 210 \times \frac{\pi}{180} = \frac{7\pi}{6} = 3.665$ rad

   $270° = 270 \times \frac{\pi}{180} = \frac{3\pi}{2} = 4.712$ rad

   $330° = 330 \times \frac{\pi}{180} = \frac{11\pi}{6} = 5.7596$ rad

   $360° = 360 \times \frac{\pi}{180} = 2\pi = 6.283$ rad

Aktiviti Penerokaan 2 (Halaman 194)

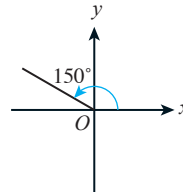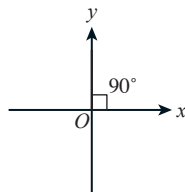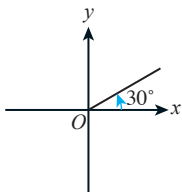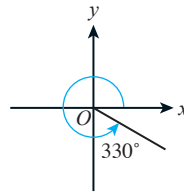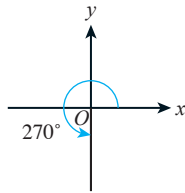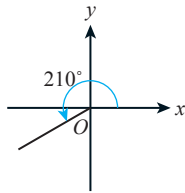| Lajur A	Lajur B
$\sin \theta = \dfrac{y}{\sqrt{x^2 + y^2}}$	$\sin(90° - \theta) = \dfrac{x}{\sqrt{x^2 + y^2}}$
$\kos \theta = \dfrac{x}{\sqrt{x^2 + y^2}}$	$\kos(90° - \theta) = \dfrac{y}{\sqrt{x^2 + y^2}}$
$\tan \theta = \dfrac{y}{x}$	$\tan(90° - \theta) = \dfrac{x}{y}$
$\kot \theta = \dfrac{x}{y}$	$\kot(90° - \theta) = \dfrac{y}{x}$
$\sek \theta = \dfrac{\sqrt{x^2 + y^2}}{x}$	$\sek(90° - \theta) = \dfrac{\sqrt{x^2 + y^2}}{y}$
$\kosek \theta = \dfrac{\sqrt{x^2 + y^2}}{y}$	$\kosek(90° - \theta) = \dfrac{\sqrt{x^2 + y^2}}{x}$

Latihan Kendiri 6.2

1. $QR = \sqrt{25 - 2} = \sqrt{23}$

(a) $\kot R = \dfrac{\sqrt{23}}{\sqrt{2}} = \sqrt{\dfrac{23}{2}}$

(b) $\sin^2 R = \left(\dfrac{\sqrt{2}}{5}\right)^2 = \dfrac{2}{25}$

(c) $\dfrac{\kos R - \sin R}{\kosek R} = \dfrac{\dfrac{\sqrt{23}}{5} - \dfrac{\sqrt{2}}{5}}{\dfrac{5}{\sqrt{2}}} = \dfrac{\dfrac{\sqrt{23} - \sqrt{2}}{5}}{\dfrac{5}{\sqrt{2}}}$

$= \left(\dfrac{\sqrt{23} - \sqrt{2}}{5}\right) \times \dfrac{\sqrt{2}}{5}$

$= \dfrac{\sqrt{46} - 2}{25}$

2. (a) $\sin \alpha = \dfrac{2}{\sqrt{13}}$

(b) $\kos^2 \alpha = \left(\dfrac{3}{\sqrt{13}}\right)^2 = \dfrac{9}{13}$

(c) $\kot \alpha = \dfrac{3}{2}$

(d) $\kosek \alpha = \dfrac{\sqrt{13}}{2}$

(e) $\dfrac{4 - \sek^2 \alpha}{2 - \sek \alpha} = \dfrac{4 - \left(\dfrac{\sqrt{13}}{3}\right)^2}{2 - \left(\dfrac{\sqrt{13}}{3}\right)}$

$= \left(\dfrac{36 - 13}{9}\right) \times \dfrac{3}{6 - \sqrt{13}}$

$= \dfrac{23}{3\left(6 - \sqrt{13}\right)}$

3. (a) $90° - 54° = 36°$

(b) $90° - 5° \, 17' \, 14'' = 84° \, 42' \, 46''$

(c) $\dfrac{\pi}{2} - \dfrac{\pi}{5} = \dfrac{5\pi - 2\pi}{10}$

$\qquad\qquad = \dfrac{3\pi}{10}$

4. (a) $\sin 57° = \kos (90° - 57°)$

$\qquad\qquad = \kos 33°$

$\qquad\qquad = 0.839$

(b) $\tan 57° = \kot (90° - 57°)$

$\qquad\qquad = \kot 33°$

$\qquad\qquad = \dfrac{\kos 33°}{\sin 33°}$

$\qquad\qquad = \dfrac{0.839}{0.545}$

$\qquad\qquad = 1.539$

(c) $\sek 57° = \dfrac{1}{\kos 57°}$

$\qquad\qquad = \dfrac{1}{\sin (90° - 57°)}$

$\qquad\qquad = \dfrac{1}{\sin 33°}$

$\qquad\qquad = \dfrac{1}{0.545}$

$\qquad\qquad = 1.835$

Perbincangan (Halaman 196)

Contoh: Cari nilai sin 2 rad.

Kaedah Asas: Tukarkan sudut ke unit darjah.

$$\sin 2 \text{ rad} = \sin \left(2 \times \dfrac{180}{\pi}\right)°$$

$$= \sin 114.59°$$

$$= 0.9093$$

Penggunaan kalkulator bergantung pada model kalkulator yang digunakan.

Kuiz Pantas (Halaman 198)

sin $(-A)$	$-\sin (A)$
kos $(-A)$	kos A
tan $(-A)$	$-\tan A$
kot $(-A)$	$-\kot A$
sek $(-A)$	sek A
kosek $(-A)$	$-\kosek A$

Aktiviti Penerokaan 3 (Halaman 198)

1.
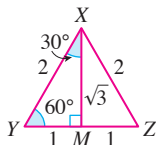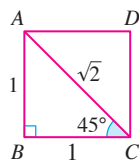
2. (a) $AC = \sqrt{2}$ (b) $YM = 1$ (c) $XM = \sqrt{3}$

    (d) $\angle ACB = 45°$ (e) $\angle XYZ = 60°$ (f) $\angle MXY = 30°$

3.

Sudut / Nisbah		sin	kos	tan	kosek	sek	kot
30°	$\dfrac{\pi}{6}$	$\dfrac{1}{2}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{3}}$	2	$\dfrac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\dfrac{\pi}{4}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\dfrac{\pi}{3}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{2}$	$\sqrt{3}$	$\dfrac{2}{\sqrt{3}}$	2	$\dfrac{1}{\sqrt{3}}$

Latihan Kendiri 6.3

1. (a) $\tan 165.7° = -0.2549$

(b) $\text{kot}\,(-555°) = \dfrac{1}{\tan\,(-555°)}$

$\qquad\qquad\quad = -3.7321$

(c) $\text{kosek}^2\,(-1.2\text{ rad}) = \dfrac{1}{\sin^2\,(-1.2\text{ rad})}$

$\qquad\qquad\qquad\quad = 1.1511$

(d) $\text{sek}\left(-\dfrac{16}{9}\pi\right) = \dfrac{1}{\text{kos}\left(-\dfrac{16}{9}\pi\right)}$

$\qquad\qquad\quad = 1.3054$

2. (a) $\sin 330° = \text{koordinat-}y$

$\qquad\qquad = -\dfrac{1}{2}$

(b) $\tan\left(\dfrac{2}{3}\pi\right) = \tan 120°$

$\qquad\qquad = -\dfrac{\text{koordinat-}y}{\text{koordinat-}x}$

$\qquad\qquad = -\dfrac{\left(\dfrac{\sqrt{3}}{2}\right)}{\left(\dfrac{1}{2}\right)}$

$\qquad\qquad = -\sqrt{3}$

(c) $\text{kot}\left(\dfrac{7}{6}\pi\right) = \text{kot } 210°$

$\qquad\qquad = -\dfrac{\text{koordinat-}x}{\text{koordinat-}y}$

$\qquad\qquad = \left(-\dfrac{\sqrt{3}}{2}\right) \times (-2)$

$\qquad\qquad = \sqrt{3}$

(d) $\text{kos } 600° = \text{kos }(600° - 360°)$

$\qquad\qquad = \text{kos } 240°$

$\qquad\qquad = \text{koordinat-}x$

$\qquad\qquad = -\dfrac{1}{2}$

(e) $\text{kosek}\left(-\dfrac{7}{2}\pi\right) = \text{kosek }(-630°)$

$\qquad\qquad = \text{kosek }(-270°)$

$\qquad\qquad = \dfrac{1}{\sin\,(-270°)}$

$\qquad\qquad = \dfrac{1}{\text{koordinat-}y}$

$\qquad\qquad = 1$

(f) $\sin\left(\dfrac{\pi}{2}\right) - (\text{sek } 3\pi) = \sin 90° - \text{sek } 540°$

$\qquad\qquad\qquad\qquad = \sin 90° - \text{sek } 540°$

$\qquad\qquad\qquad\qquad = \sin 90° - \text{sek } (540° - 360°)$

$\qquad\qquad\qquad\qquad = \sin 90° - \dfrac{1}{\text{kos } 180°}$

$\qquad\qquad\qquad\qquad = \text{koordinat-}y - \dfrac{1}{\text{koordinat-}x}$

$\qquad\qquad\qquad\qquad = 1 - (-1)$

$\qquad\qquad\qquad\qquad = 2$

3. (a) $360° - 335° = 25°$

Sudut tirus sepadan $= 25°$

(b) $\pi - \left(\dfrac{2\pi}{3}\right) = \dfrac{\pi}{3}$

Sudut tirus sepadan $= \dfrac{\pi}{3}$

(c) $\left(\dfrac{7\pi}{3}\right) - 2\pi = \dfrac{\pi}{3}$

Sudut tirus sepadan $= \dfrac{\pi}{3}$

(d) $710° - 360° = 350°$

Sudut tirus sepadan $= 360° - 350°$

$\qquad\qquad\qquad\qquad = 10°$

4. (a) $\text{sek } 150° = -\text{sek } 30°$

$\qquad\qquad = -\dfrac{2}{\sqrt{3}}$

(b) $\text{kosek } 240° = -\text{kosek } (240° - 180°)$

$\qquad\qquad\quad = -\text{kosek } 60°$

$\qquad\qquad\quad = -\dfrac{2}{\sqrt{3}}$

(c) $\text{kot } 315° = -\text{kot } (360° - 315°)$

$\qquad\qquad = -\text{kot } 45°$

$\qquad\qquad = -1$

(d) $\sin 45° + \text{kos } 225° = \dfrac{1}{\sqrt{2}} + (-\text{kos } (225° - 180°))$

$\qquad\qquad\qquad\qquad = \dfrac{1}{\sqrt{2}} + (-\text{kos } 45°)$

$\qquad\qquad\qquad\qquad = \dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}$

$\qquad\qquad\qquad\qquad = 0$

(e) $\text{sek } 60° + 2\text{ kosek } 30° = 2 + 2(2)$

$\qquad\qquad\qquad\qquad\qquad = 6$

(f) $\text{sek } \pi + \text{kos } \dfrac{\pi}{2} = -1 + 0$

$\qquad\qquad\qquad\quad = -1$

Latihan Formatif 6.2

1.

(a) $\text{kot } x = \dfrac{1}{3t}$

(b) $\text{sek } (90° - x) = \text{kosek } x$

$\qquad\qquad\qquad = \dfrac{\sqrt{1 + 9t^2}}{3t}$

(c) kosek $(180° − x)$ = kosek x

$$= \frac{\sqrt{1 + 9t^2}}{3t}$$

2.

(a) kot $\theta = \dfrac{1}{3}$

(b) tan $(\pi + \theta)$ = tan θ
  $= 3$

(c) sin $(−\theta)$ = −sin θ
  $= -\left(-\dfrac{3}{\sqrt{10}}\right)$
  $= \dfrac{3}{\sqrt{10}}$

3. (a) 2 sin 45° + kos 585° = $2\left(\dfrac{1}{\sqrt{2}}\right)$ + kos (585° − 360°)

$$= \frac{2}{\sqrt{2}} + \text{kos } 225°$$

$$= \frac{2}{\sqrt{2}} + \left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}} \text{ atau } \frac{\sqrt{2}}{2}$$

(b) tan 210° − kot (−240°) = tan 30° − (−kot 60°)

$$= \frac{1}{\sqrt{3}} - \left(-\frac{1}{\sqrt{3}}\right)$$

$$= \frac{2}{\sqrt{3}}$$

(c) kosek $\dfrac{5}{6}\pi$ + sin $\dfrac{1}{6}\pi$ = kosek 150° + sin 30°

$$= \text{kosek } 30° + \frac{1}{2}$$

$$= 2 + \frac{1}{2}$$

$$= \frac{5}{2}$$

(d) tan 2π − 6 kosek $\dfrac{3}{2}\pi$ = tan 360° − 6 kosek 270°
  $= 0 − 6(−1)$
  $= 6$

4. (a) sin 137° = sin (180° − 137°)
  $= \text{sin } 43°$
  $= 0.6820$

(b) sek 24° = $\dfrac{1}{\text{kos } 24°}$
  $= 1.095$

(c) tan 224° = tan (224° − 180°)
  $= \text{tan } 44°$
  $= 0.9656$

(d) kot 15° = kot (180° + 15°)
  $= \text{kot } 195°$
  $= 3.732$

5. (a) $\sin 135° = \dfrac{\sqrt{2}}{2}$

(b) $\sek 135° = \dfrac{1}{\kos 135°}$

$\qquad = -\dfrac{2}{\sqrt{2}}$

$\qquad = -\sqrt{2}$

(c) $\kot 45° = 1$

(d) $\kosek (-45°) = -\sqrt{2}$

Aktiviti Penerokaan 4 (Halaman 201)

2.

