



KEMENTERIAN PENDIDIKAN MALAYSIA  
Jabatan Pendidikan Negeri Pulau Pinang

PULAU PINANG PENERAJU TRANSFORMASI PENDIDIKAN NEGARA

# MODUL GMAT + PENANG

FORMAT BAHARU KSSM  
MATEMATIK TAMBAHAN 2021

## SET ITEM-ITEM TAMBAHAN

TINGKATAN 5



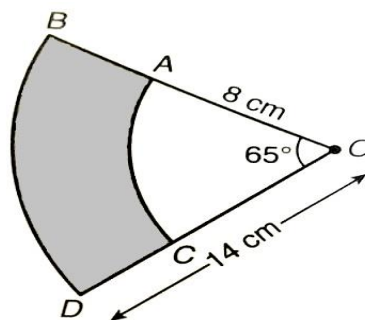
# BAB 1

## SUKATAN MEMBULAT CIRCULAR MESUARE

1. Dalam rajah 1,  $OAC$  dan  $OBD$  ialah sektor dua bulatan sepusat yang berpusat  $O$ . Cari perimeter rantau berlengk ABDC.

*In the diagram 1,  $OAC$  and  $OBD$  are sectors of two concentric circle with centre  $O$ . Find the perimeter of the shaded region  $ABDC$ .*

[5 markah]  
[5 marks]



Rajah 1  
Diagram 1

**Penyelesaian 1 :**

$$\begin{aligned}65^\circ &= 1.134 \text{ rad} \\ \text{Panjang lengkok } AC &= 8 \times 1.134 \\ &= 9.072 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Panjang lengkok } BD &= 14 \times 1.134 \\ &= 15.88 \text{ cm}\end{aligned}$$

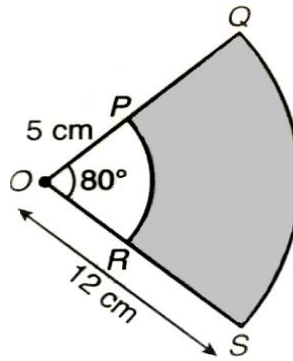
$$\begin{aligned}AB &= CD \\ &= 14 - 8 \\ &= 6 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Perimeter of } ABCD &= AB + \text{arc } BD + DC + \text{arc } AC \\ &= 6 + 15.88 + 6 + 9.072 \\ &= 36.95 \text{ cm}\end{aligned}$$

2. Dalam rajah 2,  $OPR$  dan  $OQS$  ialah dua sektor bulatan sepusat yang berpusat  $O$ . Cari perimeter rantau berlorek  $PQSR$ .

*In the diagram 2,  $OPR$  dan  $OQS$  are sectors of two concentric circles with centre  $O$ . Find the perimeter of the shaded region  $PQSR$ .*

[5 markah]  
[5 marks]



Rajah 2  
Diagram 2

**Penyelesaian 2 :**

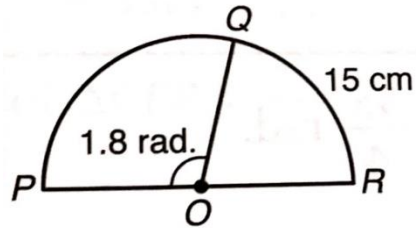
$$\begin{aligned} 80^\circ &= 1.396 \text{ rad} \\ \text{Length of arc } PR &= 5 \times 1.396 \\ &= 6.98 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Length of arc } QS &= 12 \times 1.396 \\ &= 16.75 \text{ cm} \end{aligned}$$

$$\begin{aligned} PQ &= RS \\ &= 12 - 5 \\ &= 7 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Therefore, perimeter of } PQRS &= 6.98 + 16.75 + 7 + 7 \\ &= 37.73 \text{ cm} \end{aligned}$$

3. Rajah 3 menunjukkan sebuah semibulatan berpusat  $O$ . Cari  
*Diagram 3 shows a semicircle with centre  $O$ . Find*



Rajah 3  
*Diagram 3*

- i) jejari bulatan  
*the radius of the circle*
- ii) panjang lengkok  $PQ$   
*the length of the arc  $PQ$*

(Guna / use  $\pi = 3.142$ )

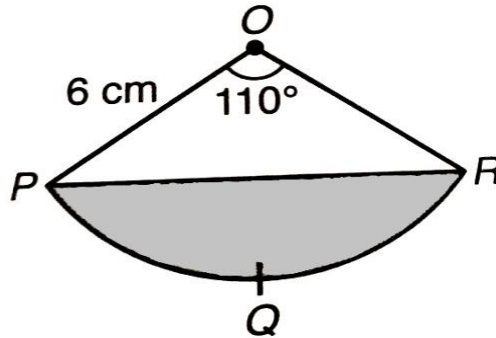
[4 markah]

[4 marks]

**Penyelesaian 3 :**

- i)  $OR = \frac{15}{\pi - 1.8}$   
Radius = 11.18cm
- ii) Panjang lengkok  $PQ$   
=  $11.18 \times 1.8$   
= 20.12cm

4. Cari luas tembereng berlorek bagi sektor Rajah 4  
*Find the area of the shaded segment of the sector in Diagram 4.*



Rajah 4  
 Diagram 4

[5 markah]  
 [5 marks]

**Penyelesaian 4 :**

$$110^\circ = 1.920 \text{ rad}$$

Luas sektor  $OPQR$

$$= \frac{1}{2}(6)^2(1.920)$$

$$= 34.56\text{cm}^2$$

Luas segitiga  $OPR$

$$= \frac{1}{2}(6)^2 \sin 110^\circ$$

$$= 16.91\text{cm}^2$$

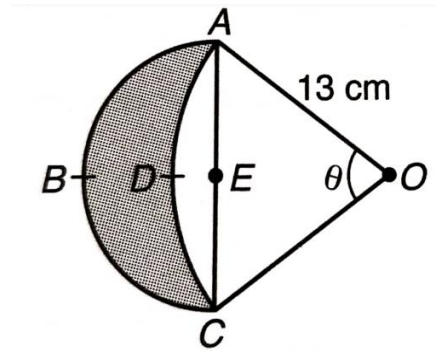
Luas tembereng berlorek PQR

$$= \frac{1}{2}(6)^2(1.920) - \frac{1}{2}(6)^2 \sin 110^\circ$$

$$= 34.56 - 16.91$$

$$= 17.65 \text{ cm}^2$$

5. Dalam Rajah 5,  $ABC$  ialah sebuah semibulatan berpusat  $E$  dan berjari 5 cm.  $OADC$  ialah sebuah sector berpusat  $O$ . Cari  
*In the Diagram 5,  $ABC$  is a semicircle with the centre  $E$  and radius 5 cm.  $OADC$  is a sector with centre  $O$ . Find*



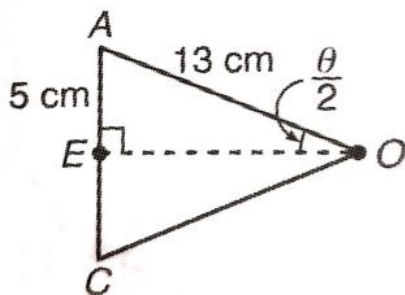
Rajah 5  
 Diagram 5

- i) Nilai  $\theta$  dalam radian  
*The value of  $\theta$  in radians.*
- ii) Luas rantau berlorek  
*The area of the shaded region.*

[6 markah]  
 [6 marks]

**Penyelesaian 5 :**

i)



Segitiga  $OAE$   
 $\sin \theta/2 = AE / OA$   
 $= 5/13$   
 $\theta/2 = 0.3948$   
 $\theta = 0.7896 \text{ rad}$

ii) Dalam Segitiga  $OAE$

$$OE^2 = 13^2 - 5^2$$

$$OE = 12\text{cm}$$

Luas semibulatan  $ABC$

$$= \frac{1}{2} \pi (5)^2$$

$$= 39.27\text{cm}^2$$

Luas Segitiga  $OAC$

$$= \frac{1}{2} (2 \times 5)(12)$$

$$= 60\text{cm}^2$$

Luas sector  $OADC$

$$= \frac{1}{2} (13)^2 (0.7896)$$

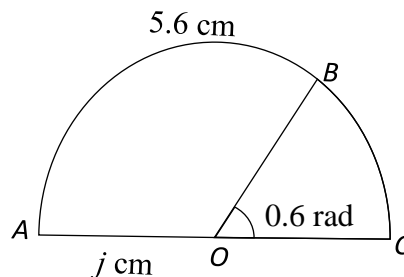
$$= 66.72 \text{ cm}^2$$

Luas rantau berlerek

$$= \frac{1}{2} \pi (5)^2 - [\frac{1}{2} (13)^2 (0.7896) - \frac{1}{2} (2 \times 5)(12)]$$

$$= 39.27 - 66.72 + 60$$

$$= 32.55 \text{ cm}^2$$



Rajah 6  
Diagram 6

6. Rajah 6 di atas menunjukkan semibulatan berpusat  $O$  dan berjajari  $j$  cm. Diberi  $\angle BOC = 0.6$  rad dan panjang lengkok  $AB$  ialah 5.6 cm. Hitung  
*The diagram 6 above shows a semicircle with centre  $O$  and a radius of  $j$  cm. Given that  $\angle BOC = 0.6$  rad and the arc lengths  $AB$  is 5.6 cm. Calculate*

- (a) nilai  $j$ ,  
*the value of  $j$ ,*
- (b) luas, dalam  $\text{cm}^2$ , sektor  $BOC$   
*area, in  $\text{cm}^2$ , of sector  $BOC$*

[ Guna  $\pi = 3.142$  ]

[ Use  $\pi = 3.142$  ]

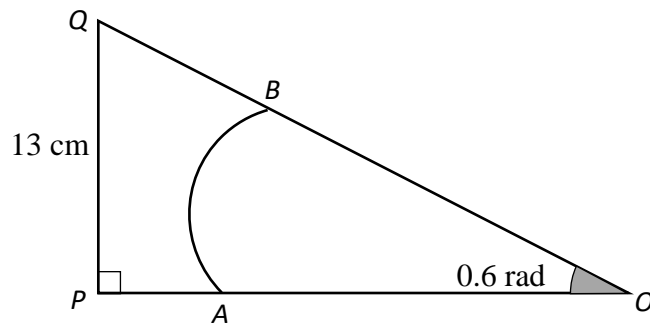
[4 markah]

[4 marks]

**Penyelesaian 6 :**

(a)  $5.6 = j (3.142 - 0.6)$   
 $j = 2.203 \text{ cm}$

(b)  $L = \frac{1}{2} (2.203)^2 (0.6)$   
 $= 1.456 \text{ cm}^2$



Rajah 7  
Diagram 7

7. Rajah 7 di atas menunjukkan sektor  $AOB$  berpusat  $O$  dengan luas  $75 \text{ cm}^2$ .  
Diberi  $\angle AOB = 0.6 \text{ rad}$  dan  $PQ = 13 \text{ cm}$ , cari  
*The diagram 7 above shows a sector  $AOB$  with centre  $O$  and an area of  $75 \text{ cm}^2$ .  
Given that  $\angle AOB = 0.6 \text{ rad}$  and  $PQ = 13 \text{ cm}$ , calculate*

- (a) jejari, dalam cm, sektor  $AOB$ ,  
*the radius, in cm, of sector  $AOB$ ,*
- (b) panjang  $BQ$ , dalam cm.  
*the length of  $BQ$ , in cm.*

[ Guna  $\pi = 3.142$  ]  
[ Use  $\pi = 3.142$  ]

[4 markah]  
[4marks]

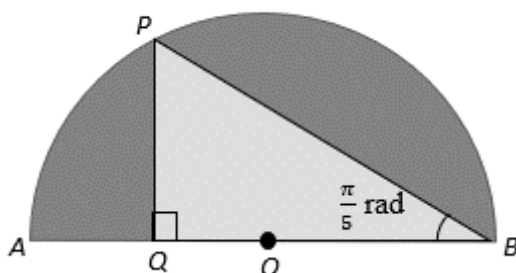
**Penyelesaian 7 :**

(a)  $75 = \frac{1}{2} j^2 (0.6)$   
 $j = 15.81 \text{ cm}$

(b)  $\sin 34.373 = \frac{13}{OQ}$   
 $OQ = 23.026 \text{ cm}$



$$BQ = 23.026 - 15.81 = 7.216 \text{ cm}$$



Rajah 8  
Diagram 8

8. Dalam rajah 8 di atas,  $AOBP$  ialah semibulatan dengan pusat  $O$  dan berjajari 15 cm. Diberi nisbah  $QO : OB = 1 : 2$  dan  $\angle QBP = \frac{\pi}{5}$  rad, cari luas kawasan berwarna biru.

[ Guna  $\pi = 3.142$  ]

*In the diagram 8 above,  $AOBP$  is a semicircle with centre  $O$  and a radius of 15 cm. Given that  $QO : OB = 1 : 2$  and  $\angle QBP = \frac{\pi}{5}$  rad, find the area of the blue shaded region.*

[ Use  $\pi = 3.142$  ]

[4 markah]

[4 marks]

**Penyelesaian 8 :**

$$\text{Luas semibulatan/ Area of semicircle} = \frac{1}{2} (15)^2 (3.142) = 353.475 \text{ cm}^2$$

$$QB = 15 + 7.5 = 22.5 \text{ cm}$$

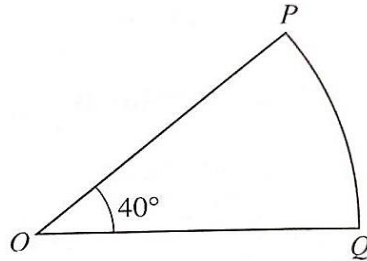
$$\tan 36^\circ = \frac{PQ}{22.5}$$

$$PQ = 16.35 \text{ cm}$$

$$\text{Luas segitiga } PQB / \text{Area of triangle } PQB = \frac{1}{2} \times 16.35 \times 22.5 = 183.9375 \text{ cm}^2$$

$$\begin{aligned} \text{Luas kawasan berwarna biru/ Area of the blue shaded region} \\ = 353.475 - 183.9375 \\ = 169.5375 \text{ cm}^2 \end{aligned}$$

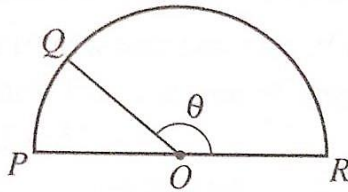
9. (a) Rajah 9a menunjukkan sektor  $OPQ$  dengan pusat  $O$ .  
*Diagram 9a shows a sector  $OPQ$  with centre  $O$ .*



Rajah 9a  
 Diagram 9a

Panjang lengkok  $PQ$  ialah 16 cm, cari  
*The length of arc  $PQ$  is 16 cm, find*

- (i) panjang dalam cm,  $OP$ ,  
*the length in cm, of  $OP$ ,*
  - (ii) Luas, dalam  $\text{cm}^2$ , sektor  $OPQ$ . (Guna  $\pi = 3.142$ )  
*the area, in  $\text{cm}^2$ , of the sector  $OPQ$ . (Use  $\pi = 3.142$ )*
- (b) Rajah 9b menunjukkan  $OPQR$  adalah semi-bulatan dengan pusat  $O$ .  
*Diagram 9b shows a semi-circle  $OPQR$  with centre  $O$ .*



Rajah 9b  
 Diagram 9b

Diberi jumlah panjang diameter dan panjang lengkok  $QR$  adalah dua kali ganda perimeter  $OPQ$  sektor minor, cari nilai  $\theta$ , dalam radian. (Guna  $\pi = 3.142$ )  
*Given that the sum of the length of the diameter and the length of arc  $QR$  is twice the perimeter of the minor sector  $OPQ$ , find the value of  $\theta$ , in radian. (use  $\pi = 3.142$ )*

[8 markah]

[8 marks]

**Penyelesaian 9 :**

$$(a) \quad (i) \quad s = r\theta \quad OP = r$$

$$16 = r(0.6982)$$

$$r = 22.92 \text{ cm}$$

$$(ii) \quad \text{Luas sector } OPQ = \frac{1}{2} \times (22.92)^2 \times (0.6982)$$

$$= 183.39 \text{ cm}^2$$

$$(b) \quad POR + QR = 2(PO + OQ + QP)$$

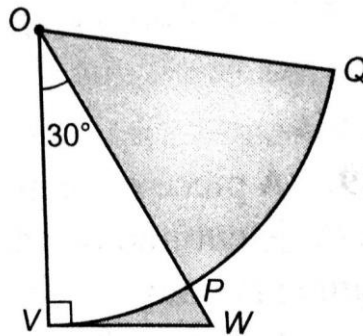
$$r + r + r\theta = 2[r + r + r(\pi - \theta)]$$

$$2r + r\theta = 4r + 2r\pi - 2r\theta$$

$$3r\theta = 2r + 2r\pi$$

$$\theta = \frac{2r + 2r\pi}{3r}$$

$$\theta = 2.7613 \text{ rad}$$



Rajah 10  
Diagram 10

10. Rajah 10 menunjukkan sektor  $VOQ$  dengan pusat  $O$  dan jejari 15 cm. Diberi bahawa panjang lengkok  $PQ$  ialah 13 cm. Hitung  
The diagram 10 shows a sector  $VOQ$  with centre  $O$  and radius 15 cm. It is given that the length of arc  $PQ$  is 13 cm. Calculate

- (a)  $\angle POQ$ , dalam radian  
 $\angle POQ$ , in radians
- (b) luas kawasan berlerek, dalam  $\text{cm}^2$   
the area of the shaded region, in  $\text{cm}^2$

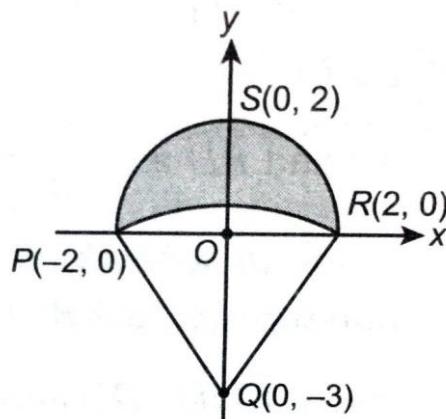
[6 markah]  
[6 marks]

**Penyelesaian 10 :**

$$\begin{aligned} \text{(a)} \quad s &= j\theta \\ 13 &= 15 \theta \\ \theta &= \frac{13}{15} \text{ rad} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \frac{1}{2} (15^2) \left( \frac{13}{15} \right) \\ & \left( \frac{1}{2} \times 15 \times 15 \tan 30 \right) \\ & \frac{1}{2} \times 15^2 \times \frac{\pi}{6} \end{aligned}$$

$$\begin{aligned} \text{Luas} &= \frac{1}{2} (15^2) \left( \frac{13}{15} \right) + \left[ \left( \frac{1}{2} \times 15 \times 15 \tan 30 \right) - \left( \frac{1}{2} \times 15^2 \times \frac{\pi}{6} \right) \right] \\ &= 103.55 \text{ cm}^2 \end{aligned}$$



Rajah 11  
Diagram 11

11. Dalam rajah 11 di sebelah, *PSR* ialah sebuah semibulatan berpusat *O* dan *PQR* ialah sebuah sektor bulatan berpusat *Q*. Hitung  
*In the diagram 11 on the right, PSR is a semicircle with centre O and PQR is a sector of a circle with centre Q. Calculate*

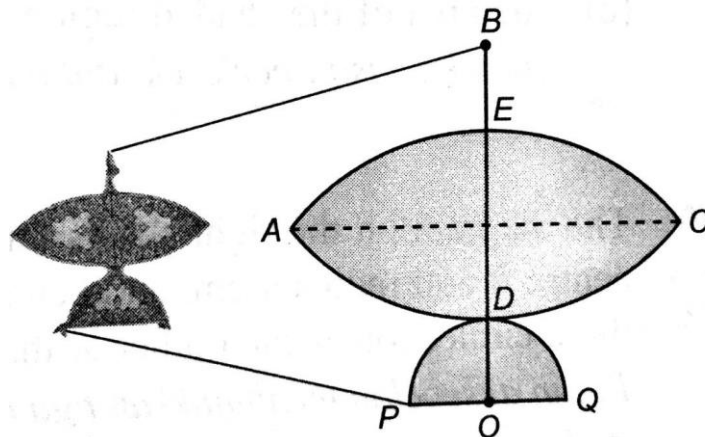
- $\angle PQR$ , dalam radian  
 $\angle PQR$ , in radians
- perimeter kawasan berlorek, dalam unit  
*the perimeter of the shaded region, in units*
- luas kawasan berlorek, dalam unit<sup>2</sup>  
*the area of the shaded region, in units<sup>2</sup>*

[4 markah]  
[4 marks]