$x°$	0°	30°	60°	90°	120°	150°	180°	210°	240°	270⁰	300°	330°	360°
x radian	0	$\dfrac{\pi}{6}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$	$\dfrac{2\pi}{3}$	$\dfrac{5\pi}{6}$	π	$\dfrac{7}{6}\pi$	$\dfrac{4}{3}\pi$	$\dfrac{3}{2}\pi$	$\dfrac{5}{3}\pi$	$\dfrac{11}{6}\pi$	2π
$y = \sin x$	0	0.5	0.87	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0
$y = \kos x$	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0	0.5	0.87	1
$y = \tan x$	0	0.58	1.73	∞	−1.73	−0.58	0	0.38	1.73	∞	−1.73	−0.58	0

3. Kumpulan I: $y = \sin x$ untuk $0° \leqslant x \leqslant 360°$ atau $0 \leqslant x \leqslant 2\pi$.
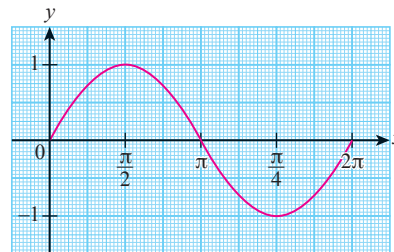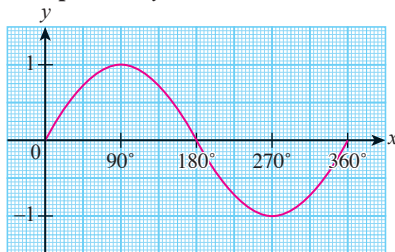
Kumpulan II: $y = \kos x$ untuk $0° \leqslant x \leqslant 360°$ atau $0 \leqslant x \leqslant 2\pi$.
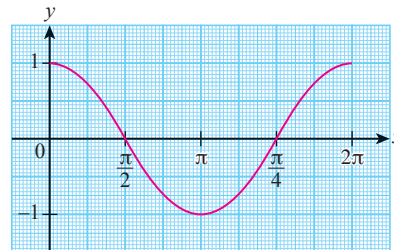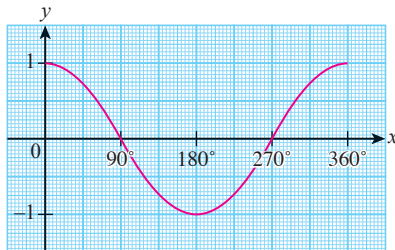
Kumpulan III: $y = \tan x$ untuk $0° \leqslant x \leqslant 360°$ atau $0 \leqslant x \leqslant 2\pi$.
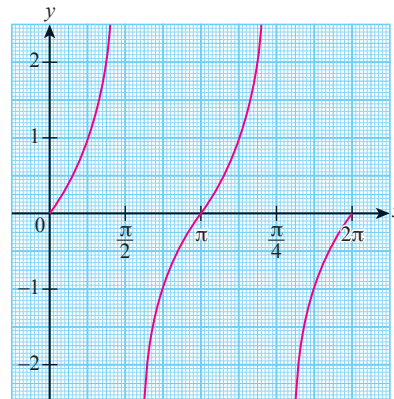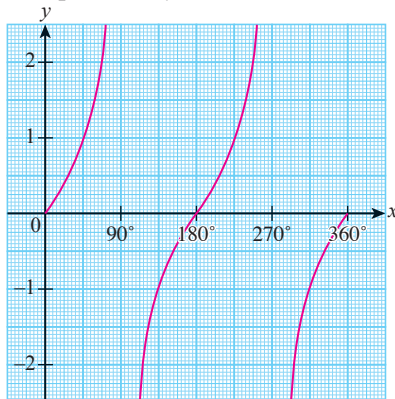
4.

	Pintasan-y	Pintasan-x	Nilai maksimum bagi y	Nilai minimum bagi y	Amplitud	Satu kala
I $y = \sin x$	0	0, 180°, 360°	1	−1	1	360°
II $y = \kos x$	1	90°, 270°	1	−1	1	360°
III $y = \tan x$	0	0, 180°, 360°	Tiada	Tiada	Tiada	180°

Perbincangan (Halaman 202)

Amplitud: Jarak dari titik maksimum atau titik minimum graf daripada paksi-x atau garis keseimbangan.

Kala: Panjang mengufuk yang sepadan dengan satu kitaran graf fungsi trigonometri.

Kitaran: Satu putaran lengkap satu graf fungsi trigonometri.

Asimptot: Garis lurus yang makin didekati oleh suatu lengkungan tetapi tidak pernah dipotong.

Aktiviti Penerokaan 5 (Halaman 203)

1.

$x°$	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
x rad	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7}{6}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{11}{6}\pi$	2π
$y = \sin x$	0	0.5	0.87	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0
$y = 3 \sin x$	0	1.5	2.6	3	2.6	1.5	0	−1.5	−2.6	−3	−2.6	−1.5	0
$y = 3 \sin 2x$	0	2.6	2.6	0	−2.6	−2.6	0	2.6	2.6	0	−2.6	−2.6	0
$y = 3 \sin 2x + 1$	1	3.6	3.6	1	−1.6	−1.6	1	3.6	3.6	1	−1.6	−1.6	1

2. (a) $y = \sin x$ dan $y = 3 \sin x$ untuk $0° \leqslant x \leqslant 360°$ atau $0 \leqslant x \leqslant 2\pi$.
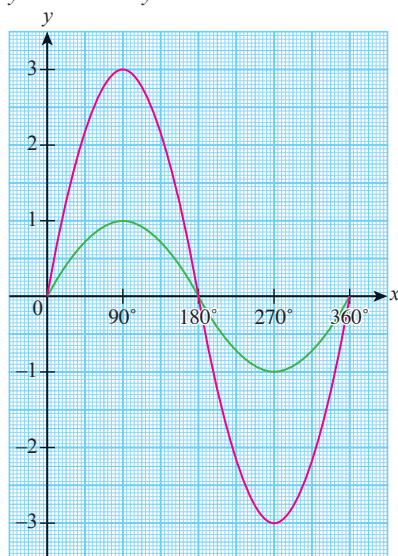
(b) $y = \sin x$ dan $y = 3 \sin 2x$ untuk $0° \leq x \leq 360°$ atau $0 \leq x \leq 2\pi$.
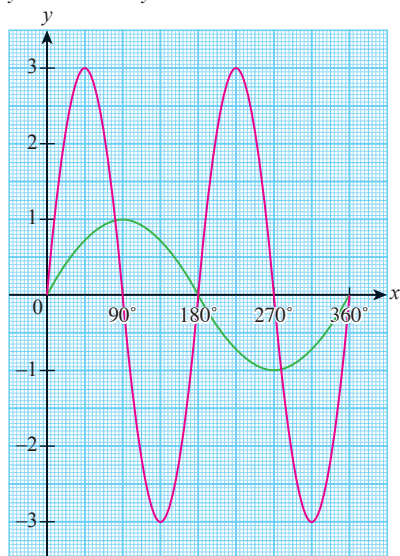
(c) $y = \sin x$ dan $y = 3 \sin 2x + 1$ untuk $0° \leq x \leq 360°$ atau $0 \leq x \leq 2\pi$.
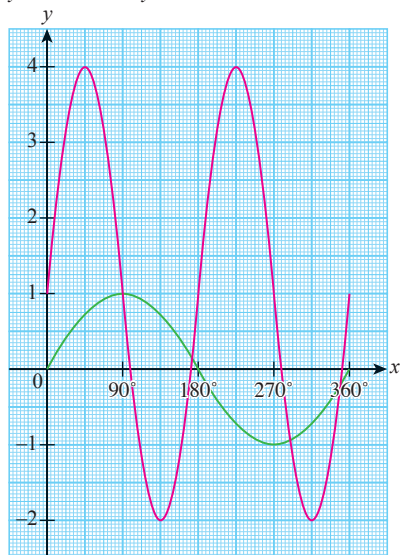
Latihan Kendiri 6.4

1. (a) $y = 1 - 3 \sin 2x$ untuk $-90° \leq x \leq 180°$.
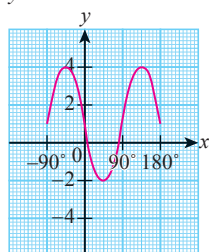
(b) $f(x) = -\left|\tan 2x\right| + 1$ untuk $0 \leqslant x \leqslant \pi$.
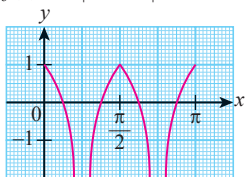
2. (a) $y = \tan x + 3, 0 \leqslant x \leqslant 2\pi$

(b) $y = 2 \kos 3x - 1, 0° \leqslant x \leqslant 360°$

3. (a) $A =$ amplitud $= 3$

$\dfrac{360°}{B} = 90°, B = 4$

Nilai minimum $= -3 + C = -2$

$\qquad\qquad\qquad C = 1$

(b) Graf bagi $y = 3 \sin 4x + 1$
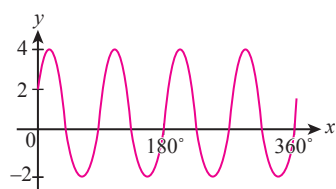
4.

Fungsi	Amplitud	Bilangan Kitaran/ Kala	Translasi	Lakaran Graf
1. $y = \dfrac{3}{2} \sin 3x$	$\dfrac{3}{2}$	3	0	
2. $y = \left|\tan 2x\right| + 1$	Tiada	4	1	

Latihan Kendiri 6.5

1. (a) (i) $y = \dfrac{1}{2} \sin 2x, 0° \leqslant x \leqslant 360°$
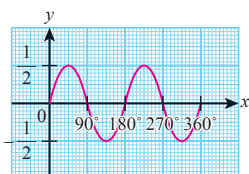
(ii) $y = 2 - \text{kos } x, 0° \leqslant x \leqslant 360°$
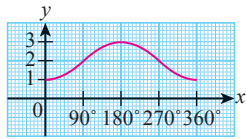
(iii) $y = -\tan 2x + 1, 0° \leqslant x \leqslant 360°$
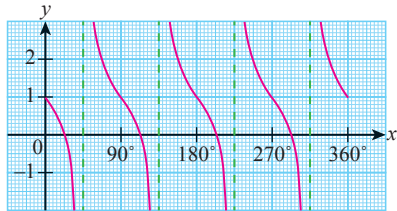
(b) (i) $y = 3 \text{ kos } 2x, 0 \leqslant x \leqslant 2\pi$
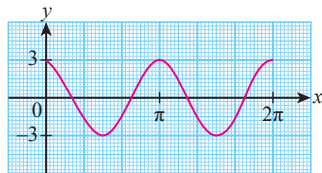
(ii) $y = -3 \sin x + 2, 0 \leqslant x \leqslant 2\pi$
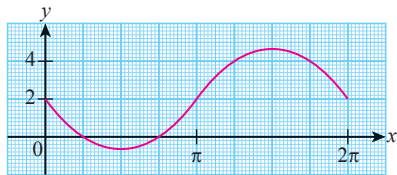
(iii) $y = |\tan x| - 1, 0 \leqslant x \leqslant 2\pi$
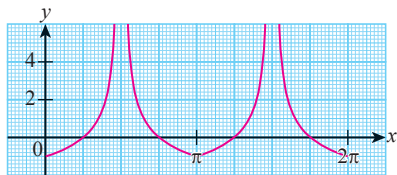
2. $y = -2 |\sin 2x| + 1$ bagi $0 \leqslant x \leqslant 2\pi$.
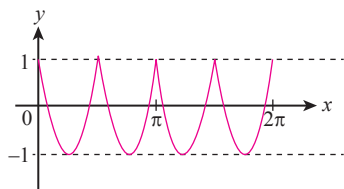
3. Lakar graf $y = \frac{3}{2}$ kos $3x$ dan $y = \frac{x}{\pi} + 1$ bagi $0 \leqslant x \leqslant \frac{\pi}{2}$.

3 kos $2x = \frac{2x}{\pi} + 2$
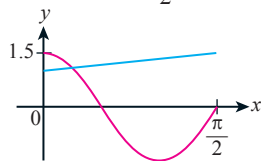
$\frac{3}{2}$ kos $3x = \frac{x}{\pi} + 1$

x	0	$\frac{\pi}{2}$
y	1	$\frac{3}{2}$

Bilangan penyelesaian = 1

4. $x - 2\pi \left| \text{kos } 2x \right| = 0$

$x = 2\pi \left| \text{kos } 2x \right|$

$\frac{x}{2\pi} = \left| \text{kos } 2x \right|$

Lakarkan dalam julat $0 \leqslant x \leqslant \pi$.

$y = \left| \text{kos } 2x \right|$ dan $y = \frac{x}{2\pi}$

x	0	π
y	1	$\frac{1}{2}$
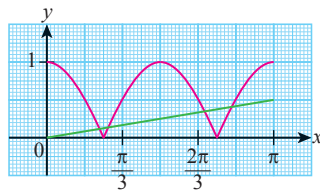
Bilangan penyelesaian = 4

Latihan Formatif 6.3

1. kos $\frac{\pi}{2}x + \frac{1}{4} = 0$

kos $\frac{\pi}{2}x = -\frac{1}{4}$

Lukis graf $y = $ kos $\frac{\pi}{2}x$ dan $y = -\frac{1}{4}$.

Daripada graf, $x = 1.0$ dan 3.0.
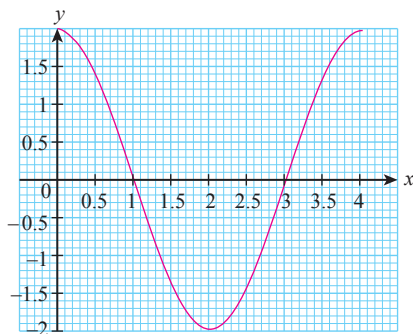
2. $30 \tan x - 6x + 5\pi = 0$

$$30 \tan x = 6x - 5\pi$$

$\div 6:$ $\quad 5 \tan x = x - \dfrac{5\pi}{6}$

$y = x - \dfrac{5\pi}{6}$

x	0	$\dfrac{5\pi}{6}$
y	-2.62	0

$x = \dfrac{63\pi}{60}$ atau 3.30 rad
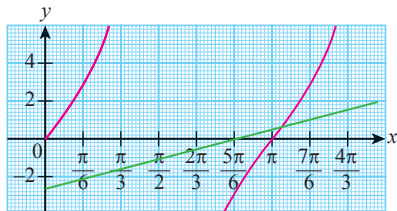
3.
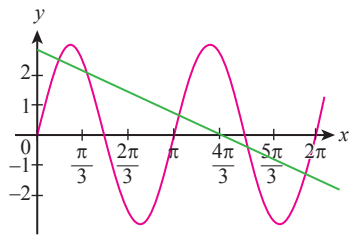
$$3\pi \sin 2x + 2x = 3\pi$$
$$3\pi \sin 2x = 3\pi - 2x$$
$$3 \sin 2x = 3 - \dfrac{2x}{\pi}$$

$y = 3 - \dfrac{2x}{\pi}$

x	0	π
y	3	1

Bilangan penyelesaian $= 5$

4.

$x - 2\pi \left| \text{kos } 2x \right| = 0$

$\quad 2\pi \left| \text{kos } 2x \right| = x$

$\qquad \left| \text{kos } 2x \right| = \dfrac{x}{2\pi}$

$y = \dfrac{x}{2\pi}$
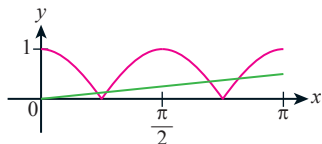
x	0	π
y	0	0.5

Bilangan penyelesaian $= 4$

5.
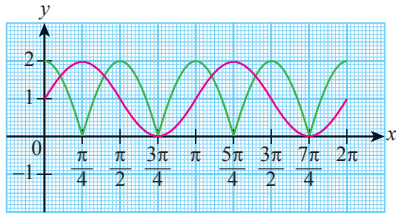
Titik persilangan: $(0.322, 1.6)$, $(1.249, 1.6)$, $\left(\frac{3\pi}{4}, 0\right)$, $(3.463, 1.6)$, $(4.391, 1.6)$, $\left(\frac{7\pi}{4}, 0\right)$

6.

$|kos\ x| = k - 3$
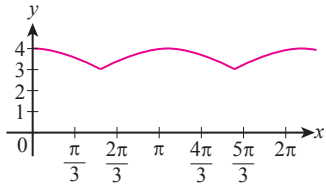
$3 + |kos\ x| = k$

Tiada punca nyata jika $k < 3$ dan $k > 4$.

7. (a) $y = -2\ kos\ \frac{3x}{2}$ bagi $0 \leqslant x \leqslant 2\pi$.

(b)

$2\ kos\ \frac{3x}{2} + \frac{\pi}{2x} = 0$

$-2\ kos\ \frac{3x}{2} = \frac{\pi}{2x}$
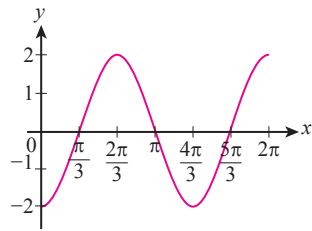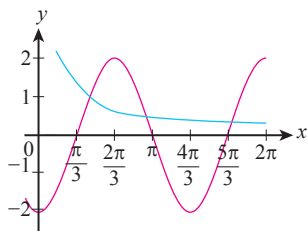
Lukis $y = \frac{\pi}{2x}$

x	0	π	2π
y	∞	0.5	0.25

Bilangan penyelesaian $= 3$

Aktiviti Penerokaan 6 (Halaman 211)

Kumpulan 1

(a)

Nisbah trigonometri	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\text{kosek}\,\theta$	$\text{sek}\,\theta$	$\text{kot}\,\theta$
Nilai	$\dfrac{m}{p}$	$\dfrac{n}{p}$	$\dfrac{m}{n}$	$\dfrac{p}{m}$	$\dfrac{p}{n}$	$\dfrac{n}{m}$

Nisbah trigonometri	$\sin^2\theta$	$\cos^2\theta$	$\tan^2\theta$	$\text{kosek}^2\,\theta$	$\text{sek}^2\,\theta$	$\text{kot}^2\,\theta$
Nilai	$\dfrac{m^2}{p^2}$	$\dfrac{n^2}{p^2}$	$\dfrac{m^2}{n^2}$	$\dfrac{p^2}{m^2}$	$\dfrac{p^2}{n^2}$	$\dfrac{n^2}{m^2}$

(b)

$m^2 + n^2 = p^2$ $\div m^2:\ \dfrac{m^2}{p^2} + \dfrac{n^2}{p^2} = \dfrac{p^2}{p^2}$ $\sin^2\theta + \cos^2\theta = 1$	$m^2 + n^2 = p^2$ $\div n^2:\ \dfrac{m^2}{n^2} + \dfrac{n^2}{n^2} = \dfrac{p^2}{n^2}$ $\tan^2\theta + 1 = \text{sek}^2\theta$	$m^2 + n^2 = p^2$ $\div m^2:\ \dfrac{m^2}{m^2} + \dfrac{n^2}{m^2} = \dfrac{p^2}{m^2}$ $1 + \text{kot}^2\theta = \text{kosek}^2\theta$

Kumpulan 2

(a) $x = \cos\theta$
$\quad y = \sin\theta$

Nilai	x^2	y^2	$\dfrac{n^2}{m^2}$	$\dfrac{1}{y^2}$	$\dfrac{1}{x^2}$	$\dfrac{y^2}{x^2}$
Nisbah trigonometri	$\sin^2\theta$	$\cos^2\theta$	$\tan^2\theta$	$\text{kosek}^2\,\theta$	$\text{sek}^2\,\theta$	$\text{kot}^2\,\theta$

(b)

$x^2 + y^2 = 1$ $\cos^2\theta + \sin^2\theta = 1$	$x^2 + y^2 = 1$ $\div x^2:\ \dfrac{x^2}{x^2} + \dfrac{y^2}{x^2} = \dfrac{1}{x^2}$ $1 + \tan^2\theta = \text{sek}^2\theta$	$x^2 + y^2 = 1$ $\div y^2:\ \dfrac{x^2}{y^2} + \dfrac{y^2}{y^2} = \dfrac{1}{y^2}$ $\text{kot}^2\theta + 1 = \text{kosek}^2\theta$

Latihan Kendiri 6.6

1. (a) $\cos^2 80° + \sin^2 80° = 1$
   (b) $\text{sek}^2 173° - \tan^2 173° = 1$
   (c) $1 - \cos^2 45° = \sin^2 45°$
   $$= \left(\frac{\sqrt{2}}{2}\right)^2$$
   $$= \frac{2}{4}$$
   $$= \frac{1}{2}$$
   (d) $\text{kosek}^2 \dfrac{8}{5}\pi - \text{kot}^2 \dfrac{8}{5}\pi = 1$

2. (a) $\text{sek}^2\theta = \dfrac{1}{\cos^2\theta}$
   $$= \frac{1}{m^2}$$
   (b) $\sin^2\theta + \cos^2\theta = 1$
   $$\sin^2\theta = 1 - \cos^2\theta$$
   $$\sin^2\theta = 1 - m^2$$

(c) $\text{kot}^2\,\theta = \text{kosek}^2\,\theta - 1$

$$= \frac{1}{\sin^2\theta} - 1$$

$$= \frac{1}{1 - m^2} - 1$$

$$= \frac{1 - (1 - m^2)}{1 - m^2}$$

$$= \frac{m^2}{1 - m^2}$$

3. $\text{kosek}^2\,\theta = 1 + \text{kot}^2\,\theta$

$$= 1 + \frac{1}{9}$$

$$= \frac{10}{9}$$

$\text{kosek}^2\,\theta = \dfrac{1}{\sin^2\theta}$

$\sin^2\theta = \dfrac{9}{10}$

$\sin\theta = \sqrt{\dfrac{9}{10}}$

$$= \frac{3}{\sqrt{10}}$$

$\text{sek}^2\,\theta = 1 + \tan^2\theta$

$$= 1 + 9$$

$$= 10$$

$\text{sek}^2\,\theta = \dfrac{1}{\text{kos}^2\,\theta}$

$\text{kos}^2\,\theta = \dfrac{1}{10}$

$\text{kos}\,\theta = \dfrac{1}{\sqrt{10}}$

4. (a) $\sin^2 A = \dfrac{p^2}{q^2}$

$1 - \text{kos}^2 A = \dfrac{p^2}{q^2}$

(b) $\text{kot}^2 A = \left(\sqrt{\dfrac{q^2 - p^2}{p}}\right)^2$

$\text{kosek}^2 A - 1 = \dfrac{q^2 - p^2}{p^2}$

(c) $-\tan^2 A = -\left(\dfrac{p}{\sqrt{q^2 - p^2}}\right)^2$

$1 - \text{sek}^2 A = -\dfrac{p^2}{q^2 - p^2}$
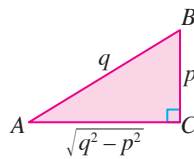
Latihan Kendiri 6.7

1. (a) Sebelah kiri $= 3\sin^2 A - 2$

$$= 3(1 - \text{kos}^2 A) - 2$$

$$= 3 - 3\text{kos}^2 A - 2$$

$$= 1 - 3\text{kos}^2 A$$

$$= \text{sebelah kanan}$$

(b) Sebelah kanan $= \dfrac{1 - \sin^4 A}{\kos^4 A}$

$\qquad\qquad = \dfrac{(1 + \sin^2 A)(1 - \sin^2 A)}{\kos^4 A}$

$\qquad\qquad = \dfrac{(1 + \sin^2 A)(\kos^2 A)}{(\kos^2 A)^2}$

$\qquad\qquad = \dfrac{1 + \sin^2 A}{\kos^2 A}$

$\qquad\qquad = \sek^2 A + \tan^2 A$

$\qquad\qquad = 1 + \tan^2 A + \tan^2 A$

$\qquad\qquad = 1 + 2\tan^2 A$

$\qquad\qquad = $ sebelah kiri

(c) Sebelah kiri $= \sek A \kosek A - \tan A$

$\qquad\qquad = \dfrac{1}{\kos A \sin A} - \dfrac{\sin A}{\kos A}$

$\qquad\qquad = \dfrac{1 - \sin^2 A}{\kos A \sin A}$

$\qquad\qquad = \dfrac{\kos^2 A}{\kos A \sin A}$

$\qquad\qquad = \dfrac{\kos A}{\sin A}$

$\qquad\qquad = \kot A$

$\qquad\qquad = $ sebelah kanan

(d) Sebelah kanan $= \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

$\qquad\qquad = \dfrac{1 - \left(\dfrac{\sin^2 A}{\kos^2 A}\right)}{\sek^2 A}$

$\qquad\qquad = \dfrac{\kos^2 A - \sin^2 A}{\kos^2 A} \times \dfrac{\kos^2 A}{1}$

$\qquad\qquad = \kos^2 A - \sin^2 A$

$\qquad\qquad = $ sebelah kiri

(e) Sebelah kiri $= \kot^2 \theta - \tan^2 \theta$

$\qquad\qquad = \dfrac{\kos^2 \theta}{\sin^2 \theta} - \dfrac{\sin^2 \theta}{\kos^2 \theta}$

$\qquad\qquad = \dfrac{\kos^4 \theta - \sin^4 \theta}{\sin^2 \theta \kos^2 \theta}$

$\qquad\qquad = \dfrac{(\kos^2 \theta + \sin^2 \theta)(\kos^2 \theta - \sin^2 \theta)}{\sin^2 \theta \kos^2 \theta}$

$\qquad\qquad = \dfrac{1(\kos^2 \theta - \sin^2 \theta)}{\sin^2 \theta \kos^2 \theta}$

$\qquad\qquad = \dfrac{\kos^2 \theta}{\sin^2 \theta \kos^2 \theta} - \dfrac{\sin^2 \theta}{\sin^2 \theta \kos^2 \theta}$

$\qquad\qquad = \kosek^2 \theta - \sek^2 \theta$

$\qquad\qquad = $ sebelah kanan

(f) Sebelah kiri $= \dfrac{\sin^2 \theta}{1 + \kos \theta}$

$\qquad\qquad = \dfrac{1 - \kos^2 \theta}{1 + \kos \theta}$

$\qquad\qquad = \dfrac{(1 + \kos \theta)(1 - \kos \theta)}{1 + \kos \theta}$

$\qquad\qquad = 1 - \kos \theta$

$\qquad\qquad = $ sebelah kanan

(g) Sebelah kiri $= \tan^2 \theta (\text{kosek}^2 \theta - 1)$

$= \tan^2 \theta (\text{kot}^2 \theta)$

$= \tan^2 \theta \left(\dfrac{1}{\tan^2 \theta} \right)$

$= 1$

$=$ sebelah kanan

(h) Sebelah kiri $= \dfrac{1 - 2 \sin^2 \theta}{\text{kos } \theta - \sin \theta}$

$= \dfrac{\sin^2 \theta + \text{kos}^2 \theta - 2 \sin^2 \theta}{\text{kos } \theta - \sin \theta}$

$= \dfrac{\text{kos}^2 \theta - \sin^2 \theta}{\text{kos } \theta - \sin \theta}$

$= \dfrac{(\text{kos } \theta - \sin \theta)(\text{kos } \theta + \sin \theta)}{\text{kos } \theta - \sin \theta}$

$= \text{kos } \theta + \sin \theta$

$=$ sebelah kanan

Latihan Formatif 6.4

1. (a) $\text{sek}^2 \theta = p$

$\tan^2 \theta = \text{sek}^2 \theta - 1$

$= p - 1$

(b) $\text{kos}^2 \theta = \dfrac{1}{\text{sek}^2 \theta}$

$= \dfrac{1}{p}$

(c) $\sin^2 \theta = 1 - \text{kos}^2 \theta$

$= 1 - \dfrac{1}{p}$

$= \dfrac{p - 1}{p}$

2. (a) $\sin^2 100° + \text{kos}^2 100° = 1$

(b) $\tan^2 3 \text{ rad} - \text{sek}^2 3 \text{ rad} = \tan^2 3 \text{ rad} - (1 + \tan^2 3 \text{ rad})$

$= -1$

(c) $1 + \tan^2 120° = \text{sek}^2 120°$

$= (-\text{sek } 60°)^2$

$= 4$

(d) $1 + \text{kot}^2 225° = \text{kosek}^2 225°$

$= (-\text{kosek } 45°)^2$

$= 2$

3. (a) Sebelah kiri $= \dfrac{\tan^2 x}{1 + \tan^2 x}$

$= \dfrac{\tan^2 x}{\text{sek}^2 x}$

$= \left(\dfrac{\sin^2 x}{\text{kos}^2 x} \right) \times \text{kos}^2 x$

$= \sin^2 x$

$=$ sebelah kanan

(b) Sebelah kanan $= 9 \text{ sek}^2 x - 4 \tan^2 x$

$= 9(1 + \tan^2 x) - 4 \tan^2 x$

$= 9 + 9 \tan^2 x - 4 \tan^2 x$

$= 9 + 5 \tan^2 x$

$= 9 + 5(\text{sek}^2 x - 1)$

$= 9 + 5 \text{ sek}^2 x - 5$

$= 5 \text{ sek}^2 x + 4$

$=$ sebelah kiri

(c) Sebelah kiri $= \dfrac{\sin \theta}{1 + \text{kos } \theta} + \dfrac{1 + \text{kos } \theta}{\sin \theta}$

$\qquad\qquad = \dfrac{\sin^2 \theta + (1 + \text{kos } \theta)^2}{\sin \theta(1 + \text{kos } \theta)}$

$\qquad\qquad = \dfrac{\sin^2 \theta + 1 + 2 \text{ kos } \theta + \text{kos}^2 \theta}{\sin \theta(1 + \text{kos } \theta)}$

$\qquad\qquad = \dfrac{1 + 1 + 2 \text{ kos } \theta}{\sin \theta(1 + \text{kos } \theta)}$

$\qquad\qquad = \dfrac{2 + 2 \text{ kos } \theta}{\sin \theta(1 + \text{kos } \theta)}$

$\qquad\qquad = \dfrac{2(1 + \text{kos } \theta)}{\sin \theta(1 + \text{kos } \theta)}$