**Penyelesaian 11 :**

- (a)  $\tan \alpha = \frac{2}{3}$   
 $\alpha = 0.588 \text{ rad}$   
 $\angle PQR = 2 \times 0.588 \text{ rad}$   
 $= 1.176 \text{ rad}$
- (b) Panjang lengkok  $PR = \sqrt{13} \times (1.176 \text{ rad})$   
 Perimeter  $PSR = 3.142 \times 2$   
 Perimeter = Panjang lengkok  $PR +$  Perimeter  $PSR$   
 $= 10.524 \text{ unit}$
- (c) Luas semibulatan  $= \frac{1}{2} \times 3.142 \times 2^2$   
 Luas sektor  $PQR = \frac{1}{2} \times (\sqrt{13})^2 \times 1.176$

$$\begin{aligned} \text{Luas kawasan berlerek} &= \text{Luas semibulatan} - \text{luas sektor } PQR - \text{luas segitiga } PQR \\ &= \left[ \frac{1}{2} \times 3.142 \times 2^2 \right] - \left[ \frac{1}{2} \times (\sqrt{13})^2 \times 1.176 \right] \left[ \frac{1}{2} \times 4 \times 3 \right] \\ &= 4.64 \text{ unit}^2 \end{aligned}$$



Rajah 12  
 Diagram 12

12. Rajah 12 menunjukkan sebuah rangka wau kucing dengan garis lurus  $OB$  sebagai paksi simetrinya.  $OPDQ$  ialah semibulatan berpusatkan  $O$  dan berjejari 20 cm.  $AEC$  dan  $ADC$  masing-masing ialah dua lengkok yang sama panjang dengan pusat  $O$  dan  $B$ . Jejari bagi kedua-dua lengkok  $AEC$  dan  $ADC$  ialah 60 cm. Diberi  $AC = 90 \text{ cm}$ , hitung

*The diagram 12 shows a fram of wau kucing with a straight line  $OB$  as its symmetrical axis.  $OPDQ$  is a semicircle with centre  $O$  and radius 20 cm.  $AEC$  and  $ADC$  are two arcs of equal length with centres  $O$  and  $B$  respectively. The radii of both arcs  $AEC$  and  $ADC$  are 60 cm. Given  $AC = 90 \text{ cm}$ , calculate*

- (a)  $\angle ABC$  , dalam radian  
 $\angle ABC$  , in radians
- (b) perimeter kawasan berlorek, dalam cm  
*the perimeter of the shaded region, in cm.*
- (c) luas kawasan berlorek, dalam  $\text{cm}^2$   
*the area of the shaded region, in  $\text{cm}^2$*

[8 markah]

[8 marks]

**Penyelesaian 12 :**

(a)  $\angle ABC = 2 \times \sin^{-1}\left(\frac{45}{60}\right)$   
 $= 1.696 \text{ rad}$

(b) Panjang lengkok =  $60 \times 1.696$

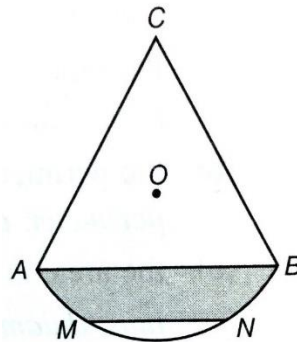
Semibulatan =  $3.142 \times 20$

Perimeter = Panjang lengkok  $AEC$  + Panjang lengkok  $ADC$  + Semibulatan  
 $= (60 \times 1.696) + (60 \times 1.696) + [(3.142 \times 20) + 20 + 20]$   
 $= 101.76 + 101.76 + 102.84$   
 $= 306.36 \text{ cm}$

(c) Luas segitiga =  $\frac{1}{2} \times 60^2 \times \sin 1.696$

Luas sektor  $AEC = \frac{1}{2} \times 60^2 \times 1.696$

Luas = luas semi bulatan + (  $2 \times$  luas sektor  $AEC$ )  
 $= \frac{1}{2} \times 3.142 \times 20^2 + [ 2 \times \left(\frac{1}{2} \times 60^2 \times 1.696\right) - \left(\frac{1}{2} \times 60^2 \times \sin 1.696\right)]$   
 $= 628.4 + 2533.78$   
 $= 3162.18 \text{ cm}^2$



Rajah 13  
Diagram 13

13. Rajah 13 menunjukkan sebuah segitiga sama sisi  $ABC$  dan satu lengkok  $AMNB$  bagi bulatan dengan pusat  $O$ . Diberi bahawa garis  $MN$  selari dengan garis  $AB$ , garis  $ON$  selari dengan  $BC$  dan garis  $OM$  selari dengan garis  $AC$ . Jika  $AC = 8$  cm, hitung  
The diagram 13 shows an equilateral triangle  $ABC$  and an arc  $AMNB$  of a circle with centre  $O$ . It is given that line  $MN$  is parallel to line  $AB$ , line  $ON$  is parallel to line  $BC$  and line  $OM$  is parallel to line  $AC$ . If  $AC = 8$  cm, calculate

- (a) panjang lengkok  $AM$ , dalam cm  
the length of arc  $AM$ , in cm
- (b) panjang lengkok minor  $MN$ , dalam cm  
the length of minor arc  $MN$ , in cm
- (c) luas kawasan berlorek, dalam  $\text{cm}^2$   
the area of the shaded region, in  $\text{cm}^2$

[8 markah]  
[8 marks]

**Penyelesaian 13 :**

$$(a) \sin 60^\circ = \frac{4}{OA}$$

$$OA = 4.619$$

$$\text{Panjang lengkok } AM = 4.619 \left(\frac{\pi}{6}\right)$$

$$= 2.419 \text{ cm}$$

$$(b) \text{ Panjang lengkok minor } MN = 4.619 \times \left(\frac{\pi}{3}\right)$$

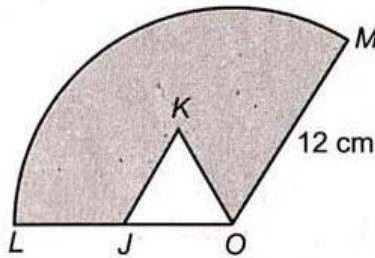
$$= 4.838 \text{ cm}$$

$$(c) \text{ Luas tembereng } MN = \frac{1}{2} \times 4.619^2 \left(\frac{\pi}{3} - \sin 60^\circ\right)$$

$$\text{Luas segitiga } OAB = \frac{1}{2} \times 4.619^2 \times \sin 120^\circ$$

Luas kawasan berlerek

$$\begin{aligned}
 &= \text{luas sektor } OAMNB - \text{luas tembereng } MN - \text{luas segitiga } OAB \\
 &= \left[ \frac{1}{2} \times 4.619^2 \times \frac{2\pi}{3} \right] - \left[ \frac{1}{2} \times 4.619^2 \left( \frac{\pi}{3} - \sin 60^\circ \right) \right] - \frac{1}{2} \times 4.619^2 \times \sin 120^\circ \\
 &= 22.342 - 1.933 - 9.238 \\
 &= 11.171 \text{ cm}^2
 \end{aligned}$$



Rajah 14  
Diagram 14

14. Rajah 14 menunjukkan sebuah sektor  $LOM$  dan sebuah segi tiga  $JOK$ . Diberi bahawa  $\angle LOM = 2.269$  rad dan  $JK = OK = OJ = JL$ . Hitung  
Diagram 14 shows a sector  $LOM$  and a triangle  $JOK$ . Given  $\angle LOM = 2.269$  rad and  $JK = OK = OJ = JL$ . Calculate

- perimeter, dalam  $cm$ , bagi seluruh rajah,  
*the perimeter, in  $cm$ , of the whole diagram,*
- luas, dalam  $cm^2$ , bagi kawasan berlerek.  
*the area, in  $cm^2$ , of the shaded region.*

[6 markah]  
[6 marks]

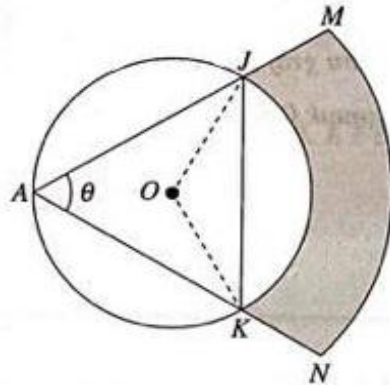
**Penyelesaian 14 :**

- Perimeter  

$$= 12 + 12 + 12 \left( 130^\circ \times \frac{3.142}{180^\circ} \right) = 51.23$$
- Luas kawasan berlerek  

$$\begin{aligned}
 &= \frac{1}{2} (12)^2 (2.269) - \frac{1}{2} (6)^2 \sin 60^\circ \\
 &= 163.38 - 15.59 \\
 &= 147.79 \text{ cm}^2
 \end{aligned}$$





Rajah 15  
Diagram 15

15. Rajah 15 menunjukkan sebuah bulatan berpusat  $O$  dan sektor bulatan berpusat  $A$ . Panjang lengkok  $MN$  dan lengkok  $JK$  masing-masing ialah  $15\text{ cm}$  dan  $12\text{ cm}$ .  
Diberi  $AO = 8\text{ cm}$ ,  $JK = 10\text{ cm}$  dan  $\theta = 1.36\text{ rad}$ .

Diagram 15 shows a circle with centre  $O$  and a sector of a circle with centre  $A$ .  
The lengths of arc  $MN$  and  $JK$  are  $15\text{ cm}$  and  $12\text{ cm}$  respectively. Given  $AO = 8\text{ cm}$ ,  
 $JK = 10\text{ cm}$  and  $\theta = 1.36\text{ rad}$ .

[Guna/Use  $\pi = 3.142$ ]

- Cari  $\angle JOK$ , dalam radian,  
Find  $\angle JOK$ , in radians,
- hitung nilai  $OJ$ ,  
calculate the value of  $OJ$ ,
- Hitung luas, dalam  $\text{cm}^2$ , kawasan berlorek.  
Calculate the area, in  $\text{cm}^2$ , of the shaded region.

[10 markah]

[10 marks]

**Penyelesaian 15 :**

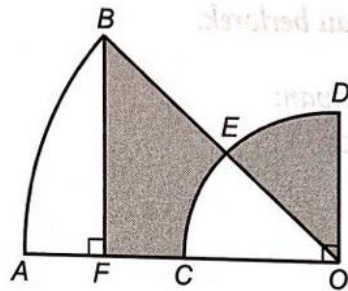
- $\angle JOK$   
 $= 1.36 \times 2$   
 $= 2.72\text{ rad}$
- $\frac{5}{OJ} = \sin 1.36$   
 $OJ = 5.113\text{ cm}$
- Luas sektor  $AMN$   
 $= \frac{1}{2}(11.03)^2(1.36)$   
 $= 82.73\text{ cm}^2$

Luas tembereng  $JK$   
 $= \frac{1}{2} (5.113)^2 (2.72 - \sin 2.72)$   
 $= 30.21 \text{ cm}^2$

Luas segi tiga  
 $= \frac{1}{2} (8)^2 \sin 1.36$   
 $= 31.29 \text{ cm}^2$

Luas kawasan berlerek  
 $= 82.73 - 30.21 - 31.29$   
 $= 21.23 \text{ cm}^2$

16. Rajah 16 menunjukkan sektor  $AOB$  berpusat  $O$ .  $COD$  adalah sukuan bulatan berpusat  $O$  dan berjari  $5 \text{ cm}$ .  
*The diagram 16 shows a sector  $AOB$  with centre  $O$ .  $COD$  is a quadrant of a circle with centre  $O$  and radius of  $5 \text{ cm}$ .*



Rajah 16  
 Diagram 16

Diberi bahawa  $CE = ED$ ,  $AC = CO$  dan  $BF = 7 \text{ cm}$ . Cari  
*Given that  $CE = ED$ ,  $AC = CO$  and  $BF = 7 \text{ cm}$ . Find*  
 [Guna/Use  $\pi = 3.142$ ]

- (a)  $\angle AOB$ , dalam radian,  
 *$\angle AOB$ , in radians,*
- (b) perimeter, dalam  $\text{cm}$ , bagi kawasan berlerek,  
*the perimeter, in  $\text{cm}$ , of the shaded region,*

[6 markah]  
 [6 marks]

**Penyelesaian 16 :**

<p>(a) <math>\sin \angle AOB = \frac{7}{10}</math>  <math>\angle AOB = 0.7755 \text{ rad}</math></p> <p>(b) Panjang <math>FC</math>  <math>= \sqrt{51} - 5</math>  <math>= 2.141 \text{ cm}</math>          Lengkuk <math>CED</math></p>	<p><math>= \frac{\pi}{2} (5)</math>  <math>= 7.855 \text{ cm}</math>          Perimeter rantau berlerek  <math>= 7 + 2.141 + 7.855 + 5 + 5</math>  <math>+ 5</math>  <math>= 32 \text{ cm}</math></p>
--	---

1. Diberi fungsi  $h(x) = kx^3 - 2x^2 + 7x$ , dengan keadaan  $k$  ialah pemalar.  
Given a function of  $h(x) = kx^3 - 2x^2 + 7x$ , where  $k$  is a constant.

Cari

Find

(a)  $h'(x)$

- (b) nilai bagi  $k$  jika  $h''(1) = h'(-1)$   
the value of  $k$  if  $h''(1) = h'(-1)$

[5 markah]

[5 marks]

**Penyelesaian 1 :**

(a)  $h(x) = kx^3 - 2x^2 + 7x$   
 $h'(x) = 3kx^2 - 4x + 7$

b)  $h'(-1) = 3k(-1)^2 - 4(-1) + 7$   
 $h'(-1) = 3k + 11$

$$h''(x) = 6kx - 4x$$

$$h''(1) = 6k(1) - 4(1)$$

$$h''(1) = 6k - 4$$

$$6k - 4 = 3k + 11$$

$$k = 5$$

2. Diberi bahawa  $y = 3x^2 - 4x + 3$ . Cari
- nilai  $\frac{dy}{dx}$  pada titik (1, 2),
  - perubahan kecil bagi  $x$  apabila  $y$  bertambah daripada 2 kepada 2.01.

*Given that  $y = 3x^2 - 4x + 3$ . Find*

- the value of  $\frac{dy}{dx}$  at point (1, 2),*
- the small change in  $x$  when  $y$  increases from 2 to 2.01.*

[6 markah]

[6 marks]

**Penyelesaian 2 :**

$$\begin{aligned} \text{a)} \quad y &= 3x^2 - 4x + 3 \\ \frac{dy}{dx} &= 6x - 4 \end{aligned}$$

Gantikan  $x = 1$

$$\begin{aligned} \frac{dy}{dx} &= 6(1) - 4 \\ \frac{dy}{dx} &= 2 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad \delta y &= 2.01 - 2.00 \\ \delta y &= 0.01 \\ \delta y &= \frac{dy}{dx} \times \delta x \end{aligned}$$

$$\begin{aligned} 0.01 &= 2 \times \delta x \\ \delta x &= \frac{0.01}{2} \\ \delta x &= 0.005 \end{aligned}$$

3. Kecerunan bagi lengkung  $y = \frac{1}{x^2 + 2x}$  pada titik A ialah sifar. Cari koordinat titik A.

*The gradient of the curve  $y = \frac{1}{x^2 + 2x}$  at point A is zero. Find the coordinates of point A.*

[5 markah]

[5 marks]

**Penyelesaian 3 :**

$$y = \frac{1}{x^2 + 2x}$$
$$= (x^2 + 2x)^{-1}$$

$$\frac{d}{dx} = (-1)(x^2 + 2x)^{-2}(2x + 2)$$
$$= \frac{-2(x+1)}{(x^2 + 2x)^2}$$

Apabila  $\frac{d}{dx} = 0, \frac{-2(x+1)}{(x^2 + 2x)^2} = 0$

$$-2(x+1) = 0$$
$$x = -1$$

4. Cari kecerunan bagi lengkung  $y = \frac{3x}{x+1}$  pada titik (1,2). Seterusnya, cari persamaan normal kepada lengkung pada titik itu.

*Find the gradient of the curve  $y = \frac{3x}{x+1}$  at point (1,2). Next, find the normal equation to the curve at that point.*

[6 markah]

[6 marks]

**Penyelesaian 4 :**

$$y = \frac{3x}{x+1}$$

$$\begin{aligned} \frac{d}{dx} &= \frac{(x+1)(3) - 3x(1)}{(x+1)^2} \\ &= \frac{3x+3-3x}{(x+1)^2} \\ &= \frac{3}{(x+1)^2} \end{aligned}$$

Apabila  $x = 1$ ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{3}{(1+1)^2} \\ &= \frac{3}{4} \end{aligned}$$

Maka kecerunan pada titik (1,2)  $= \frac{3}{4}$

Kecerunan normal pada titik (1,2)  $= -\frac{1}{\frac{3}{4}}$

$$= -\frac{4}{3}$$

Persamaan normal kepada lengkung pada titik (1,2)

$$y - 2 = -\frac{4}{3}(x - 1)$$

$$3y - 6 = -4x + 4$$

$$3y + 4x = 10$$

5. Diberi bahawa  $L = 7t - 2t^2$  dan  $x = 4 + 5t$ .

*Given that  $L = 7t - 2t^2$  and  $x = 4 + 5t$ .*

(a) Ungkapkan  $dL/dx$  dalam sebutan  $t$

*Express  $dL/dx$  in terms of  $t$*

(b) Cari perubahan kecil dalam  $x$  apabila  $L$  berubah daripada 5 kepada 5.3 pada  $t = 1$

*Find the small change in  $x$  when  $L$  changes from 5 to 5.3 at  $t = 1$*

[8 markah]

[8 marks]

**Penyelesaian 5 :**

a)  $L = 7t - 2t^2$  \_\_\_\_\_(1)  
 $x = 4 + 5t$  \_\_\_\_\_(2)

Daripada (2) :  $5t = x - 4$

$$t = \frac{1}{5}(x - 4)$$

Gantikan  $t = \frac{1}{5}(x - 4)$  ke dalam (1)

$$\begin{aligned} L &= 7\left[\frac{1}{5}(x - 4)\right] - 2\left[\frac{1}{5}(x - 4)\right]^2 \\ &= \frac{1}{25}(35x - 140 - 2x^2 + 16x - 32) \\ &= \frac{1}{25}(51x - 2x^2 - 172) \end{aligned}$$

$$\begin{aligned} \frac{dL}{dx} &= \frac{1}{25}(51 - 4x) \\ &= \frac{1}{25}[51 - 4(4 + 5t)] \\ &= \frac{1}{25}[35 - 20t] \end{aligned}$$

b) Apabila  $t = 1$  ,  $x = 4 + 5(1)$   
 $= 9$

$$\begin{aligned} \delta L &\approx \frac{dL}{dx} \times \delta x \\ &= \frac{1}{25}(51 - 4x) \delta x \end{aligned}$$

Di beri bahawa  $\delta L = 0.3$

Maka, 
$$0.3 = \frac{1}{25}[51 - 4(9)] \delta x$$

$$\begin{aligned}\delta x &\approx \frac{7.5}{15} \\ &= 0.5\end{aligned}$$

6. Diberi kecerunan normal kepada suatu lengkung ialah  $\frac{5}{2x-5}$ . Lengkung itu melalui titik  $P\left(\frac{1}{2}, 4\right)$  dan mempunyai titik minimum  $(k, 2)$ .

*Given that the gradient of normal to the curve is  $\frac{5}{2x-5}$ . The curve passes through point  $P\left(\frac{1}{2}, 4\right)$  and the minimum point is  $(k, 2)$ .*

Hitung  
Calculate

(a) persamaan tangen kepada lengkung itu pada titik  $P$ ,  
*the equation of tangent to the curve at point  $P$ ,*

(b) nilai  $k$ .  
*value of  $k$ .*

[6 markah]

[6 marks]

**Penyelesaian 6 :**

(a) 
$$m_{normal} = \frac{5}{2x-5}$$
$$\frac{dy}{dx} = -\left(\frac{2x-5}{5}\right)$$

$$\begin{aligned}x = \frac{1}{2}, \quad \frac{dy}{dx} &= -\left(\frac{2\left(\frac{1}{2}\right)-5}{5}\right) \\ &= \frac{4}{5}\end{aligned}$$

$$\begin{aligned}y - 4 &= \frac{4}{5}\left(x - \frac{1}{2}\right) \\ y &= \frac{4}{5}x + \frac{18}{5}\end{aligned}$$

(b) 
$$\frac{dy}{dx} = 0, \quad -\left(\frac{2x-5}{5}\right) = 0$$
$$x = \frac{5}{2}$$
$$k = \frac{5}{2}$$



7. Diberi persamaan suatu lengkung ialah  $y=2x(1-x)^4$  dan lengkung itu melalui P(2, 4). Cari  
*It is given the equation  $y=2x(1-x)^4$  and the curve passes through P(2, 4).*  
*Find*

- (a) kecerunan lengkung pada titik P.  
*the gradient of the curve at point P.*
- (b) persamaan garis normal kepada lengkung pada titik P.  
*the equation of the normal to the curve at point P.*

[7 markah]

[7 marks]

**Penyelesaian 7 :**

(a)  $\frac{dy}{dx} = [2x \times 4(1-x)^3 \times (-1)] + [(1-x)^4 \times 2]$   
 $= -8x(1-x)^3 + 2(1-x)^4$

Pada titik P(2, 4),  $x = 2$

Maka kecerunannya  $= -8(2)(1-2)^3 + 2(1-2)^4$   
 $= 18$

(b)  $M_N \times M_L = -1$   
 $M_N \times 18 = -1$   
 $M_N = -\frac{1}{18}$

$\frac{y-y_1}{x-x_1} = -\frac{1}{\frac{dy}{dx}}$  ATAU  $\frac{y-y_1}{x-x_1} = M_N$

$\frac{y-4}{x-2} = -\frac{1}{18}$

$x + 18y - 74 = 0$

8. Lengkung  $y = x^3 - 6x^2 + 9x + 1$  melalui titik  $A (2, 3)$  dan mempunyai dua titik pusingan,  $P (3, 1)$  dan  $Q$ .

*The curve  $y = x^3 - 6x^2 + 9x + 1$  passes through the point  $A (2, 3)$  and has two turning points  $P (3, 1)$  and  $Q$ .*

Cari  
Find

- (a) kecerunan lengkung itu pada  $A$ .  
*the gradient of the curve at  $A$ .*
- (b) persamaan normal kepada lengkung itu pada  $A$ .  
*the equation of the normal to the curve at  $A$ .*
- (c) koordinat  $Q$  dan tentukan sama ada  $Q$  adalah titik maksimum atau titik minimum  
*the coordinates of  $Q$  and determine whether  $Q$  is the maximum or the minimum point.*

[7 markah]  
[7 marks]

**Penyelesaian 8 :**

(a)  $\frac{dy}{dx} = 3x^2 - 12x + 9$

At  $A (2, 3)$ ,  $x = 2$ .

Gradient of the curve at  $A = 3(2^2) - 12(2) + 9 = -3$

(b) Gradient of the normal to the curve at  $A = \frac{1}{3}$

The equation of the normal to the curve at  $A$  is

$$y - 3 = \frac{1}{3}(x - 2)$$

$$x - 3y + 7 = 0$$

(c) At the turning points,  $\frac{dy}{dx} = 0$ .

$$3x^2 - 12x + 9 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$

$$x = 1 \text{ or } 3$$

At  $P (3, 1)$ ,  $x = 3$ .

At  $Q$ ,  $x = 1$ .

$$y = 1^3 - 6(1^2) + 9(1) + 1 = 5$$

Hence, the coordinates of  $Q$  are  $(1, 5)$ .

$$\frac{d^2y}{dx^2} = 6x - 12$$

At  $Q(1, 5)$ ,  $\frac{d^2y}{dx^2} = 6(1) - 12$

$$= -6 (< 0)$$

Hence,  $Q (1, 5)$  is the maximum point.

9. Diberi  $h = 2x - 5$  dan  $y = -\frac{4}{h}$ , cari

Given that  $h = 2x - 5$  and  $y = -\frac{4}{h}$ , find

(a)  $\frac{dy}{dx}$  dalam sebutan  $x$ .  
 $\frac{dy}{dx}$  in terms of  $x$ .

(b) kadar perubahan bagi  $x$  apabila kadar perubahan bagi  $h$  ialah 4 unit  $s^{-1}$ .  
the rate of change of  $x$  when  $h$  changes at a rate of 4 units  $s^{-1}$ .

(c) perubahan kecil dalam  $y$  apabila  $x$  menyusut dari 2 kepada 1.98.

the small change in  $y$  when  $x$  decreases from 2 to 1.98.

[7 markah]

[7 marks]

### Penyelesaian 9 :

$$\begin{aligned} \text{(a)} \quad \frac{dh}{dx} &= 2 \\ y &= -\frac{4}{h} = -4h^{-1} \\ \frac{dy}{dh} &= 4h^{-2} = \frac{4}{h^2} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dh} \times \frac{dh}{dx} \\ \frac{dy}{dx} &= \frac{4}{h^2} \times 2 \\ &= \frac{8}{h^2} \\ &= \frac{8}{(2x-5)^2} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{dx}{dt} &= \frac{dx}{dh} \times \frac{dh}{dt} \\ &= \frac{1}{2} \times 4 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \delta y &= \frac{dy}{dx} \times \delta x \\ \delta y &= \frac{8}{(2x-5)^2} \times (1.98 - 2) \\ &= \frac{8}{(2(2)-5)^2} \times (-0.02) \\ &= -0.16 \end{aligned}$$

10. (a) Diberi  $y = 2p - p^2$  dan  $x = 3 - p$ . Cari  $\frac{dy}{dx}$  dalam sebutan  $x$   
*Given  $y = 2p - p^2$  and  $x = 3 - p$ . Find  $\frac{dy}{dx}$  in terms of  $x$*

(b) Diberi bahawa  $y$  bertambah dengan kadar malar  $0.6 \text{ unit s}^{-1}$ , cari kadar perubahan  $x$  apabila  $x = 1$ .

*Given that  $y$  increases at a constant rate of  $0.6 \text{ unit s}^{-1}$ , find the rate of change of  $x$  when  $x = 1$ .*

(c) Cari perubahan kecil dalam  $y$  apabila  $p$  menyusut daripada 2 kepada 1.96.  
*Find the small change in  $y$  when  $p$  decreases from 2 to 1.96.*

[10 markah]

[10 marks]

**Penyelesaian 10 :**

(a)  $\frac{dy}{dp} = 2 - 2p$

$$x = 3 - p$$

$$p = 3 - x$$

$$\frac{dp}{dx} = -1$$

$$\frac{dy}{dx} = \frac{dy}{dp} \times \frac{dp}{dx}$$

$$= (2 - 2p)(-1)$$

$$= 2p - 2$$

$$= 2(3 - x) - 2$$

$$= 4 - 2x$$

(b)  $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$$0.6 = (4 - 2x) \times \frac{dx}{dt}$$

$$0.6 = (4 - 2(1)) \times \frac{dx}{dt}$$

$$0.6 = 2 \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = 0.3$$

(c)  $\delta y = \frac{dy}{dp} \times \delta p$

$$= (2 - 2p)(1.96 - 2)$$

$$= (2 - 2(2))(-0.04)$$

$$= 0.08$$

11. (a) Diberi titik  $K(2, -5)$  terletak pada lengkung  $y = 2x^2 - 5x$ . Cari,  
*Given the point  $K(2, -5)$  lies on the curve  $y = 2x^2 - 5x$ . Find,*

(i) kecerunan tangen kepada lengkung pada titik  $K$ .  
*the gradient of the tangent to the curve at point  $K$ .*

(ii) persamaan normal kepada lengkung pada titik  $K$ .  
*the equation of the normal to the curve at point  $K$ .*

(b) (i) Diberi  $h(x) = (1 - 2x)^3$ , nilaikan  $h'(1)$ .  
*Given  $h(x) = (1 - 2x)^3$ , evaluate  $h'(1)$ .*

(ii) Diberi garis lurus  $2x + 3py = 15$  adalah berserenjang dengan tangen kepada lengkung  $y = \frac{x}{2x-1}$  pada titik  $(2, 3)$ . Cari nilai  $p$ .  
*Given a straight line  $2x + 3py = 15$  is perpendicular to the tangent of a curve  $y = \frac{x}{2x-1}$ . Find the value of  $p$ .*

[10 markah]

[10 marks]

### Penyelesaian 11 :

(a) (i)  $\frac{dy}{dx} = 4x - 5$

Pada titik  $K(2, -5)$ , kecerunan tangen  $= 4(2) - 5$   
 $= 3$

(ii) Kecerunan normal,  $m_{normal} = -\frac{1}{3}$

$$\frac{y - (-5)}{x - 2} = -\frac{1}{3}$$

$$3y = -x - 13$$

(b) (i)  $h'(x) = -6(1 - 2x)^2$   
 $h'(1) = -6(1 - 2(1))^2$   
 $= -6$

(ii)  $\frac{dy}{dx} = \left(\frac{x}{2x-1}\right) dx$   
 $= \frac{-1}{(2x-1)^2}$

Pada titik  $(2, 3)$ , kecerunan tangen  $= \frac{-1}{(2(2)-1)^2}$   
 $= -\frac{1}{9}$

$\therefore$  Kecerunan garis normal  $= 9$

Daripada persamaan garis yang berserenjang dengan tangen (Garis normal):

$$2x + 3py = 15$$

$$y = \frac{-2}{3p}x + \frac{5}{p}$$

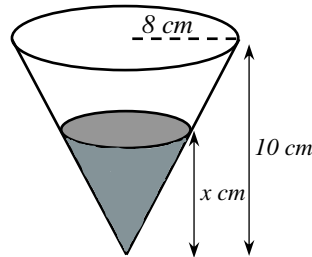
$\therefore$  Kecerunan garis normal  $\frac{-2}{3p} = 9$

$$p = -\frac{2}{27}$$

12. (a) Cari terbitan pertama bagi  $y = 4x^2$  dengan menggunakan prinsip pertama.  
*Find the first derivative of  $y = 4x^2$  by using the first principle.*

(b) Rajah di bawah menunjukkan sebuah bekas berbentuk kon tegak dengan ketinggiannya 10 cm. Air dituangkan ke dalam bekas itu dengan keadaan tinggi,  $x$  cm, paras air bertambah pada kadar tetap  $0.5 \text{ cms}^{-1}$ .

*Diagram below shows a right conical container with a height of 10 cm. Water is poured into the container such that the height,  $x$  cm, of the water level is increasing at a constant rate of  $0.5 \text{ cms}^{-1}$ .*



- (i) Tunjukkan bahawa isi padu  $V \text{ cm}^3$ , ruang yang tidak berisi air diberi oleh,  
*Show that the volume,  $V \text{ cm}^3$ , of the space that is not filled with water is given by,*

$$V = \frac{16}{3}\pi \left(40 - \frac{x^3}{25}\right)$$

- (ii) Hitung kadar perubahan dalam isi padu ruang yang tidak berisi air apabila  $x = 5$ .  
*Calculate the rate of change in the volume of the space that is not filled with water when  $x = 5$ .*

[10 markah]

[10 marks]

### Penyelesaian 12 :

$$\begin{aligned} \text{(a)} \quad y &= f(x) = 4x^2 \\ \delta y &= f(x + \delta x) - f(x) \\ \delta y &= 4(x + \delta x)^2 - 4x^2 \\ &= 8x(\delta x) + 4(\delta x)^2 \end{aligned}$$

$$\begin{aligned} \frac{\delta y}{\delta x} &= 8x + 4\delta x \\ \frac{dy}{dx} &= \lim_{\delta x \rightarrow 0} (8x + 4\delta x) \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{(b) (i) Isi padu kon} &= \frac{1}{3}\pi j^2 t \\ j &= \frac{4}{5}x \\ V &= \frac{1}{3}\pi(8^2)(10) - \frac{1}{3}\pi \left(\frac{4}{5}x\right)^2 x \\ &= \frac{16}{3}\pi \left(40 - \frac{x^3}{25}\right) \quad (\text{Terbukti}) \end{aligned}$$

$$(ii) \frac{dV}{dx} = \frac{16}{3} \pi \left( \frac{3x^2}{25} \right)$$

$$= \frac{16}{25} \pi x^2$$

$$\frac{dx}{dt} = 0.5 \text{ cms}^{-1}$$

Apabila  $x = 5$

$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$$

$$= \frac{16}{25} \pi (5)^2 \times 0.5$$

$$= -8\pi \text{ cm}^3 \text{ s}^{-1}$$

13. (a) Cari nilai had  $\frac{3n-2}{n+1}$ .

*Find the value of  $\lim_{n \rightarrow \infty} \frac{3n-2}{n+1}$ .*

(b) Diberi bahawa  $y = \sqrt[3]{x}$ . Guna kaedah pembezaan untuk mencari nilai hampir bagi  $\sqrt[3]{29}$ .

*Given that  $y = \sqrt[3]{x}$ . Use the method of differentiation to find an approximate value of  $\sqrt[3]{29}$ .*

(c) Sebuah tangki berbentuk silinder dengan tapak bulatan berjejari 0.5 dipenuhi dengan  $h$  m turpentin. Jika turpentin itu menyejat pada kadar  $0.001 \text{ m}^3 \text{ s}^{-1}$ , cari kadar perubahan dalam ketinggian paras turpentin. Beri jawapan anda dalam sebutan  $\pi$ .

*A cylindrical tank with a circular base of radius 0.5 m is filled with  $h$  m of turpentine. If the turpentine is evaporating at a uniform rate of  $0.001 \text{ m}^3 \text{ s}^{-1}$ , find the rate of change in the level of turpentine Give your answer in terms of  $\pi$ .*

[7 markah]

[7 marks]

**Penyelesaian 13 :**

$$\begin{aligned} \text{(a) } \lim_{n \rightarrow \infty} \frac{\frac{3n-2}{n} - \frac{n}{n+1}}{\frac{n}{n+1}} &= \lim_{n \rightarrow \infty} \frac{3 - \frac{2}{n}}{1 + \frac{1}{n}} \\ &= \frac{3 - \lim_{n \rightarrow \infty} \frac{2}{n}}{1 + \lim_{n \rightarrow \infty} \frac{1}{n}} \\ &= \frac{3-0}{1+0} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{(b) } y &= \sqrt[3]{x} = x^{\frac{1}{3}} \\ \frac{dy}{dx} &= \frac{1}{3} x^{-\frac{2}{3}} \\ &= \frac{1}{3\sqrt[3]{x^2}} \end{aligned}$$

Untuk mencari  $\sqrt[3]{29}$ , biar  $x$  menokok dari 27 kepada 29,  $\delta x = 2$

$$\begin{aligned} x = 27, \frac{dy}{dx} &= \frac{1}{3\sqrt[3]{27^2}} \\ &= \frac{1}{27} \end{aligned}$$

$$\begin{aligned} \delta y &= \frac{dy}{dx} \times \delta x \\ &= \frac{1}{27} \times 2 \\ &= 0.07407 \end{aligned}$$

$$\begin{aligned} \sqrt[3]{29} &= 3 + 0.07407 \\ &= 3.07407 \\ &= 3.074 \end{aligned}$$

$$\begin{aligned} \text{(c) } v &= 0.25\pi h \\ \frac{dv}{dh} &= 0.25\pi \\ -0.001 &= (0.25\pi) \times \frac{dh}{dt} \\ \frac{dh}{dt} &= \frac{1}{250\pi} \end{aligned}$$



14. (a) Cari nilai bagi had  $\frac{x^2}{-9-x^2}$

Find the value of  $\lim_{x \rightarrow \infty} \frac{x^2}{-9-x^2}$

(b) Cari  $\frac{dy}{dx}$  bagi  $y = 8x^2 + 6x$  dengan menggunakan prinsip pertama.

Find  $\frac{dy}{dx}$  for  $y = 8x^2 + 6x$  by first principle.

[5 markah]

[5 marks]

**Penyelesaian 14 :**

(a)

$$\text{had } \frac{x^2}{-9-x^2} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2}}{\frac{-9}{x^2} - \frac{x^2}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{\frac{-9}{x^2} - 1}$$

$$= \frac{1}{0-1}$$

$$= -1$$

(b)  $y = 8x^2 + 6x$

$$y + \delta y = 8(y + \delta x)^2 + 6(x + \delta x)$$

$$\delta y = 13x + 8\delta x + 6$$

$$\frac{dy}{dx} = 13x + 6$$

15. (a) Cari nilai bagi  $f''(5)$ , jika  $f(x) = 6x^3 - 5x^2 + 9x$ .

*Find the value of  $f''(5)$ , if  $f(x) = 6x^3 - 5x^2 + 9x$ .*

- (b) Bezakan  $y = \left(\frac{1}{x} + 2\right) \left(\frac{1}{x} - 3\right)$  terhadap  $x$ .

*Differentiate  $y = \left(\frac{1}{x} + 2\right) \left(\frac{1}{x} - 3\right)$  respect to  $x$ .*

[5 markah]  
[5 marks]

**Penyelesaian 15 :**

(a)  $f(x) = 6x^3 - 5x^2 + 9x$   
 $f'(x) = 18x^2 - 10x + 9$   
 $f''(x) = 36x - 10$   
 $f(5) = 170$

(b)  $y = \left(\frac{1}{x} + 2\right) \left(\frac{1}{x} - 3\right)$   
 $= -2x^{-3} + x^{-2}$   
 $= \frac{-2}{x^3} + \frac{1}{x^2}$

16. (a) Cari nilai pemalar  $m$  dengan keadaan  $\frac{d}{dx} \left(\frac{3x-4}{5-2x}\right) = \frac{m}{(5-2x)^2}$ .

*Find the constant of  $m$  if  $\frac{d}{dx} \left(\frac{3x-4}{5-2x}\right) = \frac{m}{(5-2x)^2}$ .*

- (b) Bezakan  $f(x) = x^2 \sqrt{1 - 3x^2}$  terhadap  $x$ .

*Differentiate  $f(x) = x^2 \sqrt{1 - 3x^2}$ , respect to  $x$ .*

[6 markah]  
[6 marks]

**Penyelesaian 16 :**

(a) {guna petua hasil bahagi}  
 $f'(x) = \frac{7}{(5-2x)^2}$   
 $m = 7$

- (b) {guna petua hasil darab}

$$f'(x) = \frac{3x^2 - 12x^2}{\sqrt{1-3x^2}}$$

17. (a) Jika  $a = t^3 + 2t^2 + 3t + 4$ , cari nilai – nilai  $t$  dengan keadaan  $\frac{da}{dt} = \frac{d^2a}{dt^2}$ .

*If  $a = t^3 + 2t^2 + 3t + 4$ , find the values of  $t$  which is  $\frac{da}{dt} = \frac{d^2a}{dt^2}$ .*

(b) Diberi  $x = t^2 + 3$  dan  $\frac{dy}{dt} = 14 t^3$ , cari

*Given  $x = t^2 + 3$  and  $\frac{dy}{dt} = 14 t^3$ , cari*

(i)  $\frac{dx}{dt}$ ,

(ii)  $\frac{dy}{dx}$ , dalam sebutan  $x$   
*in term of  $x$*

[6 markah]  
[6 marks]

**Penyelesaian 17 :**

(a)

$$\frac{da}{dt} = 3t^2 + 4t + 3$$

$$\frac{d^2a}{dt^2} = 6t + 4 \quad \dots\dots\dots [1 \text{ m}]$$

$$3t^2 + 4t + 3 = 6t + 4$$

$$3t^2 - 2t - 1 = 0 \quad \dots\dots\dots [1 \text{ m}]$$

$$t = \frac{-1}{3}, \quad t = 1 \quad \dots\dots\dots [1 \text{ m}]$$

(b) (i)  $\frac{dx}{dt} = 2t \dots\dots\dots [1 \text{ m}]$

(b) (ii)  $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \dots\dots\dots [1 \text{ m}]$

$$= 14 t^3 \times \frac{1}{2t} \dots\dots\dots [1 \text{ m}]$$

18. (a) Jika  $xy - 4x^2 = 12$ , tunjukkan bahawa  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y$ .

If  $xy - 4x^2 = 12$ , show that  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y$ .

(b) Diberi  $y = x(3 - x)$ , ungkapkan  $y \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 12$  dalam sebutan  $x$ .

Seterusnya cari nilai  $x$  yang memuaskan  $y \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 12 = 0$

Given  $y = x(3 - x)$ , express  $y \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 12$  in term of  $x$ . Hence, find the value of  $x$  which is satisfy  $y \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 12 = 0$ .

[8 markah]

[8 marks]

### Penyelesaian 18 :

$$(a) \quad y = \frac{12}{x} + 4x$$

$$\frac{dy}{dx} = -\frac{12}{x^2} + 4$$

$$\frac{d^2y}{dx^2} = \frac{24}{x^3}$$

$$x^2 \left( \frac{24}{x^3} \right) + x \left( -\frac{12}{x^2} + 4 \right) = \frac{24}{x} - \frac{12}{x} + 4x$$

$$= \frac{12}{x} + 4x$$

$$(b) \quad y = 3x - x^2$$

$$\frac{dy}{dx} = 3 - 2x$$

$$\frac{d^2y}{dx^2} = -2$$

$$(3x - x^2)(-2) + x(3 - 2x) + 12 = -3x + 12$$

$$-3x + 12 = 0 \text{ maka } x = 4$$

19. (a) Diberi  $y = x^3 + 3x^2 - 9x + 2$ , cari koordinat titik  $R$  yang mungkin dengan keadaan  $\frac{dy}{dx} = 0$ .  
Seterusnya, cari nilai  $\frac{d^2y}{dx^2}$  di titik  $R$  itu.