$\qquad\qquad = \dfrac{2}{\sin \theta}$

$\qquad\qquad = 2 \text{ kosek } \theta$

$\qquad\qquad = \text{sebelah kanan}$

(d) Sebelah kiri $= \text{sek}^4 \theta - \text{sek}^2 \theta$

$\qquad\qquad = \text{sek}^2 \theta(\text{sek}^2 \theta - 1)$

$\qquad\qquad = (1 + \tan^2 \theta)(1 + \tan^2 \theta - 1)$

$\qquad\qquad = (1 + \tan^2 \theta)(\tan^2 \theta)$

$\qquad\qquad = \tan^4 \theta + \tan^2 \theta$

$\qquad\qquad = \text{sebelah kanan}$

4. (a) Sebelah kiri $= \dfrac{1}{1 + \text{kos } \theta} + \dfrac{1}{1 - \text{kos } \theta}$

$\qquad\qquad = \dfrac{(1 - \text{kos } \theta) + (1 + \text{kos } \theta)}{(1 + \text{kos } \theta)(1 - \text{kos } \theta)}$

$\qquad\qquad = \dfrac{2}{1 - \text{kos}^2 \theta}$

$\qquad\qquad = \dfrac{2}{\sin^2 \theta}$

$\qquad\qquad = 2 \text{ kosek}^2 \theta$

$\qquad\qquad = \text{sebelah kanan}$

(b) $2 \text{ kosek}^2 \theta = \dfrac{1}{1 + \text{kos } \theta} + \dfrac{1}{1 - \text{kos } \theta}$

$\qquad\qquad = \dfrac{1}{1 + 0.6} + \dfrac{1}{1 - 0.6}$

$\qquad\qquad = 3.125$

$\qquad \text{kosek}^2 \theta = 1.5625$

5. (a) $\sin y \tan y + \text{kos } y = \sin y\left(\dfrac{\sin y}{\text{kos } y}\right) + \text{kos } y$

$\qquad\qquad\qquad\qquad = \dfrac{\sin^2 y + \text{kos}^2 y}{\text{kos } y}$

$\qquad\qquad\qquad\qquad = \dfrac{1}{\text{kos } y}$

$\qquad\qquad\qquad\qquad = \text{sek } y$

(b) $\dfrac{\tan y + \text{kot } y}{\text{kosek } y} = \dfrac{\left(\dfrac{\sin y}{\text{kos } y} + \dfrac{\text{kos } y}{\sin y}\right)}{\dfrac{1}{\sin y}}$

$\qquad\qquad\qquad = \left(\dfrac{\sin^2 y + \text{kos}^2 y}{\text{kos } y \sin y}\right) \times \sin y$

$\qquad\qquad\qquad = \dfrac{\sin y}{\text{kos } y \sin y}$

$\qquad\qquad\qquad = \text{sek } y$

(c) $\dfrac{1 - \text{sin } y}{2 \text{ kos } y} + \dfrac{\text{kos } y}{2 - 2 \text{ sin } y} = \dfrac{(1 - \text{sin } y)^2(2) + 2(\text{kos}^2 \, y)}{2 \text{ kos } y(2 - 2 \text{ sin } y)}$

$\qquad\qquad\qquad\qquad\qquad = \dfrac{(1 - 2 \text{ sin } y + \text{sin}^2 \, y)(2) + 2 \text{ kos}^2 \, y}{2 \text{ kos } y(2 - 2 \text{ sin } y)}$

$\qquad\qquad\qquad\qquad\qquad = \dfrac{2 - 4 \text{ sin } y + 2 \text{ sin}^2 \, y + 2 \text{ kos}^2 \, y}{2 \text{ kos } y(2 - 2 \text{ sin } y)}$

$\qquad\qquad\qquad\qquad\qquad = \dfrac{4 - 4 \text{ sin } y}{2 \text{ kos } y(2 - 2 \text{ sin } y)}$

$\qquad\qquad\qquad\qquad\qquad = \dfrac{2(2 - 2 \text{ sin } y)}{2 \text{ kos } y(2 - 2 \text{ sin } y)}$

$\qquad\qquad\qquad\qquad\qquad = \text{sek } y$

Aktiviti Penerokaan 7 (Halaman 215)

A	B	sin (A + B)	sin A kos B	kos A sin B	sin A kos B + kos A sin B
10°	20°	sin 30° = 0.5	0.1632	0.3368	0.5
		Terima sebarang jawapan yang betul			

Kuiz Pantas (Halaman 217)

(a) $\text{kosek } (A + B) = \dfrac{1}{\text{sin } (A + B)}$

$\qquad\qquad\qquad\quad = -\dfrac{65}{63}$

(b) $\text{sek } (A - B) = \dfrac{1}{\text{kos } (A + B)}$

$\qquad\qquad\qquad = \dfrac{1}{\text{kos } A \text{ kos } B + \text{sin } A \text{ sin } B}$

$\qquad\qquad\qquad = \dfrac{1}{\left(\frac{4}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{2}{5}\right)\left(\frac{-12}{13}\right)}$

$\qquad\qquad\qquad = \dfrac{1}{\frac{-56}{65}}$

$\qquad\qquad\qquad = -\dfrac{65}{56}$

(c) $\text{kot } (B - A) = \dfrac{1}{\text{tan } (B - A)}$

$\qquad\qquad\qquad = \dfrac{56}{33}$

Latihan Kendiri 6.8

1. (a) Sebelah kiri $= \text{sin } (x - y) - \text{sin } (x + y)$

$\qquad\qquad\qquad = \text{sin } x \text{ kos } y - \text{kos } x \text{ sin } y - (\text{sin } x \text{ kos } y + \text{kos } x \text{ sin } y)$

$\qquad\qquad\qquad = -2 \text{ kos } x \text{ sin } y$

$\qquad\qquad\qquad = \text{sebelah kanan}$

(b) Sebelah kiri $= \tan \left(A + \dfrac{\pi}{4}\right)$

$\qquad\qquad\qquad = \dfrac{\tan A + \tan \frac{\pi}{4}}{1 - \tan A \tan \frac{\pi}{4}}$

$\qquad\qquad\qquad = \dfrac{1 + \tan A}{1 - \tan A}$

$\qquad\qquad\qquad = \text{sebelah kanan}$

(c) Sebelah kiri $= \dfrac{\text{kos } (x - y) - \text{kos } (x + y)}{\sin (x + y) + \sin (x - y)}$

$= \dfrac{\text{kos } x \text{ kos } y + \sin x \sin y - (\text{kos } x \text{ kos } y - \sin x \sin y)}{\sin x \text{ kos } y + \text{kos } x \sin y + (\sin x \text{ kos } y - \text{kos } x \sin y)}$

$= \dfrac{2 \sin x \sin y}{2 \sin x \text{ kos } y}$

$= \tan y$

$=$ sebelah kanan

(d) Sebelah kanan $= \dfrac{\text{kot } A \text{ kot } B + 1}{\text{kot } B - \text{kot } A}$

$= \dfrac{\left(\dfrac{\text{kos } A}{\sin A}\right)\left(\dfrac{\text{kos } B}{\sin B}\right) + 1}{\left(\dfrac{\text{kos } B}{\sin B}\right) - \left(\dfrac{\text{kos } A}{\sin A}\right)}$

$= \dfrac{\dfrac{\text{kos } A \text{ kos } B + \sin A \sin B}{\sin A \sin B}}{\dfrac{\sin A \text{ kos } B - \text{kos } A \sin B}{\sin A \sin B}}$

$= \dfrac{\text{kos } A \text{ kos } B + \sin A \sin B}{\sin A \text{ kos } B - \text{kos } A \sin B}$

$= \dfrac{\text{kos } (A - B)}{\sin (A - B)}$

$= \text{kot } (A - B)$

$=$ sebelah kiri

2. (a) kos $75° =$ kos $(30° + 45°)$

$=$ kos $30°$ kos $45° - \sin 30° \sin 45°$

$= \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{\sqrt{2}}{2}\right) - \left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{2}}{2}\right)$

$= \dfrac{\sqrt{6} - \sqrt{2}}{4}$

(b) kosek $105° = \dfrac{1}{\sin (60° + 45°)}$

$= \dfrac{1}{\sin 60° \text{ kos } 45° + \text{kos } 60° \sin 45°}$

$= \dfrac{1}{\left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{\sqrt{2}}{2}\right) + \left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{2}}{2}\right)}$

$= \dfrac{1}{\dfrac{\sqrt{6} + \sqrt{2}}{4}}$

$= \dfrac{4}{\sqrt{6} + \sqrt{2}}$

(c) kot $195° = \dfrac{1}{\tan (150° + 45°)}$

$= \dfrac{1}{\dfrac{\tan 150° + \tan 45°}{1 - \tan 150° \tan 45°}}$

$= \dfrac{1 - \tan 150°}{\tan 150° + 1}$

$= \dfrac{1 - (-\tan 30°)}{-\tan 30° + 1}$

$= \dfrac{1 + \left(\dfrac{1}{\sqrt{3}}\right)}{-\left(\dfrac{1}{\sqrt{3}}\right) + 1}$

$= \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$

3. $\text{kos } x = \frac{-5}{13}, 0 < x < \pi$

$\text{sin } y = -\frac{3}{5}, \frac{\pi}{2} < y < \frac{3}{2}\pi$
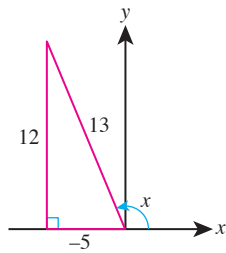
(a) $\sin (x + y) = \sin x \text{ kos } y + \text{kos } x \sin y$

$$= \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right)$$

$$= \frac{-48}{65} + \frac{15}{65}$$

$$= \frac{-33}{65}$$

(b) $\text{kos } (x - y) = \text{kos } x \text{ kos } y + \sin x \sin y$

$$= \left(\frac{-5}{13}\right)\left(\frac{-4}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{-3}{5}\right)$$

$$= \frac{20}{65} - \frac{36}{65}$$

$$= -\frac{16}{65}$$

(c) $\text{kot } (x + y) = \dfrac{1}{\tan (x + y)}$

$$= \dfrac{1}{\dfrac{\tan x + \tan y}{1 - \tan x \tan y}}$$

$$= \dfrac{1 - \tan x \tan y}{\tan x + \tan y}$$

$$= \dfrac{1 - \left(\dfrac{12}{-5}\right)\left(\dfrac{-3}{-4}\right)}{\dfrac{12}{-5} + \left(\dfrac{3}{4}\right)}$$

$$= \dfrac{1 + \left(\dfrac{36}{20}\right)}{\dfrac{33}{-20}}$$

$$= \left(\frac{20 + 36}{20}\right) \times \left(\frac{-20}{33}\right)$$

$$= -\frac{56}{33}$$

Latihan Kendiri 6.9

1. (a) $2 \sin 30° \text{ kos } 30° = \sin 2(30°)$

$$= \sin 60°$$

$$= \frac{\sqrt{3}}{2}$$

(b) $\text{kos}^2 165° - \sin^2 165° = \text{kos } 2(165°)$

$$= \text{kos } 330°$$

$$= \text{kos } 30°$$

$$= \frac{\sqrt{3}}{2}$$
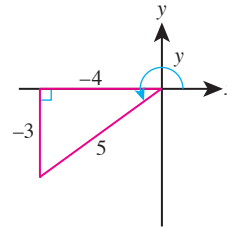
(c) $\dfrac{1 - \tan^2 75°}{2 \tan 75°} = \dfrac{1}{\tan 2(75°)}$

$\qquad\qquad\qquad = \dfrac{1}{\tan 150°}$

$\qquad\qquad\qquad = \dfrac{1}{-\tan 30°}$

$\qquad\qquad\qquad = -\sqrt{3}$

2. Sebelah kiri = kosek $2A$

$\qquad\qquad = \dfrac{1}{\sin 2A}$

$\qquad\qquad = \dfrac{1}{2 \sin A \kos A}$

$\qquad\qquad = \dfrac{1}{2} \sek A \, \text{kosek } A$

$\qquad\qquad$ = sebelah kanan

3. (a) Sebelah kiri = $\sin 2\theta(\tan \theta + \kot \theta)$

$\qquad\qquad\quad = 2 \sin \theta \kos \theta \left(\dfrac{\sin \theta}{\kos \theta} + \dfrac{\kos \theta}{\sin \theta} \right)$

$\qquad\qquad\quad = 2 \sin \theta \kos \theta \left(\dfrac{\sin^2 \theta + \kos^2 \theta}{\kos \theta \sin \theta} \right)$

$\qquad\qquad\quad = 2$

$\qquad\qquad\quad$ = sebelah kanan

(b) Sebelah kiri = $\dfrac{\sin 4x + \sin 2x}{\kos 4x + \kos 2x + 1}$

$\qquad\qquad\quad = \dfrac{2 \sin 2x \kos 2x + \sin 2x}{2 \kos^2 2x - 1 + \kos 2x + 1}$

$\qquad\qquad\quad = \dfrac{\sin 2x \, (2 \kos 2x + 1)}{\kos 2x \, (2 \kos 2x + 1)}$

$\qquad\qquad\quad = \tan 2x$

$\qquad\qquad\quad$ = sebelah kanan

(c) Sebelah kiri = kosek $2A$ + kot $2A$

$\qquad\qquad\quad = \dfrac{1}{\sin 2A} + \dfrac{\kos 2A}{\sin 2A}$

$\qquad\qquad\quad = \dfrac{1 + \kos 2A}{\sin 2A}$

$\qquad\qquad\quad = \dfrac{1 + 2 \kos^2 A - 1}{2 \sin A \kos A}$

$\qquad\qquad\quad = \dfrac{2 \kos^2 A}{2 \sin A \kos A}$

$\qquad\qquad\quad = \kot A$

(d) Sebelah kanan = $\dfrac{\kot \, x + \tan x}{\kot x - \tan x}$

$\qquad\qquad\quad = \dfrac{\dfrac{\kos x}{\sin x} + \dfrac{\sin x}{\kos x}}{\dfrac{\kos x}{\sin x} - \dfrac{\sin x}{\kos x}}$