*Given that  $y = x^3 + 3x^2 - 9x + 2$ , find the coordinate of the point  $R$  which is  $\frac{dy}{dx} = 0$ .*

*Hence, find the value of  $\frac{d^2y}{dx^2}$  at the point  $R$ .*

- (b) Kecerunan lengkung  $y = mx + \frac{n}{x^2}$  pada titik  $(-1, \frac{-7}{2})$  ialah 2.

Cari nilai  $m$  dan nilai  $n$ .

*The gradient of the curve  $y = mx + \frac{n}{x^2}$  at the point  $(-1, \frac{-7}{2})$  is 2.*

*Find the value of  $m$  and  $n$ .*

[8 markah]  
[8 marks]

**Penyelesaian 19 :**

(a)  $\frac{dy}{dx} = 3x^2 + 6x - 9$

$$x^2 + 2x - 3 = 0$$

koordinat  $R$  ialah  $(1, -3)$  dan  $(-3, 29)$

$$\frac{d^2y}{dx^2} = 12, x = 1 \qquad \frac{d^2y}{dx^2} = -12, x = -3$$

(b)  $\frac{dy}{dx} = m - \frac{2n}{x^3}$

$$2 = m + 2n \qquad \text{_____} \quad (1)$$

$$\frac{-7}{2} = -a + b \qquad \text{_____} \quad (2)$$

$$a = 3, b = \frac{-1}{2}$$

20. Diberi  $y = x^2 - 2x + 5$  tunjukkan bahawa  $4\frac{d^2y}{dx^2} + (x-1)\frac{dy}{dx} = 2y$ .

Given  $y = x^2 - 2x + 5$  show that  $4\frac{d^2y}{dx^2} + (x-1)\frac{dy}{dx} = 2y$ .

[5 markah]

[5 marks]

**Penyelesaian 20 :**

Diberi  $y = x^2 - 2x + 5$  -----(1)

$$\frac{dy}{dx} = 2x - 2$$

$$\frac{d^2y}{dx^2} = 2$$
 -----(2)

(1) x 2  $2y = 2x^2 - 4x + 10$

$$2y = (2x - 2)(x - 1) + 8$$

$$2y = (2x - 2)(x - 1) + 4(2)$$

Gantikan (2) dan (3) ke dalam (4),

$$2y = \frac{dy}{dx}(x - 1) + 4\left(\frac{d^2y}{dx^2}\right)$$

$$4\left(\frac{d^2y}{dx^2}\right) + (x - 1)\frac{dy}{dx} = 2y(\text{terbukti})$$

21. Sebuah pepejal dibentuk dengan menggabungkan sebuah hemisfera yang berjejari  $r$  cm dan ketinggian  $h$  cm. Sekiranya luas permukaan pepejal adalah  $180\pi$  cm<sup>2</sup>, kira nilai  $r$  dan  $h$  supaya isipadu pepejal tersebut adalah maksimum.

*A solid is formed by joining a hemisphere of radius  $r$  cm to a cylinder of radius  $r$  cm and height  $h$  cm. If the total surface area of the solid is  $180\pi$  cm<sup>2</sup>, calculate the value of  $r$  and of  $h$  such that the volume of the solid is a maximum.*

[8 markah]

[8 marks]

**Penyelesaian 21 :**

Luas permukaan pepejal ialah  $180\pi$  cm<sup>2</sup>

Jumlah luas permukaan = Hemisfera + silinder

$$180\pi = 2\pi r + \pi r^2 + 2\pi rh$$

$$180 = 3r^2 + 2rh$$

$$2rh = 180 - 3r^2$$

$$h = \frac{180 - 3r^2}{2r} \text{ _____ (1)}$$

Jumlah isipadu = silinder + hemisfera

$$V = \pi r^2 h + \frac{2}{3} \pi r^3 \text{ _____ (2)}$$

Ganti (1) dalam (2),

$$V = \pi r^2 \left( \frac{180 - 3r^2}{2r} \right) + \frac{2}{3} r^3$$

$$V = \frac{180\pi r - 3\pi r^3}{2} + \frac{2}{3} r^3$$

$$= \pi r^2 \left( \frac{180 - 3r^2}{2r} \right) + \frac{2}{3} r^3$$

$$= 90\pi r - \frac{5}{2} \pi r^3$$

$$\frac{dV}{dr} = 90\pi - \frac{5}{2} \pi r^2$$

$$\frac{dV}{dr} = 0, \quad 0 = 90\pi - \frac{5}{2} \pi r^2$$

$$\frac{5}{2} \pi r^2 = 90\pi$$

$$r = \pm 6$$

$$r = 6$$

$$\text{Bila } r = 6, \quad \frac{d^2V}{dr^2} = -5\pi r = -5(6)\pi = -30\pi (< 0)$$

V ialah maksimum bila  $r = 6$

$$\text{Bila } r = 6, \quad h = \frac{180 - 3(6)^2}{2(6)} = 6$$

22. Diberi persamaan suatu lengkung ialah  $y = 2x(1-x)^4$  dan lengkung itu melalui  $P(2, 4)$ . Cari

*Given the equation of a curve is  $y = 2x(1-x)^4$  and the curve passes through  $P(2,4)$ . Find*

(a) Kecerunan lengkung pada titik  $P$ .  
*The slope of the curve at point  $P$ .*

(b) Persamaan garis normal kepada lengkung pada titik  $P$ .  
*The equation of the line normal to the curve at point  $P$ .*

[7 markah]

[7 marks]

**Penyelesaian 22 :**

$$\frac{du}{dx} = 2, \quad \frac{du}{dx} = -4(1-x)^3$$

$$\frac{dy}{dx} = 2x[-4(1-x)^3] + (1-x)^4(2)$$

Apabila  $x = 2$

$$\begin{aligned} \frac{dy}{dx} &= 4[-4(-1)^3] + (-2)^4(2) \\ &= 18 \end{aligned}$$

Kecerunan normal,  $m_n$

$$m_n \times \frac{dy}{dx} = -1$$

$$m_n = \frac{-1}{18}, (2,4)$$

$$y = mx + c$$

$$4 = \frac{-1}{18}(2) + c$$

$$c = \frac{37}{9}$$

$$\therefore y = \frac{-1}{18}x + \frac{37}{9}$$



23. Lengkung  $y = x^3 - 6x^2 + 9x + 1$  melalui titik  $A(2,3)$  dan mempunyai dua titik pusingan,  $P(3,1)$  dan  $Q$ , cari

*The curve  $y = x^3 - 6x^2 + 9x + 1$  passes through point  $A(2,3)$  and has two turning points  $P(3,1)$  and  $Q$ , find*

(a) Kecerunan lengkung itu pada  $A$ .

*The slope of the curve is at  $A$ .*

(b) Persamaan normal kepada lengkung itu pada  $A$ .

*The normal equation to the curve at  $A$ .*

(c) Koordinat  $Q$  dan tentukan sama ada  $Q$  adalah titik maksimum atau titik minimum.

*The coordinates of  $Q$  and determine whether  $Q$  is the maximum point or the minimum point.*

[10 markah]

[10 marks]

**Penyelesaian 23 :**

(a)  $y = x^3 - 6x^2 + 9x + 1$

$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

Apabila  $x = 2$

$$\frac{dy}{dx} = 3(2)^2 - 12(2) + 9$$

$$x = -3$$

(b)  $-3 \times m_n = -1$

$$m_n = \frac{1}{3}$$

$$y = \frac{1}{3}x + c \rightarrow (2,3)$$

$$3 = \frac{1}{3}(2) + c$$

$$c = \frac{7}{3}$$

$$y = \frac{1}{3}x + \frac{7}{3}$$

(c)  $\frac{dy}{dx} = 0$

$$3x^2 - 12x + 9 = 0$$

$$3(x-3)(x-1) = 0$$

$$x = 3, \quad x = 1$$

$$P(3,1) \quad Q(1, \_)$$

Apabila  $x = 1$   
 $y = (1)^3 - 6(1)^2 + 9(1) + 1$   
 $= 5$

Maka  $Q(1,5)$

$$\frac{d^2y}{dx^2} = 6x - 12, x = 1$$

$$= 6(1) - 12$$

$$= -6 < 0 \text{ maksimum}$$

Maka  $Q(1,5)$  ialah titik maksimum

24. Lengkung  $y = mx + \frac{n}{2x-1}$  mempunyai titik pusingan pada titik  $(2, 7)$ .

Cari nilai  $m$  dan  $n$ .

*The curve  $y = mx + \frac{n}{2x-1}$  has turning point at  $(2, 7)$ . Find the values of  $m$  and  $n$ .*

[4 markah]

[4 marks]

**Penyelesaian 24 :**

$$\frac{dy}{dx} = m - \frac{2n}{(2x-1)^2}$$

$$\text{Substitute } x = 2 \text{ in } \frac{dy}{dx} = 0, m - \frac{2n}{[2(2)-1]^2} = 0$$

$$2n = 9m$$

$$\text{Substitute } x = 2, \text{ and } y = 7 \text{ in } y = mx + \frac{n}{2x-1}$$

$$21 = 6m + n$$

Solve simultaneous equation,  $m = 2$  and  $n = 9$ .

# BAB 3

## PENGAMIRAN INTEGRATION

1. Diberi  $\int \frac{6}{(2x-1)^2} dx = k(2-x)^n + c$ .

Given  $\int \frac{6}{(2x-1)^2} dx = k(2-x)^n + c$ .

(a) Carikan nilai k dan nilai n.  
*Find the value of k and of n.*

(b) Seterusnya cari nilai  $\int_1^2 \frac{6}{(2x-1)^2} dx$ .

Hence find the value of  $\int_1^2 \frac{6}{(2x-1)^2} dx$ .

[4 markah ]

[ 4 marks]

### Penyelesaian 1 :

1. (a)  $\int 6(2x-1)^{-2} dx$

$$\frac{6(2x-1)^{-1}}{2(-1)} + c$$

$$-3(2x-1)^{-1} + c$$

$$k = -3, n = -1$$

(b)  $\left[ \left( \frac{-3}{2x-1} \right) \right]_1^2$

$$= \left( \frac{-3}{3} \right) - \left( \frac{-3}{1} \right)$$

$$= 2$$

2. a) Diberi  $\int_2^5 f(x)dx = 6$ , cari nilai  $m$  jika  $\int_2^5 [f(x) + mx]dx = 27$ .

Given  $\int_2^5 f(x)dx = 6$ , find the value of  $m$  if  $\int_2^5 [f(x) + mx]dx = 27$ .

b)  $\int \frac{6}{5x^n} dx = \frac{m}{5x^6} + c$ , dengan keadaan  $c$ ,  $m$  dan  $n$  ialah pemalar.

Cari nilai  $m$  dan nilai  $n$ .

$\int \frac{6}{5x^n} dx = \frac{m}{5x^6} + c$ , such that  $c$ ,  $m$  dan  $n$  is a constant. Find the value of  $m$  and  $n$ .

c) Cari  $\int_4^a (x + 1) dx$  dalam sebutan  $a$ .

Find  $\int_4^a (x + 1) dx$  in terms of  $a$ .

[ 8 Markah ]

[ 8Marks ]

### Penyelesaian 2:

$$a) \int_2^5 f(x)dx + \int_2^5 mx dx = 27$$

$$6 + \left[ \frac{mx^2}{2} \right]_2^5 = 27$$

$$\frac{m(5)^2}{2} - \frac{m(2)^2}{2} = 21$$

$$m = 2$$

$$b) \int \frac{6x^{-n}}{5} dx = \frac{m}{5x^6} + c$$

$$\frac{6x^{-n+1}}{5(-n+1)} + c = \frac{m}{5x^6} + c$$

$$\frac{6}{5(-n+1)x^{n-1}} + c = \frac{m}{5x^6} + c$$

$$n = 7, m = -1$$

$$c) \left[ \frac{x^2}{2} + x \right]_4^a$$

$$\frac{(a)^2}{2} + a - 12$$

3. Fungsi kecerunan suatu lengkung ialah  $\frac{dy}{dx} = (2x - 5)$ . Diberi  $P(2,3)$  terletak pada lengkung itu.

*The gradient function of a curve is  $\frac{dy}{dx} = (2x - 5)$ . Given that  $P(2,3)$  lies on the curve.*

Cari

Find

- (a) kecerunan tangent pada titik  $P$ .  
*the gradient of the tangent at point  $P$ .*
- (b) persamaan lengkung itu.  
*the equation of the curve.*

[4 markah ]

[ 4 marks]

### Penyelesaian 3 :

- (a) Kecerunan /Gradient

$$2(2) - 5 = -1$$

- (b)  $\frac{dy}{dx} = (2x - 5)$

$$y = x^2 - 5x + c$$

$$3 = (2)^2 - 5(2) + c$$

$$c = 9$$

Persamaan lengkung ialah

*Equation of the curve is  $y = x^2 - 5x + 9$*

4. Diberi bahawa fungsi kecerunan suatu lengkung ialah  $\frac{dy}{dx} = \frac{36}{(2x+1)^3}$  dan ia melalui titik (1, 4). Cari persamaan lengkung itu.

*It is given that the gradient function of a curve is  $\frac{dy}{dx} = \frac{36}{(2x+1)^3}$  and it passes through the point (1, 4). Find the equation of the curve.*

[4 markah ]

[ 4 marks]

**Penyelesaian 4 :**

$$y = \frac{36(2x+1)^{-2}}{(-2)(2)} + c$$

$$y = \frac{-9}{(2x+1)^2} + c$$

$$4 = \frac{-9}{(2(1)+1)^2} + c$$

$$c = 5$$

$$y = \frac{-9}{(2x+1)^2} + 5$$

5. (a) Diberi  $\frac{dy}{dx} = 14$  dan  $y = 9$  apabila  $x = 3$ , ungkapkan  $y$  dalam sebutan  $x$ .

*Given  $\frac{dy}{dx} = 14$  and  $y = 9$  when  $x = 3$ , find  $y$  in terms of  $x$ .*

(b) Diberi bahawa  $\int_1^2 g(x)dx = 6$  dan  $\int_2^5 g(x) dx = 10$ ,

cari nilai bagi  $\int_1^5 5 g(x)dx$ .

*Given that  $\int_1^2 g(x)dx = 6$  and  $\int_2^5 g(x) dx = 10$ ,*

*find the value of  $\int_1^5 5 g(x)dx$ .*

[5 markah ]

[ 5 marks]

### Penyelesaian 5 :

$$(a) y = \int 14dx$$
$$y = 14x + c$$

$$9 = 14(3) + c$$

$$c = -33$$

$$y = 14x - 33$$

$$(b) 5[\int_1^2 g(x)dx + \int_2^5 g(x)dx]$$

$$5(6+10)$$

$$80$$

6. (a) Fungsi kecerunan suatu lengkung yang melalui titik (1, 2) ialah  $(3x - 4)^5$ . Cari persamaan lengkung itu.

*The gradient function of a curve passing through the point (1, 2) is  $(3x - 4)^5$ . Find the equation of the curve.*

- (b) Diberi  $\frac{d}{dx} \left[ \frac{x^2}{x-1} \right] = 2g(x)$  cari nilai bagi  $\int_2^3 \frac{1}{3} g(x) dx$ .

*Given  $\frac{d}{dx} \left[ \frac{x^2}{x-1} \right] = 2g(x)$ , find the value of  $\int_2^3 \frac{1}{3} g(x) dx$ .*

[6 markah ]

[ 6 marks]

**Penyelesaian 6 :**

$$\begin{aligned} \text{(a)} \quad y &= \frac{(3x-4)^6}{(3)(6)} + c \\ 2 &= \frac{(3(1)-4)^6}{(3)(6)} + c \\ y &= \frac{(3x-4)^6}{18} + \frac{35}{18} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \left[ \frac{x^2}{x-1} \right] &= \int 2g(x) dx \\ \int_2^3 \frac{1}{3} g(x) dx &= \frac{1}{6} \left[ \frac{x^2}{x-1} \right]_2^3 \\ &= \frac{1}{6} \left( \frac{3^2}{3-1} \right) - \frac{1}{6} \left( \frac{2^2}{2-1} \right) \\ &= \frac{1}{12} \end{aligned}$$



7. Satu lengkung mempunyai fungsi kecerunan,  $\frac{dy}{dx} = 2x + q$ , dengan keadaan  $q$  ialah pemalar. Kecerunan normal kepada lengkung itu pada titik  $(1, 5)$  ialah  $\frac{2}{3}$ . Cari

*A curve has a gradient function,  $\frac{dy}{dx} = 2x + q$ , where  $q$  is a constant. The gradient of the normal to the curve at a point  $(1, 5)$  is  $\frac{2}{3}$ . Find*

- (a) nilai  $q$ ,  
*the value of  $q$ ,*
- (b) persamaan lengkung tersebut.  
*the equation of the curve.*

[4 markah ]

[ 4 marks]

**Penyelesaian 7 :**

(a)  $2(1) + q = -\frac{3}{2}$   
 $q = -\frac{7}{2}$

(b)  $y = x^2 - \frac{7}{2}x + c$   
 $5 = (1)^2 - \frac{7}{2}(1) + c$

$$y = x^2 - \frac{7}{2}x + \frac{15}{2}$$

8. Diberi  $\int_{-2}^4 f(x)dx = -18$ , cari

Given that  $\int_{-2}^4 f(x)dx = -18$ , find

(a)  $\int_{-2}^4 \left[ \frac{2}{3} f(x) + 5 \right] dx.$

(b)  $\int_{-2}^0 f(x)dx + \int_0^4 [f(x) - x] dx$

[7 markah ]

[7 marks]

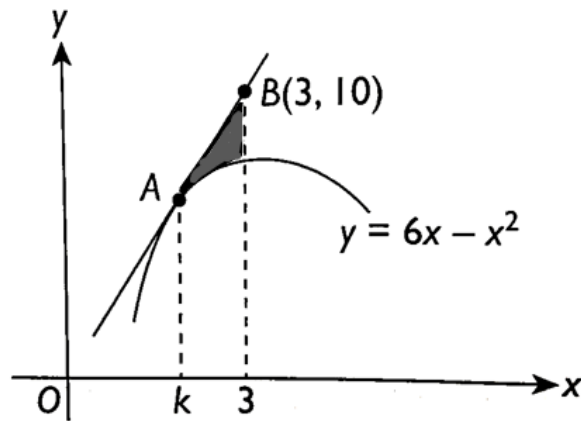
**Penyelesaian 8 :**

(a)  $\int_{-2}^4 \left[ \frac{2}{3} f(x) + 5 \right] dx$   
 $= \int_{-2}^4 \frac{2}{3} f(x) dx + \int_{-2}^4 5 dx$   
 $= \frac{2}{3} \int_{-2}^4 f(x) dx + [5x]_{-2}^4$   
 $= \frac{2}{3} (-18) + [5(4) - 5(-2)]$   
 $= -12 + 30$   
 $= 18$

(b)  $\int_{-2}^0 f(x) dx + \int_0^4 [f(x) - x] dx$   
 $= \int_{-2}^0 f(x) dx + \int_0^4 f(x) dx - \int_0^4 x dx$   
 $= \int_{-2}^4 f(x) dx - \left[ \frac{x^2}{2} \right]_0^4$   
 $= -18 - \left( \frac{4^2}{2} - 0 \right)$   
 $= -26$

9. Rajah di bawah menunjukkan lengkung  $y = 6x - x^2$  dan tangen kepada lengkung itu pada titik A melalui titik B.

The diagram shows a curve  $y = 6x - x^2$  and the tangent to the curve at point A passes point B.



- (a) Tunjukkan bahawa  $k = 2$ .  
*Show that  $k = 2$ .*
- (b) Hitung luas rantau berlorek.  
*Calculate the area of the shaded region.*

[8 markah ]

[8 marks]

**Penyelesaian 9 :**

(a) Apabila/ When  $x = k, y = 6x - x^2$

$$\therefore A(k, 6k - k^2)$$

$$\frac{dy}{dx} = 6 - 2x = 6 - 2k$$

$$\frac{10 - (6k - k^2)}{3 - k} = 6 - 2k$$

$$10 - 6k + k^2 = (6 - 2k)(3 - k)$$

$$10 - 6k + k^2 = 18 - 12k + 2k^2$$

$$k^2 - 6k + 8 = 0$$

$$(k - 2)(k - 4) = 0$$

$$k = 2 \text{ atau/or } k = 4$$

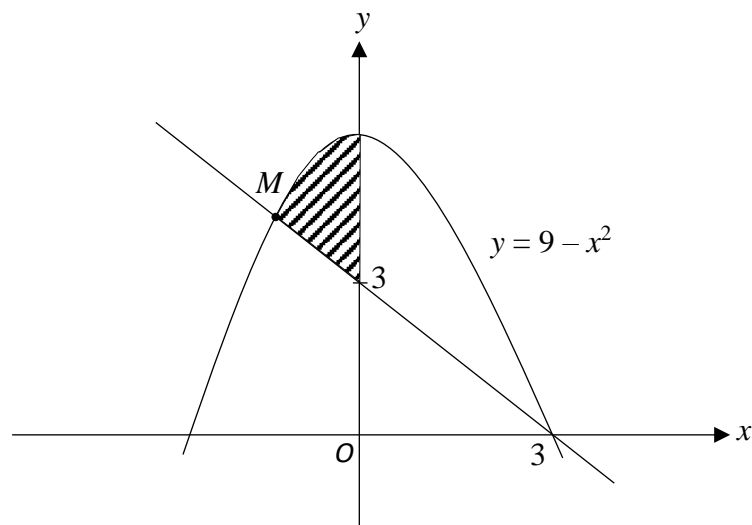
$$k < 3 \Rightarrow k = 2$$

$$A(2, 8)$$

$$\begin{aligned}
 \text{(b) Luas/ Area} &= \frac{1}{2}(8+10)(1) - \int_2^3 (6x - x^2) dx \\
 &= 9 - \left[ 3x^2 - \frac{x^3}{3} \right]_2^3 \\
 &= 9 - \left[ 3(3)^2 - \frac{3^3}{3} - \left( 3(2)^2 - \frac{2^3}{3} \right) \right] \\
 &= 9 - \left[ 27 - 9 - 12 + \frac{8}{3} \right] \\
 &= \frac{1}{3} \text{ unit}^2
 \end{aligned}$$

10. Rajah menunjukkan lengkung  $y = 9 - x^2$  yang menyilang satu garis lurus pada titik  $(3, 0)$  dan titik  $M$ .

*Diagram shows the curve  $y = 9 - x^2$  which intersects a straight line at point  $(3, 0)$  and point  $M$ .*



Hitung  
Calculate

- (a) koordinat  $M$ ,  
*the coordinates of  $M$ ,*
- (b) luas bagi rantau berlorek.  
*the area of the shaded region.*

[7 markah ]

[ 7 marks ]

**Penyelesaian 10 :**

(a) *Equation of straight line;  $y = -x + 3$*

$$9 - x^2 = -x + 3$$

$$x^2 - x - 6 = 0$$

$$(x + 2)(x - 3) = 0$$

$$x = -2, x = 3$$

$$M(-2, 5)$$

(b) 
$$\text{Area} = \int_2^0 9 - x^2 dx$$

$$= \left[ 9x - \frac{x^3}{3} \right]_{-2}^0$$

$$= 15\frac{1}{3}$$

$$\text{Area of trapezium} = \frac{1}{2} \times (5 + 3) \times 2$$

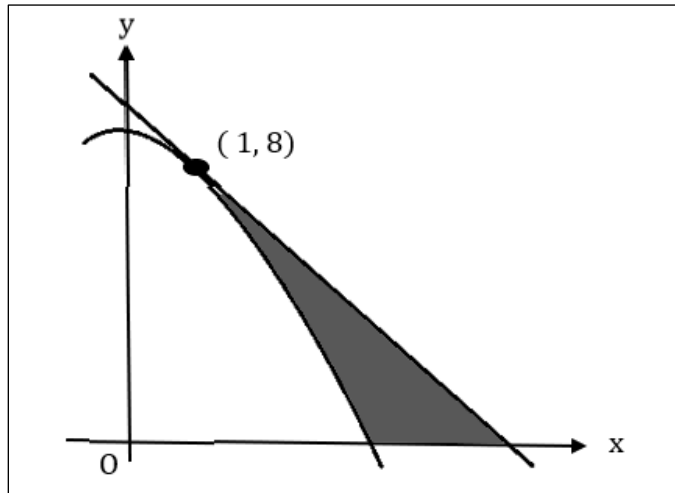
$$= 8$$

$$\text{Area of shaded region} = 15\frac{1}{3} - 8$$

$$= 7\frac{1}{3}$$

11. Rajah menunjukkan lengkung  $y = 9 - x^2$  dan garis lurus  $y = mx + c$  ialah tangen ke titik lengkung  $(1, 8)$ .

*The diagram shows the curve  $y = 9 - x^2$  and the straight line  $y = mx + c$  is a tangent to the curve at point  $(1, 8)$ .*



- (a) Cari nilai-nilai  $m$  dan  $c$ .  
*Find the values of  $m$  and  $c$ .*
- (b) Cari luas kawasan berlorek.  
*Find the area of the shaded region.*
- (c) Diberi isipadu yang dihasilkan apabila kawasan dibatasi oleh lengkung, paksi-y dan garis lurus  $y = k$ , diputar  $180^\circ$  pada paksi-y ialah  $16\pi$  unit<sup>3</sup>. Cari nilai  $k$ .  
*Given the volume generated when the region bounded by curve, y-axis and the straight lines  $y = k$ , is revolved through  $180^\circ$  about the y-axis is  $16\pi$  unit<sup>3</sup>. Find the value of  $k$ .*

[10 markah ]

[10 marks]

**Penyelesaian 11 :**

(a)  $m = \frac{dy}{dx}$  when  $x = 1$   
 $m = -2x = -2(1) = -2$

At (1,8);  $8 = -2(1) + c$   
 $c = 10$

(b)  $Area = \left(\frac{1}{2} \times 4 \times 8\right) - \int_1^3 9 - x^2 dx$   
 $= 16 - \left[\left(9x - \frac{x^3}{3}\right)\right]_1^3$   
 $= 16 - \left[\left(27 - \frac{27}{3}\right) - \left(9 - \frac{1}{3}\right)\right]$   
 $= 6\frac{2}{3}$

(c)  $Volume = \frac{\pi}{2} \int_k^9 x^2 dy$   
 $16\pi = \frac{\pi}{2} \int_k^9 9 - y dy$   
 $32 = \left[\left(9y - \frac{y^2}{2}\right)\right]_k^9$

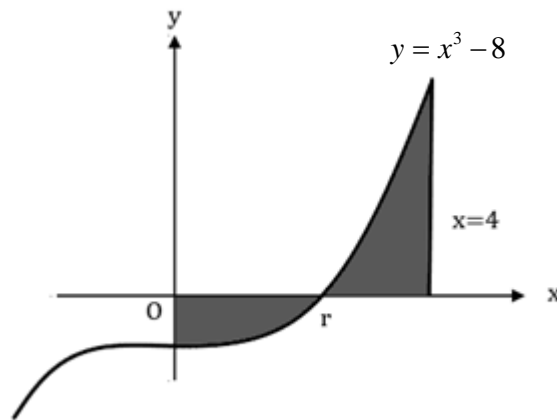
$$64 = 81 - 18k + k^2$$

$$k^2 - 18k + 17 = 0$$

$$(k - 17)(k - 1) = 0$$
$$k = 1$$

12. (a) Rajah di bawah menunjukkan lengkung bagi fungsi  $y = x^3 - 8$ .

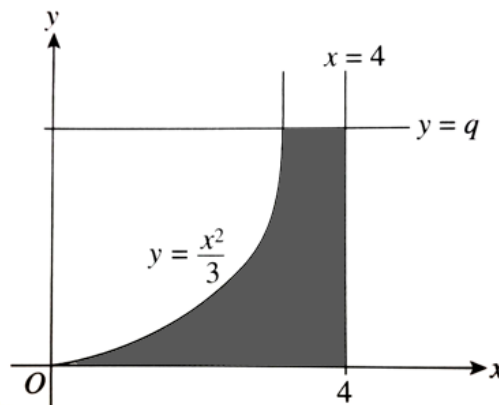
Diagram below shows the curve of the function



- (i) Cari nilai  $r$ .  
Find the value of  $r$ .
- (ii) Hitung luas kawasan berlorek.  
Calculate the area of the shaded region.

[5 markah]  
[5 marks]

- (b) Rajah di bawah menunjukkan lengkung  $y = \frac{x^2}{3}$  dan dua garis lurus,  $x = 4$  dan  $y = q$ .



Apabila kawasan berlorek diputarkan melalui  $360^\circ$  pada paksi- $y$ , isi padu janaan ialah  $34\frac{1}{2}\pi$  unit<sup>3</sup>. Jika  $q < 5$ , cari nilai  $q$ .

When the shaded region is rotated through  $360^\circ$  about the  $y$ -axis, the volume generated is  $34\frac{1}{2}\pi$  unit<sup>3</sup>. If  $q < 5$ , find the value of  $q$ .

[5 markah]  
[5 marks]



**Penyelesaian 12 :**

$$\begin{aligned} \text{(a) (i)} \quad x^3 - 8 &= 0 \\ x^3 &= 8 \\ x &= 2 \end{aligned}$$

$$\therefore r = 2$$

(ii) Luas kawasan berlorek

$$\begin{aligned} &= \left| \int_0^2 (x^3 - 8) dx \right| + \int_2^4 (x^3 - 8) dx \\ &= \left[ \left. \frac{x^4}{4} - 8x \right|_0^2 \right] + \left[ \frac{x^4}{4} - 8x \right]_2^4 \\ &= |(4-16)-0| + [(64-32)-(4-16)] \\ &= |-12| + [32 - (-12)] \\ &= 56 \text{ unit}^2 \end{aligned}$$

$$\text{(b) } y = \frac{x^2}{3} \Rightarrow x^2 = 3y$$

$$\pi(4^2)(q) - \pi \int_0^q 3y dy = 34 \frac{1}{2} \pi$$

$$16\pi q - \pi \left[ \frac{3y^2}{2} \right]_0^q = 34 \frac{1}{2} \pi$$

$$16q - \frac{3q^2}{2} = \frac{69}{2}$$

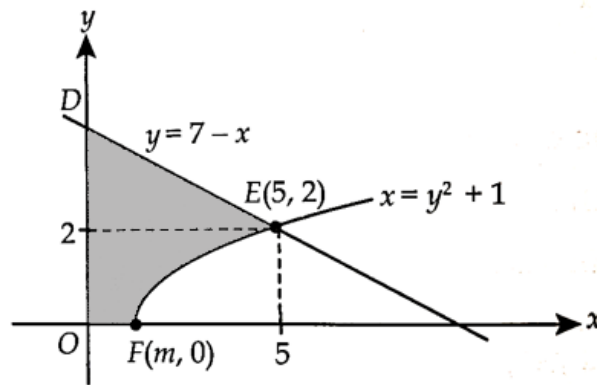
$$32q - 3q^2 = 69$$

$$3q^2 - 32q + 69 = 0$$

$$(3q - 23)(q - 3) = 0$$

Oleh sebab  $q < 5$ ,  $q = 3$

13. Rajah di bawah menunjukkan garis lurus  $DE$  bersilang dengan lengkung  $x = y^2 + 1$  pada titik  $E$ .  
 Diagram below shows the straight line  $DE$  intersects the curve  $x = y^2 + 1$  at point  $E$ .



Cari  
 Find

- (a) nilai  $m$ ,  
 the value of  $m$ ,
- (b) luas dalam unit<sup>2</sup>, bagi rantau yang berlorek,  
 the area, in units<sup>2</sup>, of the shaded region,
- (c) isi padu janaan, dalam sebutan  $\pi$ , apabila rantau yang dibatasi oleh lengkung  $x = y^2 + 1$ , garis lurus  $x = 5$  dan paksi- $x$  diputarakan melalui  $360^\circ$  pada paksi- $x$ .  
 the volume generated, in terms of  $\pi$ , when the region bounded by the curve  $x = y^2 + 1$ , the straight line  $x = 5$  and the  $x$ -axis is revolved through  $360^\circ$  about the  $x$ -axis

[10 markah ]  
 [10 marks]

**Penyelesaian 13 :**

(a) Diberi/ Given  $x = y^2 + 1$   
 $F(m, 0) : m = (0)^2 + 1$   
 $= 1$

(b) Luas kawasan berlorek/ Area of the shaded region  
 $= \frac{1}{2} \times 5 \times (7 - 2) + \int_0^2 x \, dy$

$$\begin{aligned}
&= \frac{1}{2} \times 5 \times 5 + \int_0^2 (y^2 + 1) dy \\
&= \frac{25}{2} + \left[ \frac{1}{3} y^3 + y \right]_0^2 \\
&= \frac{25}{2} + \left[ \left( \frac{1}{3} (2)^3 + (2) \right) - \left( \frac{1}{3} (0)^3 + (0) \right) \right] \\
&= \frac{25}{2} + \frac{14}{3} \\
&= \frac{103}{6} \text{ unit}^2
\end{aligned}$$

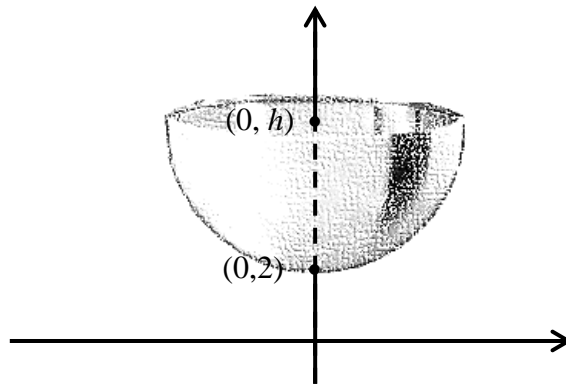
(c) Diberi/ Given  $x = y^2 + 1$   
 $y^2 = x - 1$

Isi padu/ volume

$$\begin{aligned}
&= \pi \int_1^5 y^2 dx \\
&= \pi \int_1^5 (x-1) dx \\
&= \pi \left[ \frac{1}{2} x^2 - x \right]_1^5 \\
&= \pi \left[ \left( \frac{1}{2} (5)^2 - (5) \right) - \left( \frac{1}{2} (1)^2 - (1) \right) \right] \\
&= \pi \left[ \frac{15}{2} - \left( -\frac{1}{2} \right) \right] \\
&= 8 \pi \text{ unit}^3
\end{aligned}$$

14. Mangkuk dalam Rajah 3 mempunyai permukaan dalam yang melengkung mengikut persamaan  $y = x^2 + 2$ .

*A bowl in Diagram 3 has an internal surface represented by the equation  $y = x^2 + 2$ .*



Rajah 14  
Diagram 14

Kedalaman mangkuk itu ialah  $d$  cm. Jika mangkuk itu boleh dipenuhi air sebanyak  $4.5\pi \text{ cm}^3$ , cari nilai  $d$ .

*The depth of the bowl is  $d$  cm. If the bowl can fill up with  $4.5\pi \text{ cm}^3$  of water, find the value of  $d$ .*

[5 markah]  
[5marks]

**Penyelesaian 14 :**

$$I = \pi \int_2^h (y - 2) dy = 4.5\pi \dots [1m]$$

$$\left[ \frac{y^2}{2} - 2y \right]_2^h = 4.5 \dots [1m]$$

$$\left[ \frac{h^2}{2} - 2(h) \right] - \left[ \frac{2^2}{2} - 2(2) \right] = 4.5 \dots [1m]$$

$$h^2 - 4h - 4 - 8 = 9$$

$$h = 7 \dots [1m]$$

$$\text{maka } d = 7 - 2 = 5 \dots [1m]$$

16. Diberi  $\int_3^6 g(x) dx = \frac{7}{2}$ , cari

Given that  $\int_3^6 g(x) dx = \frac{7}{2}$ , find

(a)  $\int_6^3 \frac{1}{2} g(x) dx$

(b) nilai  $q$  dengan keadaan  $\int_3^6 [g(x) - 2q] dx = 16$ .

the value of  $q$  such that  $\int_3^6 [g(x) - 2q] dx = 16$ .

[6 markah]

[6 marks]

**Penyelesaian 16 :**

(a)  $\int_6^3 \frac{1}{2} g(x) dx = -\int_3^6 \frac{1}{2} g(x) dx$

$$= -\frac{1}{2} \left( \frac{7}{2} \right)$$

$$= -\frac{7}{4}$$

(b)  $\int_3^6 [g(x) - 2q] dx = 16$

$$\int_3^6 g(x) dx - \int_3^6 2q dx = 16$$

$$\int_3^6 g(x) dx - [2qx]_3^6 = 16$$

$$\frac{7}{2} - (12q - 6q) = 16$$

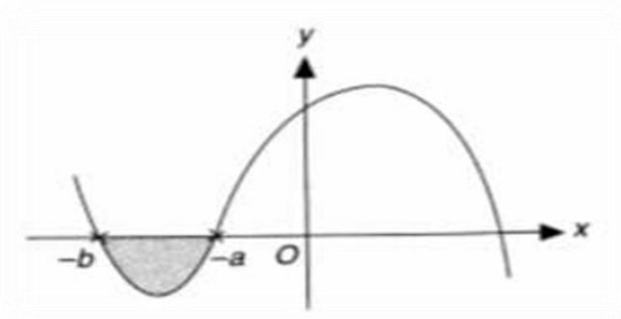
$$\frac{7}{2} - 6q = 16$$

$$6q = -\frac{25}{2}$$

$$q = -\frac{25}{12}$$

17. Rajah 17 menunjukkan kawasan berlorek yang dibatasi oleh graf  $y = h(x)$  dan paksi- $x$  dari  $x = -a$  ke  $x = -b$ .

Diagram 17 shows a shaded region bounded by the graph  $y = h(x)$  and  $x$ -axis from  $x = -a$  to  $x = -b$ .



Rajah 17  
Diagram 17

- (a) Diberi bahawa  $\frac{d}{dx}[2g(x)] = h(x)$  dan  $[g(x)]_{-a}^{-b} = -9$ , tentukan luas kawasan berlorek.

It is given  $\frac{d}{dx}[2g(x)] = h(x)$  and  $[g(x)]_{-a}^{-b} = -9$ , determine the area of the shaded region.

- (b) Graf  $y = h(x)$  melalui titik  $(4,21)$ . Diberi  $\frac{dy}{dx}[h(x)] = 3x^2 - 4x + 5$ , cari  $h(x)$  dalam sebutan  $x$ .

The graph  $y = h(x)$  passes through point  $(4,21)$ . Given  $\frac{d}{dx}[h(x)] = 3x^2 - 4x + 5$ , Find  $h(x)$  in terms of  $x$ .

[5 markah]  
[5 marks]

**Penyelesaian 17 :**

(a) Luas kawasan berlorek

$$\begin{aligned} &= \int_{-a}^{-b} h(x) dx \\ &= [2g(x)]_{-a}^{-b} \\ &= 2(-9) \\ &= -18 \end{aligned}$$

$\therefore$  luas kawasan berlorek ialah 18 unit<sup>2</sup>.

$$\begin{aligned} \text{(b) } h(x) &= \int 3x^2 - 4x + 5 dx \\ &= \frac{3x^3}{3} - \frac{4x^2}{2} + 5x + c \\ &= x^3 - 2x^2 + 5x + c \end{aligned}$$

Oleh kerana  $h(x)$  melalui titik (4,21),  $x = 4$  dan  $h(x) = 21$

$$\begin{aligned} h(x) &= x^3 - 2x^2 + 5x + c \\ 21 &= 4^3 - 2(4)^2 + 5(4) + c \\ c &= -31 \end{aligned}$$

$$\therefore h(x) = x^3 - 2x^2 + 5x - 31$$

18. (a) Diberi  $g(x) = \sqrt[5]{x} + 5$ , cari  $\int g'(x) dx$ .

*Given  $g(x) = \sqrt[5]{x} + 5$ , find  $\int g'(x) dx$ .*

(b) Cari  $\int_{-4}^a (2x+1) dx$  dalam sebutan  $a$ .

*Find  $\int_{-4}^a (2x+1) dx$  in term of  $a$ .*

[5 markah]

[5 marks]

**Penyelesaian 18 :**

$$\begin{aligned} \text{a) } \int g'(x)dx &= g(x) \dots [1m] \\ &= \sqrt[3]{x} + 5 \dots [1m] \end{aligned}$$

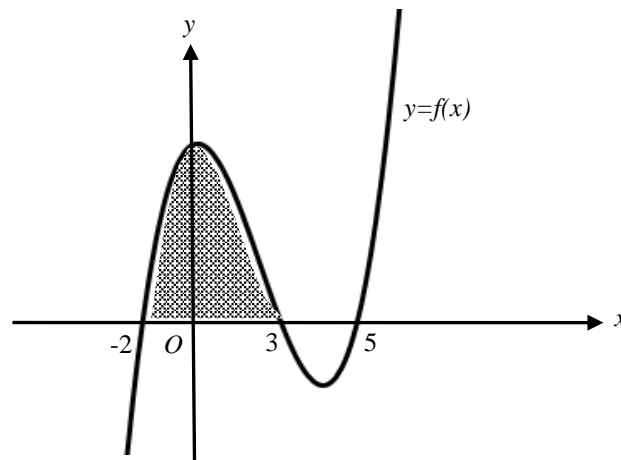
$$\begin{aligned} \text{b) } \int_{-4}^a (2x+1)dx &= [x^2 + x]_{-4}^a \dots [1m] \\ &= (a^2 + a) - ((-4)^2 + (-4)) \dots [1m] \\ &= a^2 + a - 12 \dots [1m] \end{aligned}$$

19. (a) Diberi  $\int_1^2 g(x)dx = 4$  dan  $\int_2^5 g(x)dx = 7$ , cari  $\int_1^5 [g(x) - 2x]dx$

Given that  $\int_1^2 g(x)dx = 4$  and  $\int_2^5 g(x)dx = 7$ , find  $\int_1^5 [g(x) - 2x]dx$ .