$\qquad\qquad\quad = \dfrac{\kos^2 x + \sin^2 x}{\kos^2 x - \sin^2 x}$

$\qquad\qquad\quad = \dfrac{1}{\kos 2x}$

$\qquad\qquad\quad = \sek 2x$

$\qquad\qquad\quad$ = sebelah kiri

4. Diberi $\sin x = \frac{4}{5}$ dan x adalah tirus manakala $\sin y = \frac{5}{13}$ adalah cakah.
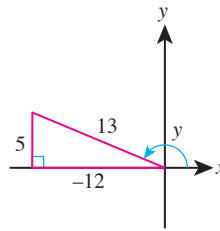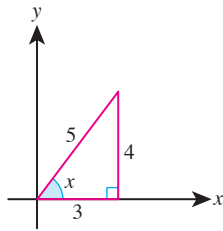
(a) $\displaystyle \text{kosek } 2x = \frac{1}{\sin 2x}$

$$= \frac{1}{2 \sin x \, \text{kos } x}$$

$$= \frac{1}{2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)}$$

$$= \frac{1}{\frac{24}{25}}$$

$$= \frac{25}{24}$$

(b) $\displaystyle \text{sek } 2y = \frac{1}{\text{kos } 2y}$

$$= \frac{1}{1 - 2 \sin^2 y}$$

$$= \frac{1}{1 - 2\left(\frac{5}{13}\right)^2}$$

$$= \frac{1}{\frac{169 - 50}{169}}$$

$$= \frac{1}{\frac{119}{169}}$$

$$= \frac{169}{119}$$

(c) $\displaystyle \sin \frac{x}{2} = \sqrt{\frac{1 - \text{kos } x}{2}}$

$$= \sqrt{\frac{1 - \frac{3}{5}}{2}}$$

$$= \frac{1}{\sqrt{5}}$$

(d) $\displaystyle \tan \frac{y}{2} = \frac{\sin y}{1 + \text{kos } y}$

$$= \frac{5}{13} \div \left(1 - \frac{12}{13}\right)$$

$$= \frac{5}{13} \div \frac{1}{13}$$

$$= \frac{5}{13} \times \frac{13}{1}$$

$$= 5$$

Latihan Formatif 6.5

1. $\tan(A + B) = 3$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 3$$

$$\tan A + \frac{1}{3} = 3\left(1 - \frac{1}{3}\tan A\right)$$

$$\tan A + \frac{1}{3} = 3 - \tan A$$

$$2\tan A = 3 - \frac{1}{3}$$

$$\tan A = \frac{4}{3}$$

2. (a) $\sin 3A = \sin(2A + A)$
$$= \sin 2A \, \text{kos}\, A + \text{kos}\, 2A \, \sin A$$
$$= 2\sin A \, \text{kos}\, A \, \text{kos}\, A + (1 - 2\sin^2 A)\sin A$$
$$= 2\sin A \, \text{kos}^2 A + \sin A - 2\sin^3 A$$
$$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$$
$$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$$
$$= 3\sin A - 4\sin^3 A$$

(b) $\text{kos}\, 3A = \text{kos}(2A + A)$
$$= \text{kos}\, 2A \, \text{kos}\, A - \sin 2A \, \sin A$$
$$= (2\,\text{kos}^2 A - 1)\text{kos}\, A - 2\sin A \, \text{kos}\, A \, \sin A$$
$$= 2\,\text{kos}^3 A - \text{kos}\, A - 2\sin^2 A \, \text{kos}\, A$$
$$= 2\,\text{kos}^3 A - \text{kos}\, A - 2(1 - \text{kos}^2 A)\text{kos}\, A$$
$$= 2\,\text{kos}^3 A - \text{kos}\, A - 2\,\text{kos}\, A + 2\,\text{kos}^3 A$$
$$= 4\,\text{kos}^3 A - 3\,\text{kos}\, A$$

3. $\sin x = \dfrac{24}{25}, \, 0 \leqslant x \leqslant \dfrac{\pi}{2}$ dan $\text{kos}\, y = \dfrac{8}{17}, \, \pi \leqslant y \leqslant 2\pi$.

(a) $\text{kos}(x + y) = \text{kos}\, x \, \text{kos}\, y - \sin x \, \sin y$
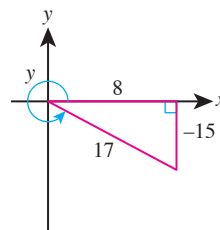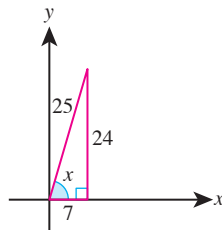
$$= \left(\frac{7}{25}\right)\left(\frac{8}{17}\right) - \left(\frac{24}{25}\right)\left(\frac{-15}{17}\right)$$

$$= \frac{56}{425} - \left(\frac{-72}{85}\right)$$

$$= \frac{416}{425}$$

(b) $\text{kosek}(x - y) = \dfrac{1}{\sin(x - y)}$

$$= \frac{1}{\sin x \, \text{kos}\, y - \text{kos}\, x \, \sin y}$$

$$= \frac{1}{\left(\frac{24}{25}\right)\left(\frac{8}{17}\right) - \left(\frac{7}{25}\right)\left(\frac{-15}{17}\right)}$$

$$= \frac{1}{\left(\frac{192}{425}\right) - \left(-\frac{21}{85}\right)}$$

$$= \frac{1}{\frac{297}{425}}$$

$$= \frac{425}{297}$$

(c) $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

$\qquad\qquad = \dfrac{\left(\dfrac{24}{7}\right) - \left(-\dfrac{15}{8}\right)}{1 + \left(\dfrac{24}{7}\right)\left(-\dfrac{15}{8}\right)}$

$\qquad\qquad = \dfrac{\dfrac{297}{56}}{-\dfrac{38}{7}}$

$\qquad\qquad = -\dfrac{297}{304}$

(d) $\sek 2y = \dfrac{1}{\kos 2y}$

$\qquad\quad = \dfrac{1}{2 \kos^2 y - 1}$

$\qquad\quad = \dfrac{1}{2\left(\dfrac{8}{17}\right)^2 - 1}$

$\qquad\quad = \dfrac{1}{-\dfrac{161}{289}}$

$\qquad\quad = -\dfrac{289}{161}$

(e) $\sin \dfrac{y}{2} = \pm\sqrt{\dfrac{1 - \kos y}{2}}$

$\qquad\quad = \pm\sqrt{\dfrac{1 - \dfrac{8}{17}}{2}}$

$\qquad\quad = \pm\sqrt{\dfrac{9}{34}}$

$\qquad\quad = \pm\dfrac{3}{\sqrt{34}}$

Ambil $\sin \dfrac{y}{2} = -\dfrac{3}{\sqrt{34}}$ kerana $\dfrac{y}{2}$ ialah sudut cakah

$\left[\pi < y < 2\pi, \text{maka } \dfrac{\pi}{2} < \dfrac{y}{2} < \pi, \text{(cakah)}\right]$

4. (a) Sebelah kiri $= \kot(x + y)$

$\qquad\qquad\quad = \dfrac{1}{\tan(x + y)}$

$\qquad\qquad\quad = \dfrac{1 - \tan x \tan y}{\tan x + \tan y}$

$\qquad\qquad\quad = \dfrac{1 - \dfrac{1}{\kot x \kot y}}{\dfrac{1}{\kot x} + \dfrac{1}{\kot y}}$

$\qquad\qquad\quad = \dfrac{\dfrac{\kot x \kot y - 1}{\kot x \kot y}}{\dfrac{\kot y + \kot x}{\kot x \kot y}}$

$\qquad\qquad\quad = \dfrac{\kot x \kot y - 1}{\kot x \kot y}$

$\qquad\qquad\quad = \text{sebelah kanan}$

(b) Sebelah kanan $= \dfrac{\kos(x-y) - \kos(x+y)}{\sin(x-y) + \sin(x+y)}$

$= \dfrac{\kos x \kos y + \sin x \sin y - (\kos x \kos y - \sin x \sin y)}{\sin x \kos y - \kos x \sin y + (\sin x \kos y + \kos x \sin y)}$

$= \dfrac{2 \sin x \sin y}{2 \sin x \kos y}$

$= \tan y$

$= $ sebelah kiri

5.
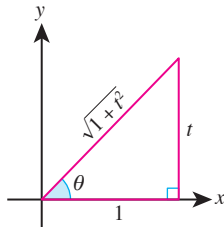
(a) $\sin 2\theta = 2 \sin \theta \kos \theta$

$= 2\left(\dfrac{t}{\sqrt{1+t^2}}\right)\left(\dfrac{1}{\sqrt{1+t^2}}\right)$

$= \dfrac{2t}{1+t^2}$

(b) $\kos 2\theta = 2 \kos^2 \theta - 1$

$= 2\left(\dfrac{1}{\sqrt{1+t^2}}\right)^2 - 1$

$= \dfrac{2}{1+t^2} - 1$

$= \dfrac{2 - (1+t^2)}{1+t^2}$

$= \dfrac{1-t^2}{1+t^2}$

(c) $\tan 2\theta = \dfrac{2 \tan \theta}{1 + \tan^2 \theta}$

$= \dfrac{2t}{1-t^2}$

(d) $\kos \theta = 1 - 2 \sin^2 \dfrac{\theta}{2}$

$2 \sin^2 \dfrac{\theta}{2} = 1 - \kos \theta$

$2 \sin^2 \dfrac{\theta}{2} = 1 - \dfrac{1}{\sqrt{1+t^2}}$

$\sin^2 \dfrac{\theta}{2} = \dfrac{\sqrt{1+t^2} - 1}{2\sqrt{1+t^2}}$

(e) $2 \kos^2 \dfrac{\theta}{2} - 1 = \kos \theta$

$2 \kos^2 \dfrac{\theta}{2} = \kos \theta + 1$

$2 \kos^2 \dfrac{\theta}{2} = \dfrac{1}{\sqrt{1+t^2}} + 1$

$\kos^2 \dfrac{\theta}{2} = \dfrac{1 + \sqrt{1+t^2}}{2\sqrt{1+t^2}}$

6. (a) Sebelah kanan $= \dfrac{\sin \theta}{1 + \text{kos } \theta}$

$$= \dfrac{2 \sin \dfrac{\theta}{2} \text{ kos } \dfrac{\theta}{2}}{1 + 2 \text{ kos}^2 \dfrac{\theta}{2} - 1}$$

$$= \dfrac{2 \sin \dfrac{\theta}{2} \text{ kos } \dfrac{\theta}{2}}{2 \text{ kos}^2 \dfrac{\theta}{2}}$$

$$= \dfrac{\sin \dfrac{\theta}{2}}{\text{kos } \dfrac{\theta}{2}}$$

$$= \tan \dfrac{\theta}{2}$$

$$= \tan \dfrac{1}{2} \theta$$

$$= \text{sebelah kiri}$$

(b) Sebelah kanan $= \dfrac{2}{1 + \text{kos } \theta}$

$$= \dfrac{2}{1 + 2 \text{ kos}^2 \dfrac{\theta}{2} - 1}$$

$$= \dfrac{2}{2 \text{ kos}^2 \dfrac{\theta}{2}}$$

$$= \text{sek}^2 \dfrac{\theta}{2}$$

$$= \text{sek}^2 \dfrac{1}{2} \theta$$

$$= \text{sebelah kiri}$$

(c) Sebelah kanan $= \dfrac{2 \tan \theta}{1 + \tan^2 \theta}$

$$= \dfrac{2 \tan \theta}{\text{sek}^2 \theta}$$

$$= \dfrac{2 \sin \theta}{\text{kos } \theta} \times \text{kos}^2 \theta$$

$$= 2 \sin \theta \text{ kos } \theta$$

$$= \sin 2\theta$$

$$= \text{sebelah kiri}$$

7. (a) $\tan \left(\theta + \dfrac{\pi}{2}\right) = \dfrac{\sin \left(\theta + \dfrac{\pi}{2}\right)}{\text{kos}\left(\theta + \dfrac{\pi}{2}\right)}$

$$= \dfrac{\sin \theta \text{ kos } \dfrac{\pi}{2} + \text{kos } \theta \sin \dfrac{\pi}{2}}{\text{kos } \theta \text{ kos } \dfrac{\pi}{2} - \sin \theta \sin \dfrac{\pi}{2}}$$

$$= \dfrac{\text{kos } \theta}{-\sin \theta}$$

$$= -\text{kot } \theta$$

(b) $\text{kos} \left(\theta + \dfrac{\pi}{2}\right) = \text{kos } \theta \text{ kos } \dfrac{\pi}{2} - \sin \theta \sin \dfrac{\pi}{2}$

$$= -\sin \theta$$

(c) $\sin \left(\theta + \dfrac{\pi}{2}\right) = \sin \theta \text{ kos } \dfrac{\pi}{2} + \text{kos } \theta \sin \dfrac{\pi}{2}$

$$= \text{kos } \theta$$

Kuiz Pantas (Halaman 223)

Nisbah	$x = \ldots$
$\sin x = 0$	$0°, 180°, 360°$
$\text{kos } x = 0$	$90°, 270°$
$\tan x = 0$	$0°, 180°, 360°$
$\sin x = 1$	$90°$
$\text{kos } x = 1$	$0°, 360°$
$\tan x = 1$	$45°, 225°$
$\sin x = -1$	$270°$
$\text{kos } x = -1$	$180°$
$\tan x = -1$	$135°, 315°$

Latihan Kendiri 6.10

1. (a) $\sin 2x = -0.4321$

   sudut rujukan, $\alpha = \sin^{-1} 0.4321$
   $= 25.60°$

   $\sin 2x$ negatif, $2x$ dalam sukuan III, sukuan IV, $0° \leqslant 2x \leqslant 720°$
   $2x = 205.6°, 334.4°, 360 + 205.6°, 360° + 334.4°$
   $= 205.6°, 334.4°, 565.6°, 694.4°$
   $x = 102.8°, 167.2°, 282.8°, 347.2°$

   (b) $\text{sek } (2x + 40°) = 2$

   $\text{kos } (2x + 40°) = \dfrac{1}{2}$

   Sudut rujukan, $\alpha = \text{kos}^{-1} \dfrac{1}{2}$
   $= 60°$

   $2x + 40°$ dalam sukuan I, sukuan IV, $40° < 2x + 40° < 760°$
   $2x + 40° = 60°, 300°, 360° + 60°, 360° + 300°$
   $= 60°, 300°, 420°, 660°$
   $2x = 20°, 260°, 380°, 620°$
   $x = 10°, 130°, 190°, 310°$

   (c) $\text{kot } \dfrac{x}{3} = 0.4452$

   $\tan \dfrac{x}{3} = \dfrac{1}{0.4452} = 2.246$

   Sudut rujukan, $\alpha = \tan^{-1} 2.246$
   $= 66°$

   $\tan \dfrac{x}{3}$ positif, $\dfrac{x}{3}$ dalam sukuan I, $0° \leqslant \dfrac{x}{3} \leqslant 120°$

   $\dfrac{x}{3} = 66°$

   $x = 198°$

   (d) $5 \tan x = 7 \sin x$

   $\dfrac{5 \sin x}{\text{kos } x} - 7 \sin x = 0$

   $5 \sin x - 7 \sin x \text{ kos } x = 0$

   $\sin x(5 - 7 \text{ kos } x) = 0$

   Jika $\sin x = 0$, $x = 0°, 180°, 360°$

   Jika $5 - 7 \text{ kos } x = 0$, $\text{kos } x = \dfrac{5}{7}$

   Sudut rujukan, $\alpha = \text{kos}^{-1} \dfrac{5}{7}$
   $= 44.42°$

   Maka, $x = 0°, 44.42°, 180°, 315.58°, 360°$

(e) $$\sin^2 x - 2 \sin x = \text{kos } 2x$$
$$\sin^2 x - 2 \sin x - (1 - 2 \sin^2 x) = 0$$
$$3 \sin^2 x - 2 \sin x - 1 = 0$$
$$(\sin x - 1)(3 \sin x + 1) = 0$$
Jika $\sin x - 1 = 0$, $\sin x = 1$, $x = 90°$

Jika $3 \sin x + 1 = 0$, $\sin x = -\dfrac{1}{3}$

Sudut rujukan, $\alpha = 19.47°$
x dalam sukuan III dan sukuan IV
$x = 199.47°, 340.53°$
Maka, $x = 90°, 199.47°, 340.53°$

(f) $\sin (x + 30°) = \text{kos } (x + 120°)$
$$\sin x \text{ kos } 30° + \text{kos } x \sin 30° - (\text{kos } x \text{ kos } 120° - \sin x \sin 120°) = 0$$
$$\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \text{kos } x - \left(-\frac{1}{2} \text{kos } x - \frac{\sqrt{3}}{2} \sin x\right) = 0$$
$$\frac{2\sqrt{3}}{2} \sin x + \text{kos } x = 0$$
$$\tan x = -\frac{1}{\sqrt{3}}$$

$\tan x$ negatif, x dalam sukuan II dan sukuan IV
$x = 150°$ dan $330°$

(g) $7 \sin x + 3 \text{ kos } 2x = 0$
$7 \sin x + 3(1 - 2 \sin^2 x) = 0$
$7 \sin x + 3 - 6 \sin^2 x = 0$
$\sin x = -0.3333, 1.5$ (abaikan)
Maka, $\sin x = -0.3333$
Sudut rujukan, $\alpha = \sin^{-1} 0.3333$
$\qquad\qquad\qquad = 19.47°$
$\sin x$ negatif, x dalam sukuan III dan sukuan IV
$x = 199.47°, 340.53°$

(h) $\sin x = 3 \sin 2x$
$\sin x - 3(2 \sin x \text{ kos } x) = 0$
$\sin x - 6 \sin x \text{ kos } x = 0$
$\sin x(1 - 6 \text{ kos } x) = 0$
Jika $\sin x = 0$, $x = 0°, 180°, 360°$
Jika $1 - 6 \text{ kos } x = 0$
$\qquad \text{kos } x = \dfrac{1}{6}$
Sudut rujukan, $\alpha = \text{kos}^{-1}\left(\dfrac{1}{6}\right)$
$\qquad\qquad\qquad = 80.41°$
kos x positif, x dalam sukuan I dan sukuan IV.
$x = 80.41°, 279.59°$
Maka, $x = 0°, 80.41°, 180°, 279.59°, 360°$

(i) $\text{kos } (x - 60°) = 3 \text{ kos } (x + 60°)$
$$\text{kos } x \text{ kos } 60° + \sin x \sin 60° - 3 (\text{kos } x \text{ kos } 60° - \sin x \sin 60°) = 0$$
$$\frac{1}{2} \text{kos } x + \frac{\sqrt{3}}{2} \sin x - 3\left(\frac{1}{2}\right) \text{kos } x + 3\left(\frac{\sqrt{3}}{2}\right) \sin x = 0$$
$$-\text{kos } x + \frac{4\sqrt{3}}{2} \sin x = 0$$
$$\frac{4\sqrt{3}}{2} \sin x = \text{kos } x$$
$$\tan x = \frac{1}{2\sqrt{3}}$$
$$x = \tan^{-1} \frac{1}{2\sqrt{3}}$$
$$= 16.10°$$

tan x positif, x dalam sukuan I dan sukuan III.

$x = 16.10°$ dan $196.10°$

2. (a) $\sin\left(2x + \dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

Sudut rujukan, $\alpha = \sin^{-1} \dfrac{\sqrt{3}}{2}$

$\qquad\qquad\qquad = \dfrac{\pi}{3}$

$\sin\left(2x + \dfrac{\pi}{6}\right)$ adalah negatif, $\left(2x + \dfrac{\pi}{6}\right)$ dalam sukuan III dan sukuan IV.

$\dfrac{\pi}{6} \leqslant \left(2x + \dfrac{\pi}{6}\right) \leqslant 4\pi + \dfrac{\pi}{6}$

$2x + \dfrac{\pi}{6} = \pi + \dfrac{\pi}{3}, 2\pi - \dfrac{\pi}{3}, 2\pi + \pi + \dfrac{\pi}{3}, 2\pi + 2\pi - \dfrac{\pi}{3}$

$2x + \dfrac{\pi}{6} = \dfrac{4\pi}{3}, \dfrac{5\pi}{3}, \dfrac{10\pi}{3}, \dfrac{11\pi}{3}$

$\qquad 2x = \dfrac{7\pi}{6}, \dfrac{3\pi}{2}, \dfrac{19\pi}{6}, \dfrac{7\pi}{2}$

$\qquad x = \dfrac{7\pi}{12}, \dfrac{3\pi}{4}, \dfrac{19\pi}{12}, \dfrac{7\pi}{4}$

(b) $3 \sin y = 2 \tan y$

$\qquad 3 \sin y - 2\left(\dfrac{\sin y}{\cos y}\right) = 0$

$3 \sin y \cos y - 2 \sin y = 0$

$\quad \sin y(3 \cos y - 2) = 0$

Jika $\sin y = 0$, $y = 0$ rad, π dan 2π.

Jika $3 \cos y - 2 = 0$, $\cos y = \dfrac{2}{3}$

Sudut rujukan, $\alpha = \cos^{-1} \dfrac{2}{3}$

$\qquad\qquad\qquad = 48.19°$

$\cos y$ positif, y dalam sukuan I dan sukuan IV

$y = 48.19° \times \dfrac{\pi}{180}, 311.81° \times \dfrac{\pi}{180}$

$y = 0$ rad, 0.2677π rad, π rad, 1.732π rad dan 2π rad.

(c) $\qquad 3 \cot^2 z - 5 \csc z + 1 = 0$

$\quad 3(\csc^2 z - 1)5 \csc z + 1 = 0$

$3 \csc^2 z - 3 - 5 \csc z + 1 = 0$

$\quad 3 \csc^2 z - 5 \csc z - 2 = 0$

$\qquad\qquad\qquad \csc z = 2, -0.3333$

Jika $\csc z = 2$, $\sin z = \dfrac{1}{2} = 0.5$

Sudut rujukan, $\alpha = \sin^{-1} 0.5$

$\qquad\qquad\qquad = \dfrac{\pi}{6}$

$\sin z$ positif, z dalam sukuan I dan sukuan II

$z = \dfrac{\pi}{6}$ rad, $\pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$ rad

Jika $\csc z = -0.3333$, $\sin z = -3$, abaikan

Maka, $z = \dfrac{\pi}{6}$ rad, $\dfrac{5\pi}{6}$ rad

(d) sin $2A$ – kos $2A = 0$

$\qquad$ sin $2A =$ kos $2A$

$\qquad$ tan $2A = 1$

tan $2A$ positif, $2A$ dalam sukuan I dan sukuan III, $0 \leqslant 2A \leqslant 4\pi$

$2A = \dfrac{\pi}{4}, \pi + \dfrac{\pi}{4}, 2\pi + \dfrac{\pi}{4}, 2\pi + \pi + \dfrac{\pi}{4}$

$\qquad = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4}$

$\quad A = \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8}, \dfrac{13\pi}{8}$

(e) kos B sin $B = \dfrac{1}{4}$

$\quad 2$ kos B sin $B = \dfrac{1}{2}$

$\quad$ sin $2B = \dfrac{1}{2}$

sudut rujukan, $\alpha = \sin^{-1} 0.5$

$\qquad\qquad\qquad = \dfrac{\pi}{6}$

sin $2B$ positif, $2B$ dalam sukuan I dan II, $0 \leqslant 2B \leqslant 4\pi$

$2B = \dfrac{\pi}{6}, \pi - \dfrac{\pi}{6}, 2\pi + \dfrac{\pi}{6}, 2\pi + \pi - \dfrac{\pi}{6}$

$\qquad = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}$

$\quad B = \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{13\pi}{12}, \dfrac{17\pi}{12}$

(f) $\quad 4$ sin $(x - \pi)$ kos $(x - \pi) = 1$

$\quad 2(2$ sin $(x - \pi)$ kos $(x - \pi)) = 1$

$\qquad\qquad\qquad$ sin $2(x - \pi) = \dfrac{1}{2}$

Sudut rujukan, $\alpha = \dfrac{\pi}{6}$

sin adalah positif, $2x - 2\pi$ dalam sukuan I dan sukuan II, $-2\pi \leqslant 2x - 2\pi \leqslant 2\pi$

$2x - 2\pi = \dfrac{\pi}{6}, \pi - \dfrac{\pi}{6}, 2\pi + \dfrac{\pi}{6}, 2\pi + \pi - \dfrac{\pi}{6}$

$\qquad\qquad = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}$

$\qquad 2x = \dfrac{\pi}{6} + 2\pi, \dfrac{5\pi}{6} + 2\pi, \dfrac{13\pi}{6} + 2\pi, \dfrac{17\pi}{6} + 2\pi$

$\qquad\qquad = \dfrac{13\pi}{6}, \dfrac{17\pi}{6}, \dfrac{25\pi}{6}, \dfrac{29\pi}{6}$

$\qquad x = \dfrac{13\pi}{12}, \dfrac{17\pi}{12}, \dfrac{25\pi}{12}, \dfrac{29\pi}{12}$

Latihan Kendiri 6.11

1. $v = \dfrac{770 \sin 135°}{\sin \theta}$

$\quad = \dfrac{770 (\sin 45°)}{\dfrac{7}{\sqrt{50}}}$

$\quad = 770 \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{50}}{7}\right)$

$\quad = 55\sqrt{100}$

$\quad = 550 \text{ kmj}^{-1}$

2. $\quad$ sek$^2 A +$ tan$^2 A = 2$

$\quad 1 +$ tan$^2 A +$ tan$^2 A = 2$

$\qquad\qquad 2$ tan$^2 A = 1$

$\qquad\qquad\quad$ tan$^2 A = \dfrac{1}{2}$

$\qquad\qquad\quad$ tan $A = \pm\dfrac{1}{\sqrt{2}}$

$\qquad\qquad\quad$ tan $A = \pm 0.7071$

3. (a) $\tan(\alpha + \beta) = \dfrac{15}{10} = 1.5$

(b) $\tan \alpha = \dfrac{8}{10} = 0.8$

(c) $\tan(\alpha + \beta) = \dfrac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

$\qquad 1.5 = \dfrac{0.8 + \tan \beta}{1 - 0.8 \tan \beta}$

$1.5 - 1.2 \tan \beta = 0.8 + \tan \beta$

$\qquad -2.2 \tan \beta = -0.7$

$\qquad\qquad \tan \beta = 0.3182$

$\tan \alpha = 0.8$

$\quad \alpha = 38.66°$

$\tan \beta = 0.3182$

$\quad \beta = 17.65°$

$\angle BAC = 180° - 90°\,(38.66° + 17.85°)$

$\qquad\quad = 33.69°$

$\angle ADB = 180° - 17.85° - 33.49°$

$\qquad\quad = 128.66°$

$\angle BDC = 180° - \alpha - 90°$

$\qquad\quad = 90° - 38.66°$

$\qquad\quad = 51.34°$

$BD = \sqrt{8^2 + 10^2}$

$\quad\ = 12.81 \text{ cm}$

$AB = \sqrt{10^2 + 15^2}$

$\quad\ = 18.03 \text{ cm}$

Latihan Formatif 6.6

1. (a) $2 \text{ kos }(x - 10°) = -1$

$\qquad \text{kos }(x - 10°) = -\dfrac{1}{2}$

Sudut rujukan, $\alpha = \text{kos}^{-1} \dfrac{1}{2}$

$\qquad\qquad\qquad\quad = 60°$

$x - 10°$ dalam sukuan II dan sukuan III

$x - 10° = 120°, 240°$

$\qquad x = 130°, 250°$

(b) $\qquad\qquad \tan^2 x = \text{sek } x + 2$

$\qquad\quad \text{sek}^2 x - 1 = \text{sek } x + 2$

$\text{sek}^2 x - \text{sek } x - 3 = 0$

$\qquad\qquad\qquad \text{sek } x = -1.303, 2.303$

Jika $\text{sek } x = -1.303$, $\text{kos } x = -0.7675$

Sudut rujukan, $\alpha = 39.87°$

x dalam sukuan II dan sukuan III

$x = 140.13°, 219.87°$

Jika $\text{sek } x = 2.303$

$\text{kos } x = \dfrac{1}{2.303}$

$\qquad\ = 0.4342$

$\quad x = 64.27°$

x dalam sukuan I dan IV

$x = 64.27°, 295.73°$

Maka, $x = 64.27°, 140.13°, 219.87°, 295.73°$

(c) $3 \sin x + 4 \kos x = 0$

$$3 \sin x = -4 \kos x$$

$$\tan x = -\frac{4}{3}$$

Sudut rujukan, $\alpha = \tan^{-1} \frac{4}{3}$

$$= 53.13°$$

$\tan x$ adalah negatif, x dalam sukuan II dan sukuan IV.