(b) Rajah 19 menunjukkan sebahagian daripada lengkung  $y = f(x)$ .

Diagram 19 shows part of the curve  $y = f(x)$ .



Rajah 19  
Diagram 19

Kawasan berlorek ditakrifkan sebagai  $\int_a^b f(x)dx = 8$ .

The shaded region is defined as  $\int_a^b f(x)dx = 8$ .

i) Nyatakan nilai  $a$  dan  $b$ .

State the value of  $a$  and of  $b$ .



- ii) Diberi bahawa luas kawasan yang dibatasi oleh  $y = f(x)$  dan paksi- $x$  dari  $x = -2$  ke  $x = 5$  ialah 10. Nyatakan  $\int_3^5 f(x)dx$ .

*Given that the area bounded by  $y = f(x)$  and the  $x$ -axis from  $x = -2$  to  $x = 5$  is 10. State  $\int_3^5 f(x)dx$ .*

[6 markah]  
[6 marks]

**Penyelesaian 19 :**

$$\begin{aligned} \text{a) } \int_1^5 [g(x) - 2x]dx &= \int_1^2 g(x)dx + \int_2^5 g(x)dx - \left[ x^2 \right]_1^5 \dots [1m] \\ &= 4 + 7 - [5^2 - 1^2] \dots [1m] \\ &= -13 \dots [1m] \end{aligned}$$

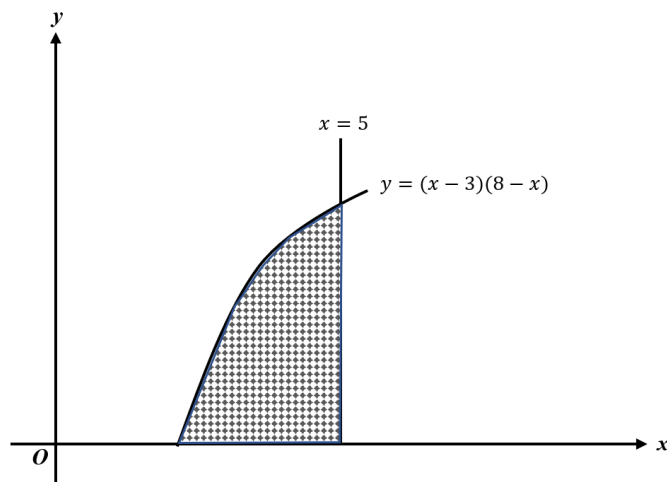
b) i)  $a = -2, b = 3 \dots [1m]$

$$\begin{aligned} \text{ii) } \int_3^5 f(x)dx &= 10 - 8 \dots [1m] \\ &= 2 \dots [1m] \end{aligned}$$

20. ( a ) Diberi  $\frac{d}{dx} [f(x)] = 3h(x)$ , cari  $\int h(x)dx$ .  
Given  $\frac{d}{dx} [f(x)] = 3h(x)$ , find  $\int h(x)dx$ .

( b ) Diberi  $\int_{-1}^2 h(x)dx = 21$ , cari  $\int_{-1}^2 \frac{h(x)}{3} dx$   
Given  $\int_{-1}^2 h(x)dx = 21$ , find  $\int_{-1}^2 \frac{h(x)}{3} dx$

( c ) Rajah menunjukkan sebahagian daripada lengkung  $y = (x - 3)(8 - x)$ .  
Diagram shows a part of the curve  $y = (x - 3)(8 - x)$ .



Cari luas bagi kawasan yang berlorek.  
*Find the area of the shaded region.*

[8 Markah]  
 [ 8 Marks]

**Penyelesaian 20 :**

a)  $f(x) = \int 3h(x)dx$   
 $\frac{f(x)}{3} = \int h(x)dx$

b)  $\frac{1}{3} \int_{-1}^2 h(x)dx$   
 7

c) (3,0)  
 $= \int_3^5 -x^2 + 11x - 24 dx$   
 $= \left[ -\frac{x^3}{3} + \frac{11x^2}{2} - 24x \right]_3^5$   
 $= \left( -\frac{(5)^3}{3} + \frac{11(5)^2}{2} - 24(5) \right) - \left( -\frac{(3)^3}{3} + \frac{11(3)^2}{2} - 24(3) \right)$   
 $= 7\frac{1}{3}$

21 (a) Diberi bahawa  $\int_2^p f(x)dx = 6$  dan  $\int_2^p [f(x)+2]dx = 10$ , cari nilai  $p$ .

*It is given that  $\int_2^p f(x)dx = 6$  dan  $\int_2^p [f(x)+2]dx = 10$ , find the value of  $p$ .*

(b) Diberi fungsi kecerunan bagi suatu lengkung ialah  $2x - k$ , dengan keadaan  $k$  ialah pemalar. Lengkung itu mempunyai titik pusingan pada (3,5).

*Given the gradient function of a curve is  $2x - k$ , where  $k$  is a constant. The curve has a turning point at (3,5).*

(i) Cari nilai  $k$ .

*Find the value of  $k$ .*

(ii) Tentukan persamaan bagi lengkung itu.

*Determine the equation of the curve.*

(iii) Nyatakan sifat titik pusingan itu.

*State the nature of the turning point.*

[10 markah]

[10 mark]

### Penyelesaian 21 :

$$\begin{aligned} \text{(a)} \quad & \int_2^p [f(x)+2]dx = 10 \\ & \int_2^p f(x)dx + \int_2^p 2dx = 10 \\ & 6 + [2x]_2^p = 10 \\ & 6 + [2p - 2(2)] = 10 \\ & 2p = 10 - 6 + 4 \\ & 2p = 8 \\ & p = 4 \end{aligned}$$

(b)

$$\text{(i)} \quad \frac{dy}{dx} = 2x - k$$

$$\text{Pada titik pusingan (3,5), } \frac{dy}{dx} = 0$$

$$\text{At the turning point (3,5), } \frac{dy}{dx} = 0$$

$$2(3) - k = 0$$

$$k = 6$$

$$(ii) \frac{dy}{dx} = 2x - 6$$

$$y = \int (2x-6)dx$$

$$= x^2 - 6x + c$$

Apabila / When  $x = 3, y = 5$ .

$$5 = 3^2 - 6(3) + c$$

$$5 = 9 - 18 + c$$

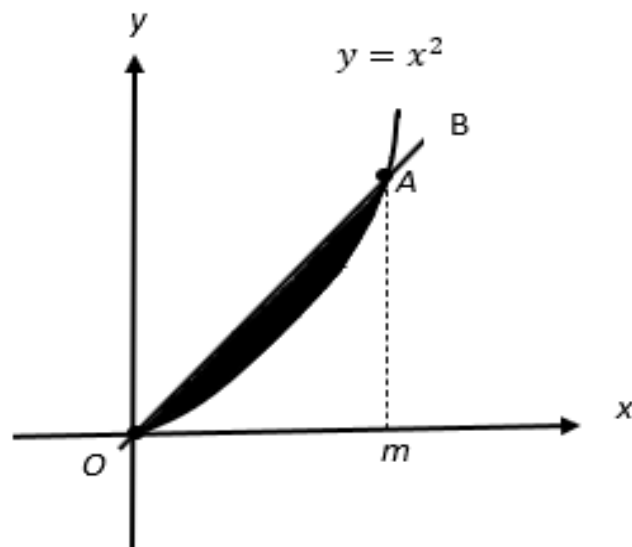
$$c = 14$$

Persamaan lengkung itu ialah  $y = x^2 - 6x + 14$ .

(iii) Oleh sebab pekali  $x^2$  bernilai positif bagi fungsi kuadratik ini maka titik pusingan itu ialah titik minimum.

22. Rajah di bawah menunjukkan suatu lengkung  $y = x^2$  bersilang dengan garis lurus  $OAB$  pada titik  $O$  dan  $A$ .

The diagram below shows the curve  $y = x^2$  intersect the straight line  $OAB$  at points  $O$  and  $A$ .



- (a) Cari nilai  $m$  jika luas rantau berlorek adalah  $\frac{4}{3}$  units<sup>2</sup>.

Find the value of  $m$  if the area of the shaded region is equal to  $\frac{4}{3}$  units<sup>2</sup>.

(b) Kira isipadu kisaran dalam sebutan  $\pi$  apabila rantau berlerek dikisarkan melalui  $360^\circ$  pada paksi-x.

*Calculate the volume of revolution in terms of  $\pi$  when the shaded region is revolved through  $360^\circ$  about the x-axis.*

[8 markah]

[8 marks]

**Penyelesaian 22 :**

$$\begin{aligned} \text{a) Luas segitiga} &= \frac{1}{2} \times m \times m^2 \\ &= \frac{m^3}{2} \end{aligned}$$

$$\text{Luas di bawah lengkung} = \int_0^m x^2 dx \quad @ \quad = \left[ \frac{x^3}{3} \right]_0^m \quad @ \quad \frac{m^3}{3} - 0 \quad @ \quad \frac{m^3}{3}$$

$$\begin{aligned} \text{Luas rantau berlerek} &= \frac{m^3}{2} - \frac{m^3}{3} = \frac{4}{3} \quad @ \quad \frac{m^3}{6} = \frac{4}{3} \\ m &= 2 \end{aligned}$$

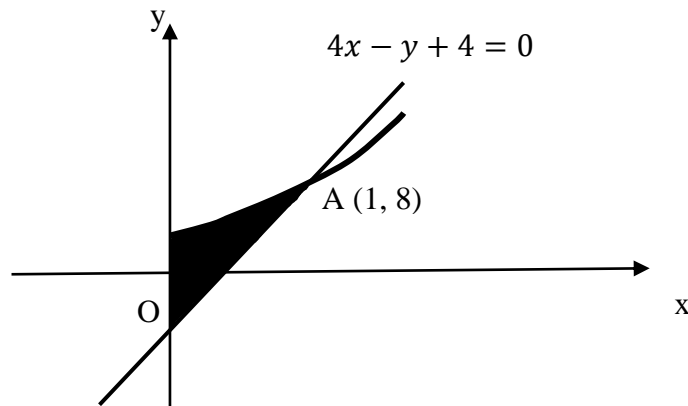
$$\begin{aligned} \text{b) Isipadu kon} &= \frac{1}{3} \times \pi \times 4^2 \times 2 \\ &= \frac{32\pi}{3} \end{aligned}$$

$$\begin{aligned} \text{Isipadu di bawah lengkung} &= \pi \int_0^2 x^4 dx \quad @ \quad = \left[ \frac{x^5}{5} \right]_0^2 \\ &= \frac{32\pi}{5} \end{aligned}$$

$$\begin{aligned} \text{Isipadu rantau berlerek} &= \frac{1}{3} \times \pi \times 4^2 \times 2 - \left[ \frac{x^5}{5} \right]_0^2 \quad @ \quad \frac{32\pi}{3} - \frac{32\pi}{5} \\ &= \frac{64\pi}{15} \end{aligned}$$

23. Rajah di bawah menunjukkan garis lurus  $4x - y + 4 = 0$  menyilang suatu lengkung di titik A.

The diagram below shows the straight line  $4x - y + 4 = 0$  intersect a curve at point A.



Diberi fungsi kecerunan lengkung itu ialah  $2x$ . Kira isipadu kisan dalam sebutan  $\pi$  apabila rantau berlorek dikisarkan melalui  $360^\circ$  pada paksi-y.

Given the gradient function of the curve is  $2x$ . Calculate the volume of revolution in terms of  $\pi$  when the shaded region is revolved through  $360^\circ$  about the y-axis.

[8 markah]

[8 marks]

**Penyelesaian 23 :**

$$y = \frac{2x^2}{2} + c \quad @ \quad y = x^2 + c \quad @ \quad 8 = 1 + c \quad @ \quad c=7$$

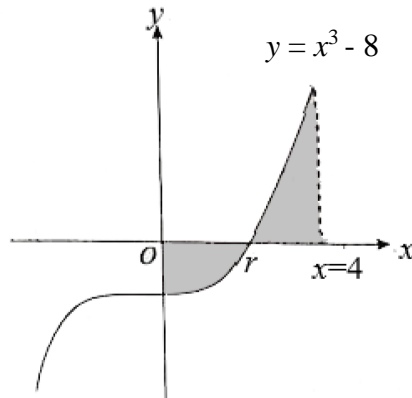
$$y = x^2 + 7$$

$$\begin{aligned} \text{Isipadu kon} &= \frac{1}{3} \times \pi \times 1^2 \times 4 \\ &= \frac{4\pi}{3} \end{aligned}$$

$$\begin{aligned} \text{Isipadu kisan di bawah lengkung} &= \pi \int_7^8 y - 7 \, dy \\ &= \pi \left[ \frac{y^2}{2} - 7y \right]_7^8 \\ &= \pi \left[ \left( \frac{8^2}{2} - 7(8) \right) - \left( \frac{7^2}{2} - 7(7) \right) \right] \\ &= \frac{\pi}{2} \quad @ \quad 0.5\pi \end{aligned}$$

$$\begin{aligned} \text{Isipadu rantau berlorek} &= \frac{4\pi}{3} - \frac{\pi}{2} \\ &= \frac{5\pi}{6} \end{aligned}$$

24. (a) Rajah 24 (a) menunjukkan lengkung bagi fungsi  $y = x^3 - 8$ .  
*Diagram 24 (a) shows the curve of the function  $y = x^3 - 8$ .*

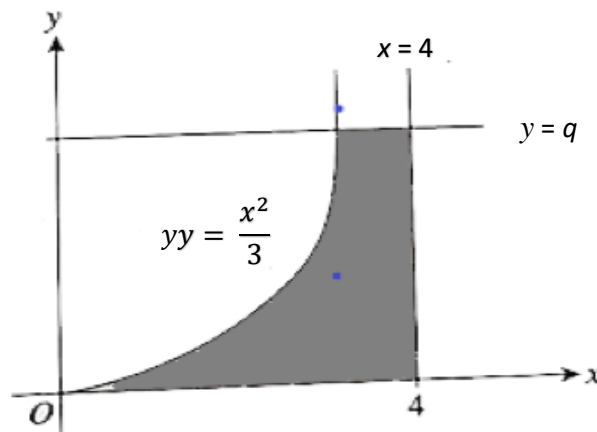


Rajah 24(a)  
 Diagram 24(a)

- (i) Cari nilai  $r$ .  
*Find the value of  $r$ .*
- (ii) Hitung luas kawasan berlorek.  
*Calculate the area of the shaded region.*

[5 markah]  
 [5 marks]

- (b) Rajah 24(b) menunjukkan lengkung  $y = \frac{x^2}{3}$  dan dua garis lurus,  $x = 4$  dan  $y = q$ .  
*Diagram 23 (b) shows a curve  $y = \frac{x^2}{3}$  and two straight lines,  $x = 4$  dan  $y = q$ .*



Rajah 24(b)  
 Diagram 24(b)

Apabila kawasan berlorek diputarkan melalui  $360^\circ$  pada paksi-y, isi padu janaan ialah  $34\frac{1}{2}\pi$  unit<sup>3</sup>. Jika  $q < 5$ , cari nilai  $q$ .

*When the shaded region is rotated through  $360^\circ$  about the y-axis, the volume generated is  $34\frac{1}{2}\pi$  unit<sup>3</sup>. If  $q < 5$ , find the value of  $q$ .*

[5 markah]  
 [5 marks]

**Penyelesaian 24 :**

(a)

$$\begin{aligned} \text{(i) } x^3 - 8 &= 0 \\ x^3 &= 8 \\ x &= 2 \\ \therefore r &= 2 \end{aligned}$$

(ii) Luas kawasan berlerek

*Area of the shaded region*

$$\begin{aligned} &= \left| \int_0^2 (x^3 - 8) dx \right| + \int_2^4 (x^3 - 8) dx \\ &= \left| \left[ \frac{x^4}{4} - 8x \right]_0^2 \right| + \left[ \frac{x^4}{4} - 8x \right]_2^4 \\ &= |(4 - 16) - 0| + [(64 - 32) - (4 - 16)] \\ &= 56 \text{ unit}^2 \end{aligned}$$

(b)  $y = \frac{x^2}{3}$   
 $x^2 = 3y$

$$\begin{aligned} \pi(4^2)(q) - \pi \int_0^q 3y \, dy &= 34 \frac{1}{2} \pi \\ 16\pi q - \pi \left[ \frac{3y^2}{2} \right]_0^q &= 34 \frac{1}{2} \pi \\ 16q - \frac{3q^2}{2} &= \frac{69}{2} \\ 32q - 3q^2 &= 69 \\ 3q^2 - 32q + 69 &= 0 \\ (3q - 23)(q - 3) &= 0 \\ q &= \frac{23}{3} \quad \text{atau} \quad q = 3 \end{aligned}$$

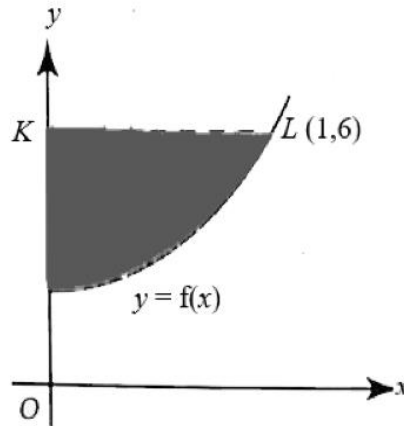
Oleh sebab  $q < 5$  maka  $q = 3$

*Since  $q < 5$  then  $q = 3$*



25. Rajah 24 menunjukkan sebahagian daripada lengkung  $y = f(x)$  yang melalui  $L(1,6)$ . Garis lurus  $KL$  adalah selari dengan paksi- $x$ .

Diagram 25 shows a part of the curve  $y = f(x)$  that passes through  $L(1,6)$ . The straight line  $KL$  is parallel to the  $x$ -axis.



Rajah 25  
Diagram 25

Lengkung itu mempunyai fungsi kecerunan  $4x$ . Cari

*The curve has a gradient function of  $4x$ . Find*

(a) persamaan lengkung itu,

*the equation of the curve,*

(b) luas, dalam  $\text{unit}^2$ , bagi rantau berlorek,

*the area, in  $\text{unit}^2$ , of the shaded region,*

(c) isi padu janaan, dalam sebutan  $\pi$ , apabila rantau berlorek diputarakan melalui  $360^\circ$  pada paksi- $y$ .

*the volume generated, in terms of  $\pi$ , when the shaded region is revolved through  $360^\circ$  about the  $y$ -axis.*

[10 markah]

[10 marks]

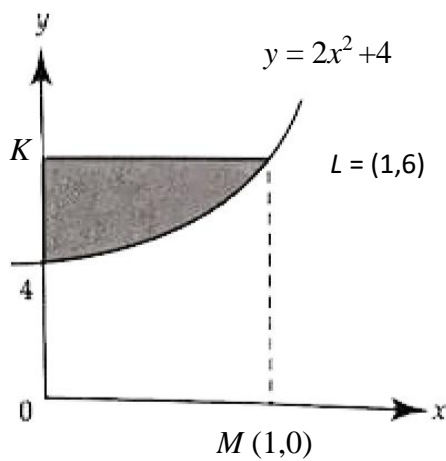
**Penyelesaian 25 :**

(a) fungsi kecerunan,  $\frac{dy}{dx} = 4x$   
 $y = \int 4x \, dx$   
 $= \frac{4x^2}{2} + c$   
 $= 2x^2 + c$

Pada (1,6) ,  $6 = 2(1)^2 + c$   
 $c = 4$

∴ Persamaan lengkung ialah  $y = 2x^2 + 4$ .

(b)



Luas kawasan berlorek

*Area of the shaded region*

= Luas segi empat OKLM  $- \int_0^1 y \, dx$

=  $(1 \times 6) - \int_0^1 (2x^2 + 4) \, dx$

=  $6 - \left[ \frac{2x^3}{3} + 4x \right]_0^1$

=  $6 - \left[ \frac{2(1)^3}{3} + 4(1) - 0 \right]$

=  $1 \frac{1}{3} \text{ unit}^2$

$$(c) y = 2x^2 + 4$$

$$x^2 = \frac{y}{2} - 2$$

$$V = \int_4^6 \pi x^2 dy$$

$$= \pi \int_4^6 \left(\frac{y}{2} - 2\right) dy$$

$$= \pi \left[\frac{y^2}{4} - 2y\right]_4^6$$

$$= \pi \left[\left(\frac{6^2}{4} - 2(6)\right) - \left(\frac{4^2}{4} - 2(4)\right)\right]$$

$$= \pi[(-3) - (-4)]$$

$$= \pi \text{ unit}^3$$

26. Diberi suatu lengkung dengan fungsi kecerunan  $g'(x) = px^2 - 2x$ , dengan keadaan  $p$  ialah pemalar. Jika  $y = 20x - 39$  ialah persamaan tangen kepada lengkung tersebut pada titik  $(2,1)$ , cari nilai  $p$ . Seterusnya, cari persamaan lengkung  $g(x)$ .