$x = 126.87°, 306.87°$

2. (a) $$\sin 2A = \sin 4A$$

$$\sin 2A = 2 \sin 2A \kos 2A$$

$$\sin 2A - 2 \sin 2A \kos 2A = 0$$

$$\sin 2A(1 - 2 \kos 2A) = 0$$

Jika $\sin 2A = 0$

$$2A = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$A = 0, \frac{\pi}{2}, \pi$$

Jika $1 - 2 \kos 2A = 0$

$$2 \kos 2A = 1$$

$$\kos 2A = \frac{1}{2}$$

Sudut rujukan, $\alpha = \frac{\pi}{3}$

$\kos 2A$ positif, $2A$ dalam sukuan I dan sukuan IV, $0 \le 2A \le 2\pi$

$$2A = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$= \frac{\pi}{3}, \frac{5\pi}{3}$$

$$A = \frac{\pi}{6}, \frac{5\pi}{6}$$

Maka, $A = 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi$

(b) $5 \kot^2 A - 4 \kot A = 0$

$\kot A(5 \kot A - 4) = 0$

Jika $\kot A = 0$, $A = 0, \pi$

Jika $5 \kot A = 4$, $\kot A = \frac{4}{5}$

$$\tan A = \frac{5}{4}$$

Sudut rujukan, $\alpha = 51.34°$

$$A = 51.34° \times \frac{\pi}{180}$$

$$= 0.2852\pi \text{ rad}$$

Maka, $A = 0 \text{ rad}, 0.2852\pi \text{ rad}, \pi \text{ rad}$

3. $\tan \theta + \kot \theta = \sek \theta \kosek \theta$

Sebelah kiri $= \tan \theta + \kot \theta$

$$= \frac{\sin \theta}{\kos \theta} + \frac{\kos \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \kos^2 \theta}{\kos \theta \sin \theta}$$

$$= \frac{1}{\kos \theta \sin \theta}$$

$$= \sek \theta \kosek \theta$$

Selesaikan: sek θ kosek θ = 4 kot θ

$$\tan \theta + \text{kot } \theta = 4 \text{ kot } \theta$$
$$\tan \theta - 3 \text{ kot } \theta = 0$$
$$\tan \theta - \frac{3}{\tan \theta} = 0$$
$$\tan^2 \theta = 3$$
$$\tan \theta = \pm\sqrt{3}$$
$$= 60°, 120°, 240°, 300°$$

4. (a) $\sin (B + C) = \sin A$, $B + C = 180° - A$

Sebelah kiri $= \sin (B + C)$
$$= \sin (180° - A)$$
$$= \sin 180° \text{ kos } A - \text{kos } 180° \sin A$$
$$= 0 - (-1) \sin A$$
$$= \sin A$$
$$= \text{sebelah kanan}$$

(b) kos $(B + C) = -$kos A, $B + C = 180° - A$

Sebelah kiri $=$ kos $(B + C)$
$$= \text{kos } (180° - A)$$
$$= \text{kos } 180° \text{ kos } A + \sin 180° \sin A$$
$$= -\text{kos } A$$
$$= \text{sebelah kanan}$$

5. $\angle BCD = \theta$

(a) kos $\theta = -$kos $\angle BCE$
$$= -\frac{8}{17}$$

(b) $\sin 2\theta = 2 \sin \theta \text{ kos } \theta$, $\sin \theta = \sin \angle BCE$
$$= \frac{15}{17}$$

Maka, $\sin 2\theta = 2 \times \frac{15}{17} \times \left(-\frac{8}{17}\right)$
$$= -\frac{240}{289}$$

(c) $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$, $\tan \theta = -\tan \angle BCE$
$$= -\frac{15}{8}$$

$$= \frac{2\left(-\frac{15}{8}\right)}{1 - \left(-\frac{15}{8}\right)^2}$$

$$= \frac{-\frac{15}{4}}{\frac{64 - 225}{64}}$$

$$= \frac{-15}{4} \times \frac{64}{-161}$$

$$= \frac{240}{161}$$

Untuk menentukan nilai θ, gunakan

kos $\theta = \frac{-8}{17}$, $\theta = 118.07°$

6. (a) $\angle CAD = 60° - \theta$

$\sin \angle CAD = \sin (60° - A)$
$$= \sin 60° \text{ kos } A - \text{kos } 60° \sin \theta$$
$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{24}{25}\right) + \left(\frac{1}{2}\right)\left(\frac{7}{25}\right)$$
$$= \frac{24\sqrt{3} - 7}{50}$$

kos $\angle CAD$ = kos $(60° - \theta)$

$\qquad$ = kos $60°$ kos θ – kos $60°$ sin θ
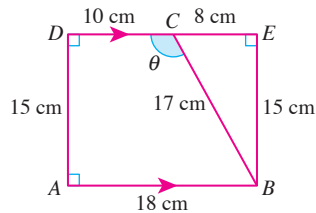
$\qquad = \left(\dfrac{1}{2}\right)\left(\dfrac{24}{25}\right) - \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{7}{25}\right)$

$\qquad = \dfrac{24 + 7\sqrt{3}}{50}$

tan $\angle CAD = \dfrac{\text{sin } \angle CAD}{\text{kos } \angle CAD}$

$\qquad = \dfrac{24\sqrt{3} - 7}{50} \times \dfrac{50}{24 + 7\sqrt{3}}$

$\qquad = \dfrac{24\sqrt{3} - 7}{24 + 7\sqrt{3}}$

ATAU

tan $\angle CAD$ = tan $(60° - \theta)$

$\qquad = \dfrac{\text{tan } 60° - \text{tan } \theta}{1 + \text{tan } 60° \text{ tan } \theta}$

$\qquad = \dfrac{\sqrt{3} - \left(\dfrac{7}{24}\right)}{1 + \sqrt{3}\left(\dfrac{7}{24}\right)}$

$\qquad = \dfrac{24\sqrt{3} - 7}{24 + 7\sqrt{3}}$

(b) Panjang kabel $AC = 25$ m (teorem pythagoras)

kos $60° = \dfrac{24}{AD}$

$\qquad AD = \dfrac{24}{\text{kos } 60°}$

Panjang kabel $AD = 48$ m

7. $L = \dfrac{p^2 \text{ sin } \beta \text{ sin } \alpha}{2 \text{ sin } (\beta + \alpha)}$

Bukti: $L = \dfrac{1}{2} rq$ sin θ,

Menggunakan petua sinus:

$\dfrac{r}{\text{sin } \alpha} = \dfrac{q}{\text{sin } \beta} = \dfrac{p}{\text{sin } \theta}$

$\qquad r = \dfrac{p \text{ sin } \alpha}{\text{sin } \theta}$

$\qquad q = \dfrac{p \text{ sin } \beta}{\text{sin } \theta}$

Maka, $L = \dfrac{1}{2}\left(\dfrac{p \text{ sin } \alpha}{\text{sin } \theta}\right) \times \left(\dfrac{p \text{ sin } \beta}{\text{sin } \theta}\right) \times (\text{sin } \theta)$

$\qquad L = \dfrac{p^2 \text{ sin } \beta \text{ sin } \alpha}{2 \text{ sin } (\beta + \alpha)}$ ...❶

Diketahui sin θ = sin $(180° - (\beta + \alpha))$

$\qquad$ = sin $(\beta + \alpha)$

Persamaan ❶: $L = \dfrac{p^2 \text{ sin } \beta \text{ sin } \alpha}{2 \text{ sin } (\beta + \alpha)}$

8. sek $\theta = t$

   (a) $\sin \theta = \dfrac{\sqrt{t^2 - 1}}{t}$
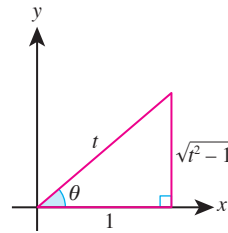
   (b) $\kos \left(\dfrac{\pi}{2} + \theta\right) = \kos \dfrac{\pi}{2} \kos \theta - \sin \dfrac{\pi}{2} \sin \theta$

        $= -\dfrac{\sqrt{t^2 - 1}}{t}$

   (c) $\tan (\pi - \theta) = \dfrac{\tan \pi - \tan \theta}{1 + \tan \pi \tan \theta}$

        $= -\tan \theta$

        $= -\sqrt{t^2 - 1}$

9. $f(x) = 1 + |\kos x|$

   (a) Julat sepadan $1 \leqslant f(x) \leqslant 2$
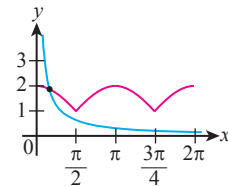
   (b) $x |\kos x| = 1 - x$

      $|\kos x| = \dfrac{1}{x} - 1$

      $1 + |\kos x| = \dfrac{1}{x}$

      Lakar $f(x) = \dfrac{1}{x}$

x	0	π	2π
$f(x)$	∞	0.32	0.16

      Bilangan penyelesaian $= 1$

Latihan Sumatif

1. (a) $0 \leqslant x \leqslant 2\pi$

   (b) $-\pi \leqslant x \leqslant \dfrac{\pi}{2}$

   (c) $\dfrac{3}{2}\pi \leqslant x \leqslant 4\pi$

2. (a) Sudut tirus $= 0 < x < \dfrac{\pi}{2}$

   (b) Sudut cakah $= \dfrac{\pi}{2} < x < \pi$

   (c) Sudut refleks $= \pi < x < 2\pi$

3. (a) $\sin \theta = \pm 0.66$

     Sudut rujukan $= \sin^{-1} 0.66 = 41.30°$

     $\sin \theta$ bernilai positif dan negatif: θ dalam semua sukuan.

     $\theta = 41.30°, 180° - 41.30°, 180° + 41.30°, 360 - 41.30°$

     $\theta = 41.30°, 138.70°, 221.30°, 318.70°$

   (b) sek $\theta = \pm 2.2727$

     $\kos \theta = \pm \dfrac{1}{2.2727}, \pm 0.44$

     Sudut rujukan $= \kos^{-1} 0.44 = 63.90°$

     $\kos \theta$ positif dan negatif, θ dalam semua sukuan

     $\theta = 63.90°, 116.10°, 243.90°, 296.10°$

   (c) kot $\theta = \pm 1.136$

     $\tan \theta = \pm 0.8803$

     Sudut rujukan $= \tan^{-1} 0.8803 = 41.36°$

     $\tan \theta$ positif dan negatif, θ dalam semua sukuan

     $\theta = 41.36°, 138.64°, 221.36°, 318.64°$

4. (a) $\sin (-120°) = -\sin 120°$

                 $= -\sin (180° - 120°)$

                 $= -\sin 60°$

                 $= -\dfrac{\sqrt{3}}{2}$

(b) $\tan 480° = \tan (480° - 360°)$
$= \tan 120°$
$= -\tan (180° - 120°)$
$= -\tan 60°$
$= -\sqrt{3}$

(c) $\sek 750° = \sek (750° - 2(360°))$
$= \sek 30°$
$= \dfrac{2}{\sqrt{3}}$

(d) $\kosek 3\pi = \kosek (3\pi - 2\pi)$
$= \kosek \pi$
$= \dfrac{1}{0}$
$= \infty$

(e) $\kot \left(-\dfrac{9}{4}\pi\right) = \kot \left(-\dfrac{9}{4}\pi - (-2\pi)\right)$
$= \kot \left(-\dfrac{\pi}{4}\right)$
$= -\kot \dfrac{\pi}{4}$
$= -1$

(f) $\kos \left(-\dfrac{8}{3}\pi\right) = \kos \left[-\dfrac{8}{3}\pi - (-2\pi)\right]$
$= \kos \left(-\dfrac{2\pi}{3}\right)$
$= -\kos \dfrac{2\pi}{3}$
$= -\kos \left(\pi - \dfrac{2\pi}{3}\right)$
$= -\kos \dfrac{\pi}{3}$
$= -\dfrac{1}{2}$

5. $\sin A = \dfrac{5}{13}$ dan $\sin B = \dfrac{4}{5}$. Cari nilai $\kos (A - B)$ dan $\tan (A + B)$

(a) A dan B adalah tirus
$\kos (A - B) = \kos A \kos B + \sin A \sin B$
$= \left(\dfrac{12}{13}\right)\left(\dfrac{3}{5}\right) + \left(\dfrac{5}{13}\right)\left(\dfrac{4}{5}\right)$
$= \dfrac{36}{65} + \dfrac{20}{65}$
$= \dfrac{56}{65}$

$\tan (A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
$= \dfrac{\dfrac{5}{12} + \dfrac{4}{3}}{1 - \left(\dfrac{5}{12}\right)\left(\dfrac{4}{3}\right)}$
$= \dfrac{\dfrac{7}{4}}{\dfrac{4}{9}}$
$= \dfrac{7}{4} \times \dfrac{9}{4}$
$= \dfrac{63}{16}$

(b) A dan B adalah cakah

kos $(A - B) = $ kos A kos $B + $ sin A sin B

$$= \left(\frac{-12}{13}\right)\left(\frac{-3}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$$

$$= \frac{36}{65} + \frac{20}{65}$$

$$= \frac{56}{65}$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{5}{-12} + \frac{4}{-3}}{1 - \left(\frac{5}{-12}\right)\left(\frac{4}{-3}\right)}$$

$$= \frac{-\frac{7}{4}}{\frac{4}{9}}$$

$$= -\frac{7}{4} \times \frac{9}{4}$$

$$= -\frac{63}{16}$$

(c) kos A dan kos B adalah negatif.

A dan B dalam sukuan II dan kedua-duanya cakah seperti dalam (b).

$$\text{kos } (A - B) = \frac{56}{65}$$
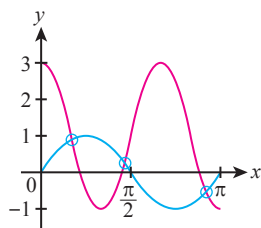
$$\tan (A + B) = -\frac{63}{16}$$

6. $y = a$ kos bx, $0 \leqslant x \leqslant 2\pi$

Graf	Persamaan	Bilangan Kitaran	Kala	Selang Kelas
I	$y = $ kos x	1	2π	$\frac{\pi}{2}$
II	$y = $ kos $2x$	2	π	$\frac{\pi}{4}$
III	$y = $ kos $\frac{1}{2}x$	$\frac{1}{2}$	4π	π

7. (a) Kala bagi $y = $ sin $2x$ ialah π.

(b) Amplitud bagi $y = 1 + 2$ kos $3x$ ialah 2. Nilai maksimum, $y = 3$, nilai minimum, $y = -1$.

(c) Lakaran graf $y = $ sin $2x$ dan $y = 1 + 2$ kos $3x$

(d) Penyelesaian bagi sin $2x - 2$ kos $3x - 1 = 0$ adalah titik persilangan

$$\text{sin } 2x = 1 + 2 \text{ kos } 3x$$

Terdapat 3 titik persilangan graf, maka terdapat 3 penyelesaian.