*Given a curve with the gradient function  $g'(x) = px^2 - 2x$ , where  $p$  is a constant.*

*If  $y = 20x - 39$  is the tangent equation to the curve at the point  $(2,1)$ , find the value of  $p$ .*

*Hence, find the equation of the curve  $g(x)$ .*

[ 5 Markah ]

[ 5 marks ]

**Penyelesaian 26 :**

$$p(2)^2 - 2(2) = 20$$

$$p = 6$$

$$g(x) = \int (6x^2 - 2x) dx$$

$$= \frac{6x^3}{3} - \frac{2x^2}{2} + c \quad c = \text{pemalar}$$

$$= 2x^3 - x^2 + c$$

$$1 = 2(2)^3 - (2)^2 + c$$

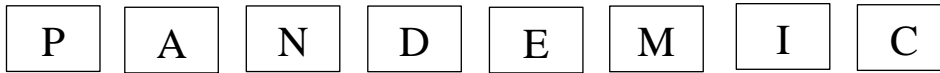
$$c = -11$$

$$\therefore g(x) = 2x^3 - x^2 - 11$$

# BAB 4

## PILIH ATUR DAN GABUNGAN PERMUTATION AND COMBINATION

1. Rajah menunjukkan lapan keping kad huruf.  
*Diagram shows eight letter cards.*



Hitung bilangan cara yang berlainan  
*Calculate the number of different ways*

- (a) untuk menyusun semua kad huruf itu dalam satu baris.  
*to arrange all the letter cards in a row.*
- (b) untuk memilih lima keping kad huruf yang terdiri daripada 2 vokal.  
*to choose five letter cards which consist of 2 vowels.*

[3 markah]  
[3 marks]

### Penyelesaian 1

- (a)  $8! = 40320$   
(b)  ${}^3C_2 \times {}^5C_3 = 30$

2. Kod dengan 7 huruf dibentuk dari perkataan 'HARMONI'. Cari bilangan kod  
*7 letter codes are formed using the letters from the word 'HARMONI'. Find the number of codes that*

- (a) bermula dengan konsonan  
*begin with a consonant*
- (b) huruf vokal dikumpul sekali  
*the vowels group together*
- (c) bermula dengan vokal dan berakhir dengan consonant.  
*Begin with a vowel and end with a consonant.*

[6 markah]  
[6 marks]

### Penyelesaian 2

(a)  $4 \times 6!$   
2880

(b)  $5! \times 3!$   
720

(c)  $3 \times 5! \times 4$   
1440

3. Berapakah nombor empat digit boleh dibentuk daripada digit-digit 2, 3, 4, 5 dan 6 jika nombor-nombor itu adalah  
*How many four digit numbers can be formed from the digits 2, 3, 4, 5 and 6 if the numbers are*

(a) kurang daripada 5 000  
*less than 5 000*

(b) nombor-nombor ganjil  
*odd numbers*

[4 markah]

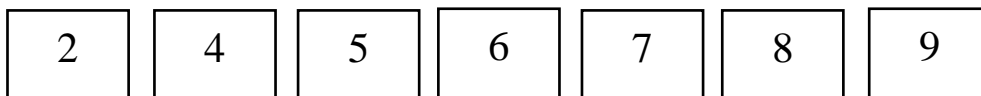
[4 marks]

### Penyelesaian 3

(a)  $3 \times {}^4P_3 = 72$

(b)  ${}^4P_3 \times 2 = 48$

4. Rajah 4 menunjukkan tujuh keping kad nombor.  
*Diagram 4 shows seven numbered cards.*



Rajah 4  
*Diagram 4*

Samuel hendak membentuk satu kata laluan empat digit untuk sistem keselamatan rumahnya dengan menggunakan empat daripada kad-kad ini. Cari  
*Samuel wants to form a four-digit password for his house security by using the four of these cards. Find*

- (a) bilangan kata laluan empat digit berlainan yang dapat dibentuk.  
*the number of different four-digit passwords that can be formed.*

- (b) bilangan kata laluan empat digit berlainan yang melebihi 6 500 yang dapat dibentuk.  
*the number of different four-digit passwords which are more than 6 500 that can be formed.*

[4 markah]  
[4 marks]

**Penyelesaian 4**

- (a)  ${}^7P_4 = 840$   
(b)  $(3 \times {}^6P_3) + (1 \times 4 \times {}^5P_2) = 440$

5. Rajah 5 menunjukkan enam keping kad nombor.  
*Diagram 5 shows six numbered cards.*



Rajah 5  
*Diagram 5*

Cari bilangan cara,  
*Find the number of ways,*

- (a) nombor yang berlainan yang dapat dibentuk,  
*the different numbers can be formed,*  
(b) nombor genap yang berlainan yang dapat dibentuk.  
*the different even numbers can be formed.*

[4 markah]  
[4 marks]

**Penyelesaian 5**

- (a)  ${}^6P_4 = 360$   
(b)  ${}^5P_3 \times 4 = 240$

6. Berapakah bilangan nombor 4 digit yang boleh dibentuk daripada digit 1, 3, 5, 7, 8 dan 9 tanpa ulangan.

*How many 4-digit number can be formed from the digits 1, 3, 5, 7, 8 and 9 without repetition.*

- (a) tanpa apa-apa syarat,  
*if there is no restriction,*
- (b) nombor genap 4-digit yang lebih daripada 7000.  
*4-digit even number greater than 7000.*

[4 markah]

[4 marks]

**Penyelesaian 6**

(a)  ${}^6P_4 = 360$

(b)  $2 \times {}^4P_2 \times 1 = 24$

7. Rajah 7 menunjukkan enam keping kad nombor.

*Diagram 7 shows six numbered cards.*



Rajah 7  
*Diagram 7*

- (a) Berapakah bilangan nombor 5 digit yang boleh dibentuk tanpa ulangan.  
*How many 5-digit number can be formed from the digits without repetition.*
- (b) Cari bilangan nombor ganjil 5 digit yang kurang daripada 50000 yang dapat dibentuk daripada digit- digit yang diberikan tanpa ulangan.  
*Find the number of 5-digit odd numbers less than 50000 can be formed from the given digits without repetition.*

[4 markah]

[4 marks]

**Penyelesaian 7**

(a)  ${}^6P_5 = 720$

(b)  $(2 \times 3 \times {}^4P_3) + (1 \times 2 \times {}^4P_3) = 192$

8. Sepuluh orang pelajar daripada Kelab Sejarah dari sekolah tertentu berminat untuk menyertai Kuiz Sejarah.

*Ten students from History Club at particular school interested to take parts in History Quiz.*

- (a) Berapa carakah pemilihan boleh dibuat jika hanya lima orang pelajar yang layak.

*How many ways the selection can be made if only five students are qualified.*

- (b) Lima orang pelajar yang terpilih disusun sebaris untuk sesi bergambar. Cari bilangan cara menyusun mereka jika dua daripada mereka hendak duduk bersebelahan antara satu sama lain.

*The five selected students are arranged in a row for a group photograph. Find the number of ways to arrange them if two of them want to sit next to each other.*

[4 markah]

[4 marks]

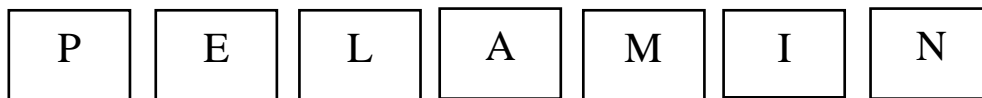
**Penyelesaian 8**

(a)  ${}^{10}C_5 = 252$

(b)  $2! \times 4! = 48$

9. Rajah 9 menunjukkan tujuh huruf ditulis di atas tujuh keping kad.

*Diagram 9 shows seven letters written on seven cards.*



Rajah 9  
*Diagram 9*

- (a) Cari bilangan susunan yang mungkin, dalam sebaris, bagi semua kad itu.

*Find the possible arrangements, in a row, of all cards.*

- (b) Cari bilangan susunan yang bermula dengan vokal dan berakhir dengan konsonan.

*Find the number of arrangements which starts with a vowel and ends with a consonant.*

[4 markah]

[4 marks]

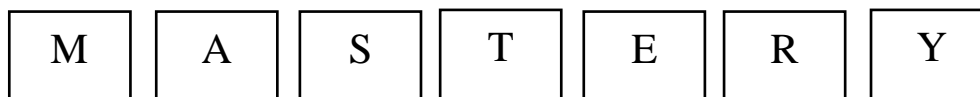
**Penyelesaian 9**

(a)  $7! = 5040$

(b)  $3 \times 4 \times {}^5P_5 = 1440$



10. Rajah 10 menunjukkan tujuh keping kad yang berlainan huruf.  
*Diagram 10 shows seven cards of different letters.*



Rajah 10  
*Diagram 10*

Suatu kod empat huruf hendak dibentuk dari kad-kad tersebut. Cari  
*A four-letter code is to be formed using four of these cards. Find*

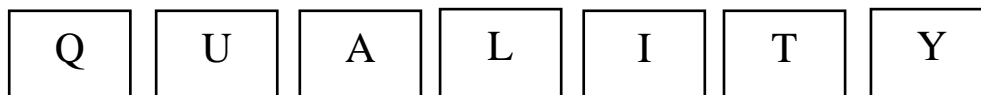
- (a) bilangan susunan yang berlainan yang dapat dibentuk,  
*the number of different arrangements that can be formed,*
- (b) bilangan susunan yang berlainan yang dapat dibentuk dengan syarat ia bermula dengan huruf vokal dan berakhir dengan huruf konsonan.  
*the number of different arrangements where it starts with a vowel and ends with a consonant.*

[4 markah]  
[4 marks]

**Penyelesaian 10**

- (a)  ${}^7P_4 = 840$   
(b)  $2 \times 5 \times {}^5P_2 = 200$

11. Rajah 11 menunjukkan tujuh keping kad huruf.  
*Diagram 11 shows seven letter cards.*



Rajah 11  
*Diagram 11*

Suatu kod tujuh huruf hendak dibentuk dengan menggunakan tujuh daripada kad-kad itu. Cari  
*A seven-letter code is formed using seven of these cards. Find*

- (a) bilangan kod tujuh huruf yang berlainan yang dapat dibentuk  
*the number of seven-letter codes that can be formed,*
- (b) bilangan kod tujuh huruf, jika huruf vokal adalah bersebelahan.  
*the number of seven-letter codes, if the vowels are side by side.*

[4 markah]  
[4 marks]

**Penyelesaian 11**

- (a)  ${}^7P_7 = 5040$   
(b)  $3! \times 5! = 720$

12. Rajah 12 menunjukkan tujuh keping kad huruf.

*Diagram 12 shows seven letter cards.*



Rajah 12  
Diagram 12

Suatu kod enam huruf hendak dibentuk dengan menggunakan enam daripada kad-kad itu. Cari

*A six-letter code is to be formed using six of these cards. Find*

- (a) bilangan kod enam huruf yang berlainan yang dapat dibentuk,  
*the number of different six-letter codes that can be formed,*
- (b) bilangan kod enam huruf yang berlainan dengan keadaan huruf U dan huruf E adalah bersebelahan.  
*the number of different six-letter codes which the letters U and E are side by side.*

[4 markah]

[4 marks]

**Penyelesaian 12**

- (a)  ${}^7P_6 = 5040$   
(b)  $2! \times {}^6P_5 = 1440$

13. Rajah 13 menunjukkan lapan keping kad huruf dan nombor yang berlainan.

*Diagram 13 shows eight cards of different letters and numbers.*



Rajah 13  
Diagram 13

- (a) Cari bilangan cara susunan yang mungkin, dalam satu baris, semua kad itu.  
*Find the number of possible arrangements, in a row, of all the cards.*
- (b) Cari bilangan cara susunan jika nombor-nombor itu disusun bersama-sama.  
*Find the number of these arrangements, if the numbers must be together.*

[4 markah]

[4 marks]

**Penyelesaian 13**

- (a)  ${}^8P_8 = 40320$   
(b)  $3! \times 6! = 4320$

14. Cari bilangan cara susunan berbeza yang mungkin bagi semua huruf perkataan MAGNIFICENT jika huruf vokal sentiasa bersama.  
*Find the the number of possible ways to arrange all the letters in the word MAGNIFICENT if the vowels are always together.*

[3 markah]  
[3 marks]

**Penyelesaian 15**

$$\frac{4!}{2!} \times \frac{8!}{2!} = 241920$$

15. Berapakah bilangan cara untuk menyusun 6 orang dalam suatu bulatan.  
*How many ways to arrange 6 people in a circle.*

[2 markah]  
[2 marks]

**Penyelesaian 16**

$$(5 - 1)! = 120$$

16. Cari bilangan cara untuk menyusun 10 butir manik berlainan warna untuk membentuk satu rantai tangan.  
*Find the num of ways to assemble 10 beads of different colour into a bracelet.*

[2 markah]  
[2 marks]

**Penyelesaian 17**

$$\frac{(10-1)!}{2} = 181440$$

17. Rajah di bawah menunjukkan lapan kod huruf. Satu kod 4 huruf akan dibentuk dengan menggunakan empat daripada kad-kad ini.  
*The diagram below shows eight letter cards. A 4-letter code is to be formed using four of these cards.*



Carikan bilangan kod 4 huruf yang berlainan  
*Find the number of different 4-letter codes*

- (a) yang dapat dibentuk.  
*that can be formed.*
- (b) yang bermula dengan satu vokal.  
*that begin with a vowel.*

[4 markah]  
 [4 marks]

**Penyelesaian 18**

- (a)  ${}^8P_4 = 1680$   
 (b)  $2 \times {}^7P_3 = 420$

18. Berapakah bilangan cara huruf-huruf dalam perkataan **PARALLEL** boleh disusun jika huruf **P** dan **R** bersama-sama.  
*How many ways can the letters in the word **PARALLEL** can be arranged if the letter **P** and **R** are together?*

[3 markah]  
 [3 marks]

**Penyelesaian 19**

$$\frac{2! \times 7!}{2! \times 3!} = 840$$

19. Cari bilangan cara 7 orang pekerja *P, Q, R, S, T, U* dan *V* di sebuah syarikat yang boleh disusun di sebuah meja bulat dengan syarat *U* dan *V* mesti bersebelahan.  
*Find the number of way to arrange 7 employees, *P, Q, R, S, T, U* and *V* from a company at a round table if *U* and *V* must be seated together.*

[3 markah]  
 [3 marks]

**Penyelesaian 20**

$$2! \times (6 - 1)! = 240$$

# BAB 5

## TABURAN KEBARANGKALIAN PROBABILITY DISTRIBUTION

1. Dalam suatu permainan catur, pemain akan diberi 1 mata jika menang.  $\frac{1}{2}$  mata akan diberi jika seri dan 0 mata akan diberi jika kalah. Lee bermain dua set permainan catur.

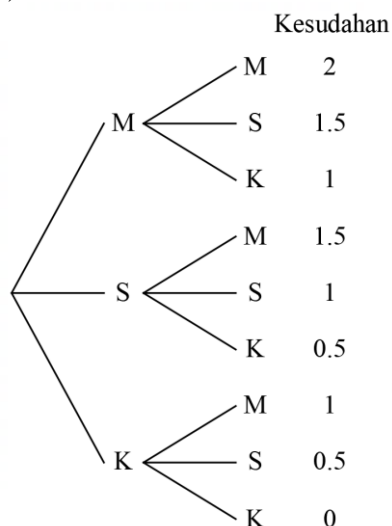
*In a chess game, player will be awarded 1 point if he wins.  $\frac{1}{2}$  point is given if he gets a draw and 0 point if he loses the game. Lee played two sets of chess games.*

- (a) Bina gambar rajah pokok untuk mewakili semua kesudahan yang mungkin.  
*Construct a tree diagram to represent all the possible outcomes.*
- (b) Jika  $X$  mewakili bilangan mata yang diperoleh Lee, senaraikan set  $X$ .  
*If  $X$  represents the number of points obtained by Lee, list the set of  $X$ .*
- (c) Lukiskan graf bagi taburan kebarangkalian  $X$ .  
*Draw a graph of the probability distribution of  $X$ .*

[6 markah]  
[6 marks]

### Penyelesaian 1

(a)



(b)  $X = \{0, 0.5, 1, 1.5, 2\}$

(c)  $P(X: r)$

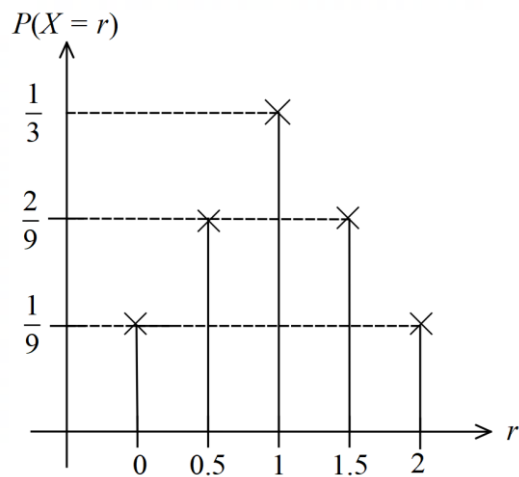
$$P(X=0) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$P(X=0.5) = 2\left(\frac{1}{3} \times \frac{1}{3}\right) = \frac{2}{9}$$

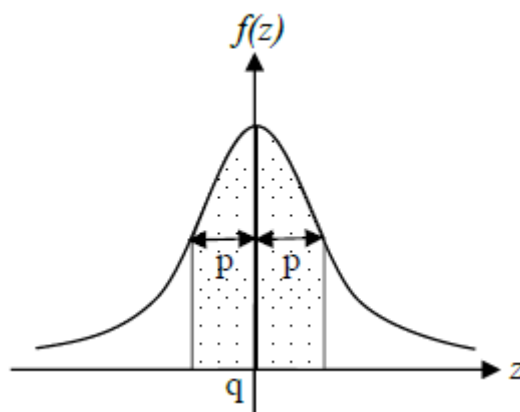
$$P(X=1) = 3\left(\frac{1}{3} \times \frac{1}{3}\right) = \frac{1}{3}$$

$$P(X=1.5) = 2\left(\frac{1}{3} \times \frac{1}{3}\right) = \frac{2}{9}$$

$$P(X=2) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$



2. (a) Rajah 2 menunjukkan graf taburan normal piawai dengan min,  $q$  dan sisihan piawai,  $p$ .  
*Diagram 2 shows standard normal distribution graph with mean,  $q$  and standard deviation,  $p$ .*



Rajah 2  
 Diagram 2

- (i) Nyatakan nilai  $p$  dan  $q$ .  
*State the value of  $p$  and  $q$ .*
- (ii) Cari luas kawasan berlorek.  
*Find the area of the shaded region.*
- (b) Markah bagi calon dalam peperiksaan Matematik Tambahan adalah bertaburan secara normal dengan min,  $\mu$  dan sisihan piawai,  $\sigma$ . Jika 5% daripada calon tersebut mendapat anugerah kepujian di mana mereka mendapat markah minimum 70 dan 35% daripada calon mendapat markah lulus sekurang-kurangnya 40. Cari nilai bagi  $\mu$  dan  $\sigma$ .  
*The marks of candidates in the Additional Mathematics examination are normally distributed with a mean,  $\mu$  and a standard deviation,  $\sigma$ . If 5% of the candidates get an honors award where they get a minimum mark of 70 and 35% of the candidates get a pass mark of at least 40. Find the values of  $\mu$  and  $\sigma$ .*

[10 markah]

[10 marks]

## Penyelesaian 2

(a) i)  $p = 1$   
 $q = 0$

ii)  $1 - 2P(Z > 1)$   
 $1 - 2(0.1587) = 0.6826$

(b)  $P\left(Z \geq \frac{70-\mu}{\sigma}\right) = 0.05$

$$P(Z \geq 1.645) = 0.05$$

$$\frac{70-\mu}{\sigma} = 1.645$$

$$\mu = 70 - 1.645\sigma$$

$$P\left(Z \geq \frac{40-\mu}{\sigma}\right) = 0.35$$

$$P(Z \geq 0.385) = 0.35$$

$$\frac{40-\mu}{\sigma} = 0.385$$

Guna persamaan serentak

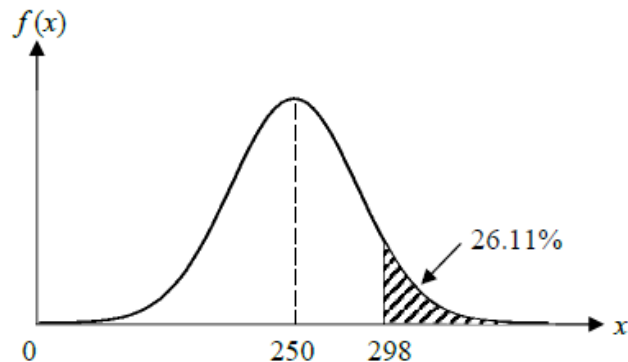
$$40 - (70 - 1.645\sigma) = 0.385\sigma$$

$$\sigma = 23.81$$

$$\mu = 70 - (1.645 \times 23.81)$$

$$\mu = 30.83$$

3. Satu soal selidik menunjukkan bahawa jumlah perbelanjaan bagi pelanggan di Pasaraya Mewah bertaburan secara normal seperti yang ditunjukkan dalam Rajah 3. *A survey shows that the expenditure of customers at Pasaraya Mewah is normally distributed as shown in Diagram 3.*