8. $A + B + C = 180°$

$\qquad C = 180° - (A + B)$

$\sin (A - B) \sin C = \sin (A - B) \sin (180° - (A + B))$

$\qquad = \sin (A - B) \sin (A + B)$

$\qquad = (\sin A \text{ kos } B - \text{kos } A \sin B)(\sin A \text{ kos } B + \text{kos } A \sin B)$

$\qquad = \sin^2 A \text{ kos}^2 B - \text{kos}^2 A \sin^2 B$

$\qquad = \sin^2 A (1 - \sin^2 B) - \sin^2 B (1 - \sin^2 A)$

$\qquad = \sin^2 A - \sin^2 B$

9. $\tan A \tan (A - 60°) + \tan A \tan (A + 60°) + \tan (A - 60°) \tan (A + 60°) = -3$

Bukti:

Sebelah kiri $= \tan A \left(\dfrac{\tan A - \tan 60°}{1 + \tan A \tan 60°} \right) + \tan A \left(\dfrac{\tan A + \tan 60°}{1 - \tan A \tan 60°} \right) + \left(\dfrac{\tan A - \tan 60°}{1 + \tan A \tan 60°} \right)\left(\dfrac{\tan A + \tan 60°}{1 - \tan A \tan 60°} \right)$

$\qquad = \dfrac{\tan^2 A - \sqrt{3} \tan A}{1 + \sqrt{3} \tan A} + \dfrac{\tan^2 A + \sqrt{3} \tan A}{1 - \sqrt{3} \tan A} + \dfrac{\tan^2 A - 3}{1 - 3 \tan^2 A}$

$\qquad = \dfrac{\left(\tan^2 A - \sqrt{3} \tan A\right)\left(1 - \sqrt{3} \tan A\right) + \left(\tan^2 A + \sqrt{3} \tan A\right)\left(1 + \sqrt{3} \tan A\right) + \tan^2 A - 3}{1 - 3 \tan^2 A}$

$\qquad = \dfrac{\tan^2 A - \sqrt{3} \tan^3 A - \sqrt{3} \tan A + 3 \tan^2 A + \tan^2 A + \sqrt{3} \tan^3 A + \sqrt{3} \tan A + 3 \tan^2 A + \tan^2 A - 3}{1 - 3 \tan^2 A}$

$\qquad = \dfrac{9 \tan^2 A - 3}{1 - 3 \tan^2 A}$

$\qquad = \dfrac{-3(1 - 3 \tan^2 x)}{1 - 3 \tan^2 x}$

$\qquad = -3$

10. $A = \text{kos}^{-1} \left(\dfrac{3}{\sqrt{10}} \right) \Rightarrow \text{kos } A = \dfrac{3}{\sqrt{10}}$

$B = \sin^{-1} \left(\dfrac{1}{\sqrt{5}} \right) \Rightarrow \sin B = \dfrac{1}{\sqrt{5}}$

$\sin (A + B) = \sin A \text{ kos } B + \text{kos } A \sin B$

$\qquad = \left(\dfrac{1}{\sqrt{10}} \right)\left(\dfrac{2}{\sqrt{5}} \right) + \left(\dfrac{3}{\sqrt{10}} \right)\left(\dfrac{1}{\sqrt{5}} \right)$

$\qquad = \dfrac{2}{\sqrt{50}} + \dfrac{3}{\sqrt{50}}$

$\qquad = \dfrac{5}{\sqrt{25 \times 2}}$

$\qquad = \dfrac{5}{5\sqrt{2}}$

$\sin (A + B) = \dfrac{1}{\sqrt{2}}$

Maka, $A + B = \dfrac{\pi}{4}$

11. (a) $\sin 2x + \sin x = 0$

$\qquad 2 \sin x \text{ kos } x + \sin x = 0$

$\qquad \sin x(2 \text{ kos } x + 1) = 0$

$\qquad \sin x = 0, x = 0, \pi, 2\pi$

$\qquad 2 \text{ kos } x + 1 = 0$

$\qquad\qquad \text{kos } x = -\dfrac{1}{2}$

$\qquad\qquad\quad x = \dfrac{2}{3}\pi, \dfrac{4}{3}\pi$

Maka, pintasan-x bagi graf ialah $0, \dfrac{2}{3}\pi, \pi, \dfrac{4}{3}\pi$ dan 2π.

(b)
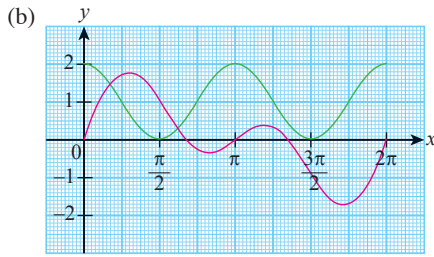
Nilai maksimum, $y = 2$

Kala $= \pi$

(c) Persilangan graf $y = \sin 2x$ dan $y = \kos 2x + 1$ ialah

$\sin 2x + \sin x = \kos 2x + 1$
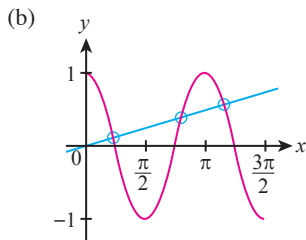
$\sin 2x + \sin x = 2 \kos^2 x - 1 + 1$

$\sin 2x + \sin x = 2 \kos^2 x$

Maka, persilangan graf adalah penyelesaian kepada $\sin 2x + \sin x = 2 \kos^2 x$.

Bilangan penyelesaian $= 2$

12. (a) Sebelah kiri $= \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$

$= \dfrac{1 - \tan^2 x}{\sek^2 x}$

$= \left(1 - \dfrac{\sin^2 x}{\kos^2 x}\right) \kos^2 x$

$= \left(\dfrac{\kos^2 x - \sin^2 x}{\kos^2 x}\right) \kos^2 x$

$= \kos^2 x - \sin^2 x$

$= \kos 2x$

$= $ sebelah kanan

(b)

Bilangan penyelesaian $= 3$

(c) $5\pi(1 - \tan^2 x) = x(1 + \tan^2 x)$

$\dfrac{1 - \tan^2 x}{1 + \tan^2 x} = \dfrac{x}{5\pi}$

$\kos 2x = \dfrac{x}{5\pi}$

Lukis $y = \dfrac{x}{5\pi}$ pada graf di (b).

Terdapat 3 penyelesaian.

13. (a) (i) $\sin (x + 30°) = 2 \kos x$

$\sin x \kos 30° + \kos x \sin 30° - 2 \kos x = 0$

$\dfrac{\sqrt{3}}{2} \sin x + \dfrac{1}{2} \kos x - 2 \kos x = 0$

$\dfrac{\sqrt{3}}{2} \sin x = \dfrac{3}{2} \kos x$

$\tan x = \dfrac{3}{\sqrt{3}}$

$= \sqrt{3}$

$x = 60°, 240°$

(ii)
$$2 \text{ sek } (x + 60°) = 5 \text{ sek } (x - 20°)$$
$$\frac{2}{\text{kos } (x + 60°)} = \frac{5}{\text{kos } (x - 20°)}$$
$$2 \text{ kos } (x - 20°) = 5 \text{ kos } (x + 60°)$$
$$2 (\text{kos } x \text{ kos } 20° + \sin x \sin 20°) = 5 (\text{kos } x \text{ kos } 60° - \sin x \sin 60°)$$
$$2 \text{ kos } x \text{ kos } 20° + 2 \sin x \sin 20° = 5 \text{ kos } x\left(\frac{1}{2}\right) - 5 \sin x\left(\frac{\sqrt{3}}{2}\right)$$
$$1.879 \text{ kos } x + 0.684 \sin x = 2.5 \text{ kos } x - 4.33 \sin x$$
$$5.014 \sin x = 0.621 \text{ kos } x$$
$$\tan x = 0.1239$$
$$x = 7.063°, 187.063°$$

(iii) $\dfrac{\tan x + \tan 15°}{1 - \tan x \tan 15°} = 2$
$$\tan (x + 15°) = 2$$
$$x + 15° = 63.43°, 243.43°$$
$$x = 48.43°, 228.43°$$

(b) (i)
$$3 \sin x = 2 \text{ kos }\left(x + \frac{\pi}{4}\right)$$
$$3 \sin x = 2\left(\text{kos } x \text{ kos } \frac{\pi}{4} - \sin x \sin \frac{\pi}{4}\right)$$
$$3 \sin x = 2 \text{ kos } x\left(\frac{1}{\sqrt{2}}\right) - 2 \sin x\left(\frac{1}{\sqrt{2}}\right)$$
$$\left(3 + \frac{2}{\sqrt{2}}\right) \sin x = \frac{2}{\sqrt{2}} \text{ kos } x$$
$$\tan x = 0.3204$$
$$x = 17.77° \times \frac{\pi}{180}, 197.77° \times \frac{\pi}{180}$$
$$= 0.0987\pi \text{ rad atau } 0.3102 \text{ rad, dan } 1.099\pi \text{ rad atau } 3.452 \text{ rad}$$

(ii)
$$2 \tan x + 3 \tan \left(x - \frac{\pi}{4}\right) = 0$$
$$2 \tan x + 3\left(\frac{\tan x - \tan \frac{\pi}{4}}{1 + \tan x \tan \frac{\pi}{4}}\right) = 0$$
$$2 \tan x + 3\left(\frac{\tan x - 1}{1 + \tan x}\right) = 0$$
$$2 \tan x(1 + \tan x) + 3 \tan x - 3 = 0$$
$$2 \tan x + 2 \tan^2 x + 3 \tan x - 3 = 0$$
$$2 \tan^2 x + 5 \tan x - 3 = 0$$
$$\tan x = 0.5, -3$$
Jika $\tan x = 0.5$, $x = 26.57°, 206.57°$
Jika $\tan x = -3$, sudut rujukan, $\alpha = 71.57°$
$$x = 108.43°, 288.43°$$
Maka, $x = 26.57°, 108.43°, 206.57°, 288.43°$
$$x = 0.4637 \text{ rad}, 1.892 \text{ rad}, 3.605 \text{ rad}, 5.034 \text{ rad}$$

(iii)
$$\tan 5x = \tan 2x$$
$$\tan (2x + 3x) = \tan 2x$$
$$\frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x} = \tan 2x$$
$$\tan 2x + \tan 3x = \tan 2x - \tan^2 2x \tan 3x$$
$$\tan 3x + \tan^2 2x \tan 3x = 0$$
$$\tan 3x(1 + \tan^2 2x) = 0$$

Jika $\tan 3x = 0$,

$$3x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$$

Jika $1 + \tan^2 2x = 0$

$$\tan^2 2x = -1$$

$$\tan 2x = \sqrt{-1} \text{, tidak mungkin}$$

Maka, $x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \pi, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}, 2\pi$

14. $g = 9.78039(1 + 0.005288 \sin \theta - 0.000006 \sin^2 2\theta)$

(a) Latitud bagi Kuala Lumpur ialah $3.1390°$ U

$\sin 3.1390° = 0.05476$

$\sin 2(3.1390°) = 0.1094$

Maka, $g = 9.78039(1 + 0.005288(0.05476) - 0.000006(0.1094))$

$= 9.78039(1 - 6.564 \times 10^{-7})$

$= 9.780 \text{ ms}^{-2}$

(b) Latitud di Khatulistiwa $= 0°$, $\sin 0° = 0$

Latitud di kutub $= 90°$, $\sin 90° = 1$

Nilai graviti di Khatulistiwa $= 9.78039(1 + 0.005288 \sin 0° - 0.000006 \sin^2 2(0°))$

$= 9.78039 \text{ m s}^{-2}$

Nilai graviti di kutub $= 9.78039(1 + 0.005288 \sin 90° - 0.000006 \sin^2 2(90°))$

$= 9.78039(1 - 0.005288)$

$= 9.8321 \text{ ms}^{-2}$

Maka, nilai graviti adalah maksimum di Kutub Utara dan Kutub Selatan bila $\theta = 90°$ U dan $90°$ S.

Nilai tersebut ialah 9.8321 ms^{-2}.

15. Koordinat P ialah (kos B, sin B)

Koordinat Q ialah (kos A, sin A)

Luas $\triangle OPQ = \dfrac{1}{2}(1)(1)(\sin (A - B)$

$= \dfrac{1}{2} \sin (A - B) \quad \dots \mathbf{❶}$

Luas $\triangle OPQ = \dfrac{1}{2} \begin{vmatrix} 0 & \text{kos } B & \text{kos } A & 0 \\ 0 & \sin B & \sin A & 0 \end{vmatrix}$

$= \dfrac{1}{2} \left| \text{kos } B \sin A - \text{kos } A \sin B \right|$

$= \dfrac{1}{2} (\sin A \text{ kos } B - \text{kos } A \sin B) \quad \dots \mathbf{❷}$

Persamaan $\mathbf{❶}$ = Persamaan $\mathbf{❷}$

Maka, $\sin (A - B) = \sin A \text{ kos } B - \text{kos } A \sin B$
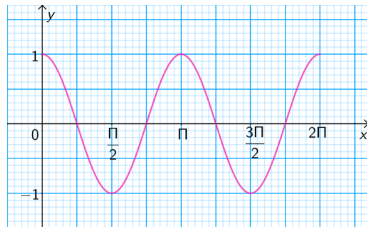
16. (a) $\dfrac{1}{\tan x + \text{kot } x} = \sin x \text{ kos } x$
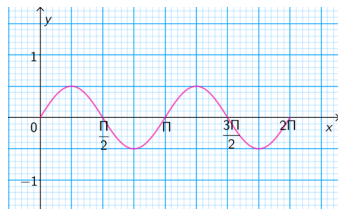
$$\text{Sebelah kiri} = \frac{1}{\tan x + \kot x}$$

$$= \frac{1}{\dfrac{\sin x}{\kos x} + \dfrac{\kos x}{\sin x}}$$

$$= \frac{1}{\dfrac{\sin^2 x + \kos^2 x}{\kos x \sin x}}$$

$$= \kos x \sin x$$

$$= \text{sebelah kanan}$$

(b) $(\sin x - \kos x)(\tan x + \kot x) = \sek x - \kosek x$

$$\text{Sebelah kiri} = (\sin x - \kos x)(\tan x + \kot x)$$

$$= (\sin x - \kos x)\left(\frac{\sin x}{\kos x} + \frac{\kos x}{\sin x}\right)$$

$$= (\sin x - \kos x)\left(\frac{\sin^2 x + \kos^2 x}{\kos x \sin x}\right)$$

$$= (\sin x - \kos x)\left(\frac{1}{\kos x \sin x}\right)$$

$$= \frac{\sin x}{\kos x \sin x} + \frac{\kos x}{\kos x \sin x}$$

$$= \sek x - \kosek x$$

$$= \text{sebelah kanan}$$

(c) $\dfrac{\kot x - \tan x}{\kot x + \tan x} = \kos^2 x - \sin^2 x$

Sebelah kiri $= \dfrac{\text{kot } x - \tan x}{\text{kot } x + \tan x}$

$= \dfrac{\dfrac{1}{\tan x} - \tan x}{\dfrac{1}{\tan x} + \tan x}$

$= \dfrac{\dfrac{1 - \tan^2 x}{\tan x}}{\dfrac{1 + \tan^2 x}{\tan x}}$

$= \dfrac{1 - \tan^2 x}{1 + \tan x}$

$= \dfrac{1 - \tan^2 x}{\text{sek}^2 x}$

$= \dfrac{1}{\text{sek}^2 x} - \dfrac{\sin^2 x}{\text{kos}^2 x \ \text{sek}^2 x}$

$= \text{kos}^2 x - \dfrac{\sin^2 x \ \text{kos}^2 x}{\text{kos}^2 x}$

$= \text{kos}^2 x - \sin^2 x$

$=$ sebelah kanan

Maka, (a) $\dfrac{1}{\tan x + \text{kot } x} = y = \sin x \ \text{kos } x$

(b) $(\sin x - \text{kos } x)(\tan x - \text{kot } x) = \text{sek } x - \text{kosek } x$

(c) $\dfrac{\text{kot } x - \tan x}{\text{kot } x + \tan x} = \text{kos}^2 x - \sin^2 x$