Rajah 3  
Diagram 3

- (a) i) Cari min dan sisihan piawai  
*Find the mean and the standard deviation.*
- ii) Jika 80 orang pelanggan telah dipilih secara rawak, cari bilangan pelanggan yang berbelanja antara RM200 dan RM265.  
*If 80 customers are chosen at random, find the number of customers who spend between RM 200 and RM 265.*
- (b) Diberi 12% pelanggan telah berbelanja lebih daripada RM  $q$ , cari nilai  $q$ . Beri jawapan anda kepada ringgit terhampir.  
*Give your answer to the nearest ringgit.*

[10 markah]

[10 mark]

### Penyelesaian 3

- (a) i)  $\mu = 250$   

$$P\left(Z > \frac{298-250}{\sigma}\right) = 0.2611$$

$$P(Z > 0.64) = 0.2611$$

$$\frac{298-250}{\sigma} = 0.64$$

$$\sigma = 75$$
- ii)  $P(200 < X < 265)$   

$$= P\left(\frac{200-250}{75} < Z < \frac{265-250}{75}\right)$$

$$= P(-0.667 < Z < 0.2)$$

$$= 1 - 0.2523 - 0.4207$$

$$= 0.327$$

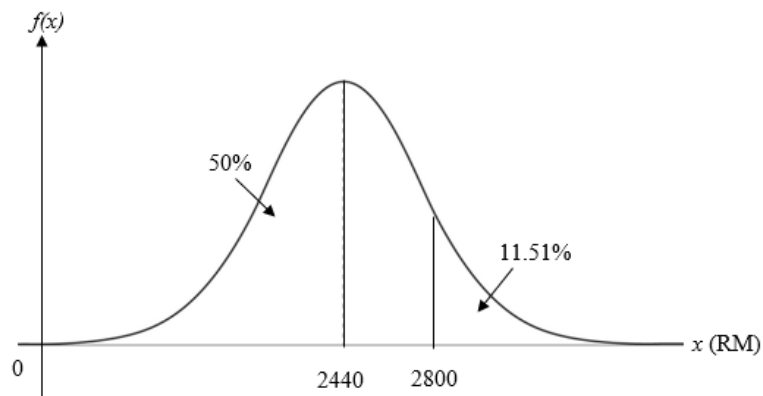
$$= 0.327 \times 80 = 26.16$$

$$= 26 \text{ orang}$$



$$\begin{aligned}
 \text{(b)} \quad P\left(Z > \frac{q-250}{75}\right) &= 0.12 \\
 P(Z > 1.175) &= 0.12 \\
 \frac{q-250}{75} &= 1.175 \\
 q &= 338
 \end{aligned}$$

4. Satu kajian menunjukkan bahawa gaji seorang siswazah adalah bertaburan secara normal seperti yang ditunjukkan dalam Rajah 4.  
*A study shows that the salary of a graduate is normally distributed as shown in the Diagram 4.*



Rajah 4.  
 Diagram 4.

- (a)(i) Cari sisihan piawai.  
*Find the standard deviation.*
- (ii) Jika 40 orang siswazah dipilih secara rawak, cari bilangan siswazah yang mempunyai gaji antara RM 2000 dan RM 2600.  
*If 40 graduates are chosen at random, find the number of graduates who have salary between RM2000 and RM2600.*
- (b) Didapati bahawa 20% siswazah mempunyai gaji kurang daripada RM $x$ . Cari nilai  $x$ .  
*It is found that 20% of the graduates have salaries of less than RM $x$ . Find the value of  $x$ .*

[10 markah]  
 [10 marks]

#### Penyelesaian 4

(a)  $X \sim N(\mu, \sigma^2)$

(i)  $\mu = 2440$

$$P(X > 2800) = 11.51\%$$

$$P\left(Z > \frac{2800-2440}{\sigma}\right) = 0.1151$$

$$P\left(Z > \frac{360}{\sigma}\right) = 0.1151$$

$$\frac{360}{\sigma} = 1.2$$

$$\sigma = 300$$

(ii)  $P(2000 < X < 2600)$

$$= P\left(\frac{2000-2440}{300} < Z < \frac{2600-2440}{300}\right)$$

$$= P(-1.467 < Z < 0.533)$$

$$= 1 - P(Z < -1.467) - P(Z > 0.533)$$

$$= 1 - 0.0711 - 0.2971$$

$$= 0.6318$$

Bilangan siswazah yang mempunyai gaji antara RM2000 dan RM2600

*The number of graduates who have salary between RM2000 and RM2600*

$$= 40 \times 0.6318$$

$$= 25$$

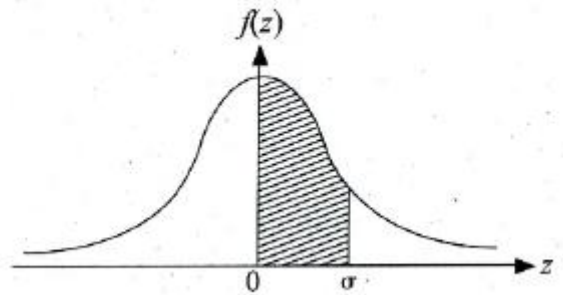
(b)  $P(X < x) = 20\%$

$$P\left(Z < \frac{x-2440}{300}\right) = 0.2$$

$$\frac{x-2440}{300} = -0.842$$

$$x = 2187.40$$

- 5.(a) Pembolehubah rawak  $X$  mengikut taburan normal dengan min,  $\mu$ , 73 dan varians,  $\sigma^2$ . Diberi bahawa  $P(X > 85) = 0.0606$ , cari nilai  $\sigma$ .  
*A random variables  $X$  has a normal distribution with mean,  $\mu$ , 73 and variance,  $\sigma^2$ . Given that  $P(X > 85) = 0.0606$ , find the value of  $\sigma$ .*
- (b) Rajah 5 menunjukkan graf taburan normal piawai dengan min,  $\mu$  dan sisihan piawai,  $\sigma$ .  
*Diagram 5 shows the standard normal distribution graph with mean,  $\mu$  and standard deviation,  $\sigma$ .*



Rajah 5  
 Diagram 5

Nyatakan  
 State

- (i) nilai  $\mu$ ,  
*The value of  $\mu$ ,*
- (ii) nilai  $\sigma$ .  
*The value of  $\sigma$ .*
- (iii) Cari luas rantau berlorek.  
*Find the area of the shaded region.*

[7 markah]  
 [7 marks]

**Penyelesaian 5**

(a)  $P(X > 85) = 0.0606$   
 $P\left(Z > \frac{85-73}{\sigma}\right) = P(Z > 1.55)$   
 $\frac{85-73}{\sigma} = 1.55$   
 $\sigma = 7.7419$

- (b) (i)  $\mu = 0$   
 (ii)  $\sigma = 1$   
 (iii) Luas rantau berlorek =  $P(Z > 0) - P(Z > 1)$   
 $= 0.5 - 0.1587$   
 $= 0.3413$

6. Di sebuah kebun, hanya tembikai yang mempunyai jisim  $x$  lebih daripada  $h$  kg digred dan dipasarkan. Jadual 6 menunjukkan gred tembikai berdasarkan jisim. Diberi bahawa jisim tembikai adalah mengikut taburan normal dengan min 5.8 kg dan sisihan piawai 1.8 kg.

*In an orchard, only watermelons with masses  $x$  greater than  $h$  kg are graded and marketed. Table 6 shows the grades of the watermelons based on their masses. It is given that the masses of the watermelons have a normal distribution with a mean of 5.8 kg and a standard deviation of 1.8 kg.*

Gred / Grade	AAA	AA
Jisim / Mass , $x$ (kg)	$x > 8$	$4 < x \leq 8$

Jadual 6

Table 6

- (a) Jika sebiji tembikai dipilih secara rawak, hitung kebarangkalian bahawa itu adalah gred AAA.

*If one watermelon is picked at random, calculate the probability that it is if grade AAA.*

- (b) Jika 95.24% daripada tembikai dipasarkan, cari nilai  $h$ .

*If 95.24% of the watermelons is marketed, find the value of  $h$ .*

[7 markah ]

[ 7 marks ]

### Penyelesaian 6

$$\begin{aligned}
 \text{(a) } P(X > 8) &= P\left(Z > \frac{8-5.8}{1.8}\right) \\
 &= P(Z > 1.222) \\
 &= 0.1108
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } P(X > h) &= 0.9524 \\
 P\left(X > \frac{h-5.8}{1.8}\right) &= 0.9524 \\
 z &= -1.669 \\
 \frac{h-5.8}{1.8} &= -1.669 \\
 h &= 2.796 @ 2.80
 \end{aligned}$$

7. Sebuah kilang menghasilkan 2500 botol kicap sehari. Jumlah kicap dalam botol adalah taburan normal dengan min 300ml dan varians  $400\text{ml}^2$ .  
*A factory produces 2500 bottles of soya bean sauce a day. The volume of the soya bean sauce in the bottle is a normal distribution with a mean of 300ml and a variance of  $400\text{ml}^2$ .*

- (a) Cari kebarangkalian bahawa isipadu botol kicap yang diambil secara rawak dari kilang adalah dalam julat 285ml dan 320ml.  
*Find the probability that the volume of a bottle of soya bean sauce taken randomly from the factory is in the range of 285ml and 320ml.*

- (b) Diberi bahawa 1 800 botol daripada 2 500 botol kicap mempunyai isipadu yang lebih besar daripada  $y$  ml, cari nilai  $y$ .  
*Given that 1800 bottles out of 2500 bottles of soya bean sauce have volume greater than  $y$  ml, find the value of  $y$ .*

[8 markah]

[8 marks]

### Penyelesaian 7

(a)  $\mu = 300 \quad \sigma = 20$   

$$P(285 < x < 320)$$

$$= P\left(\frac{285-300}{20} < z < \frac{320-300}{20}\right)$$

$$= P(-0.75 < z < 1)$$

$$= 1 - P(z < -0.75) - P(z > 1)$$

$$= 1 - 0.2266 - 0.1587$$

$$= 0.6147$$

(b)  $\frac{1800}{2500} = 0.72$   

$$P(x > y) = 0.72$$

$$P\left(z > \frac{y-300}{20}\right) = 0.72$$

$$\frac{y-300}{20} = -0.583$$

$$y = 288.34$$

8. Jisim pekerja dalam sebuah kilang adalah mengikut taburan normal dengan min,  $\mu$ , 67.58 kg dan sisihan piawai 6.8 kg.  
*The mass of the workers in a factory is normally distributed with a mean,  $\mu$  of 67.58 kg and a standard deviation of 6.8 kg.*
- (a) Hitung kebarangkalian bahawa seorang pekerja yang dipilih secara rawak daripada kumpulan itu mempunyai jisim kurang daripada 65 kg.  
*Calculate the probability that a worker chosen at random from this group has a mass of less than 65 kg.*
- (b) Jika 120 orang pekerja kilang itu mempunyai jisim antara 62 kg dan 69 kg, cari jumlah bilangan pekerja kilang itu.  
*If 120 of the workers in the factory weight between 62 kg and 69 kg., find the total number of the workers in the factory.*

[8 markah ]  
 [8 marks]

### Penyelesaian 8

$$(a) P(X < 65) = P\left(Z < \frac{65-67.58}{6.8}\right)$$

$$P(X < 65) = P(Z < -0.379)$$

$$P(X < 65) = 0.3523$$

$$(b) P(62 < X < 69)$$

$$P\left(\frac{62-67.58}{6.8} < X < \frac{69-67.58}{6.8}\right)$$

$$P(-0.8206 < X < 0.2088)$$

$$= 1 - 0.2058 - 0.4172$$

$$= 0.377$$

$$\frac{120}{n} = 0.377$$

$$n = 318.3$$

$$n = 318 \text{ or } 319 \text{ orang}$$

9. Jisim ikan yang ditangkap dari sebuah kolam ikan adalah mengikut taburan normal dengan min 2.4kg dan sisihan piawai  $k$  kg. Diberi bahawa 10.56% daripada ikan mempunyai jisim lebih daripada 3kg.

*The masses of fish caught from a pond follows a normal distribution with a mean of 2.4kg and a standard deviation of  $k$  kg. It is given that 10.56% of the fish have a mass of more than 3 kg.*

- (a) Hitung nilai  $k$ .

*Calculate the value of  $k$ .*

- (b) Diberi bilangan ikan yang ditangkap dari kolam itu ialah 1800 ekor. Cari bilangan ikan yang mempunyai jisim antara 2.0 kg hingga 3.0 kg.

*Given the number of fish caught from the pond is 1800. Find the number of fish that have the mass between 2.0kg and 3.0 kg.*

[8 markah]

[8 marks]

### Penyelesaian 9

$$X \sim N(2.4, k^2)$$

(a)  $P(X > 3) = 0.1056$

$$P\left(Z > \frac{3-2.4}{k}\right) = 0.1056$$

$$P(Z > 1.25) = 0.1056$$

$$\frac{0.6}{k} = 1.25$$

$$k = 0.48$$

(b)  $P(2.0 < X < 3.0)$

$$= P\left(\frac{2.0-2.4}{0.48} < Z < \frac{3.0-2.4}{0.48}\right)$$

$$= P(-0.8333 < Z < 1.25)$$

$$= 1 - P(Z < -0.8333) - P(Z > 1.25)$$

$$= 1 - 0.2025 - 0.1056$$

$$= 0.6919$$

Bilangan ikan / *number of fish*

$$1800 \times 0.6919 = 1245$$

10. Dalam sebuah sekolah, 500 orang pelajar menduduki ujian Matematik. Markah yang diperoleh adalah bertabur secara normal dengan min 55 dan sisihan piawai 8.  
*In a school, 500 students sat for the Mathematics test. The marks obtained follow a normal distribution with a mean of 55 and a standard deviation of 8.*

- (a) Cari bilangan pelajar yang lulus ujian itu jika markah lulus ialah 40.  
*Find the number of students who pass the test if the passing mark is 40.*
- (b) Jika 15% pelajar lulus ujian itu dengan memperoleh gred A, cari markah minimum untuk memperoleh gred A.  
*If 15% of the students pass the test with grade A, find the minimum marks to obtain grade A.*

[8 markah]

[8 marks]

### Penyelesaian 10

(a)  $\mu = 55, \sigma = 8, n = 500$

$$P(X \geq 40) = P\left(Z > \frac{40-55}{8}\right)$$

$$= P(Z > -1.875)$$

$$= 1 - P(Z > -1.875)$$

$$= 1 - 0.0303$$

$$= 0.9697$$

Bilangan murid

$$\text{Number of students} = 0.9697 \times 500 = 484.85$$

$$= 484 \text{ or } 485 \text{ orang}$$

- (b) Jadikan A sebagai markah minimum

*Let A be the minimum mark*

$$P(Z \geq A) = 0.15$$

$$P\left(Z \geq \frac{A-55}{8}\right) = 0.15$$

$$\frac{A-55}{8} = 1.036$$

$$A = 63.288$$

Markah minimum

$$\text{Minimum mark} = 63.288$$



11. Diameter buah oren dari sebuah ladang bertabur secara normal dengan min 5.0 cm dan sisihan piawai 1.2 cm. Hitung  
*The diameter of oranges from a farm have a normal distribution with a mean of 5.0 cm and a standard deviation of 1.2 cm. Calculate*
- (a) peratus sebiji oren yang dipilih secara rawak dari ladang itu mempunyai diameter lebih daripada 5.5 cm.  
*the percentage that an orange chosen at random from this farm has a diameter of more than 5.5 cm.*
- (b) nilai  $q$  jika 35% daripada oren-oren itu mempunyai diameter kurang dari  $q$  cm.  
*the value of  $q$  if 35% of the oranges have diameter less than  $q$  cm.*

[5 markah]  
 [5 marks]

### Penyelesaian 11

(a)  $Z = \frac{5.5-5.0}{1.2}$   
 $Z = 0.4167$   
 $Z = 0.4167 \times 100\%$   
 $Z = 41.67\%$

(b)  $P\left(Z < \frac{q-5.0}{1.2}\right) = 0.35$   
 $\frac{q-5.0}{1.2} = -0.385$   
 $q = 4.538$

12. Umur bagi 80 000 orang penduduk di sebuah bandar bertaburan secara normal dengan min 45 tahun dan varians 36 tahun<sup>2</sup>.  
*The ages of 80 000 residents in a town are normally distributed with a mean of 45 years and a variance of 36 years<sup>2</sup>.*

- (a) Hitung kebarangkalian bahawa umur seorang penduduk yang dipilih secara rawak adalah lebih daripada 48 tahun.  
*Calculate the probability that the age of a resident, selected randomly, is more than 48 years.*
- (b) Anggarkan bilangan penduduk yang berumur melebihi 40 tahun.  
*Estimate the number of residents who are above 40 years.*

[6 markah]  
 [6 marks]

### Penyelesaian 12

$$X \sim N(45, 36)$$

$$(a) P(X > 48) = P\left(Z > \frac{48-45}{6}\right) \\ P(Z > 0.5) = 0.3085$$

$$(b) P(X > 40) = P\left(Z > \frac{40-45}{6}\right) \\ P(Z > -0.833) = 1 - 0.2025 \\ = 0.7975$$

Bilangan penduduk yang berumur melebihi 40 tahun

*The number of residents who are above 40 years*

$$80\,000 (0.7975) = 63\,800$$

13. Jisim bagi buah durian dari sebuah ladang mempunyai taburan normal dengan min 3.1 kg dan sisihan piawai 0.7 kg. Hitung  
*The mass of durians from a farm have a normal distribution with a mean of 3.1 kg and a standard deviation of 0.7 kg. Calculate*

- (a) kebarangkalian bahawa sebiji durian yang dipilih secara rawak dari ladang ini mempunyai jisim lebih daripada 1.4 kg.  
*the probability that a durian chosen at random from this farm has a mass of more than 1.4 kg.*
- (b) nilai  $m$  jika 87% daripada durian mempunyai jisim kurang daripada  $m$  kg.  
*the value of  $m$  if 87% of the durian have masses less than  $m$  kg.*

[6 markah]

[6 marks]

### Penyelesaian 13

$$(a) \mu = 3.1, \sigma = 0.7$$

$$P(X > 1.4) = P\left(Z > \frac{1.4 - 3.1}{0.7}\right) \\ = P(Z > -2.4286) \\ = 1 - P(Z > 2.4286) \\ = 1 - 0.00757 \\ = 0.9924$$

$$(b) P(X < m) = 0.87 \\ P(Z > m) = 1 - 0.87 = 0.13 \\ P\left(Z > \frac{m-3.1}{0.7}\right) = 0.13 \\ P(Z > 1.127) = 0.13 \\ \frac{m-3.1}{0.7} = 1.127 \\ m = 3.889$$

14. Jangka hayat sejenis bateri bertaburan secara normal dengan min 150 jam dan varians 25 jam<sup>2</sup>.

*The lifespans of a type of batteries are normally distributed with a mean of 150 hours and a variance of 25 hours<sup>2</sup>.*

- (a) Cari kebarangkalian bahawa sebiji bateri yang dipilih secara rawak mempunyai jangka hayat antara 140 jam dan 148 jam.

*Find the probability that a battery selected at random has a lifespan between 140 hours and 148 hours.*

- (b) Diberi 10% daripada bateri itu mempunyai jangka hayat lebih daripada  $x$  jam. Cari nilai  $x$ .

*Given 10% of the batteries have a lifespan of more than  $x$  hours. Find the value of  $x$ .*

[10 markah]

[10 marks]

#### Penyelesaian 14

- (a)  $X \sim N(150, 25)$

$$P(140 < X < 148)$$

$$= P\left(\frac{140-150}{5} < Z < \frac{148-150}{5}\right)$$

$$= P(-2 < Z < -0.4)$$

$$= P(Z > 0.4) - P(Z > 2)$$

$$= 0.3446 - 0.0228 = 0.3218$$

- (b)  $P(X > x) = 0.1$

$$P\left(Z > \frac{x-150}{5}\right) = 0.1$$

$$\frac{x-150}{5} = 1.281$$

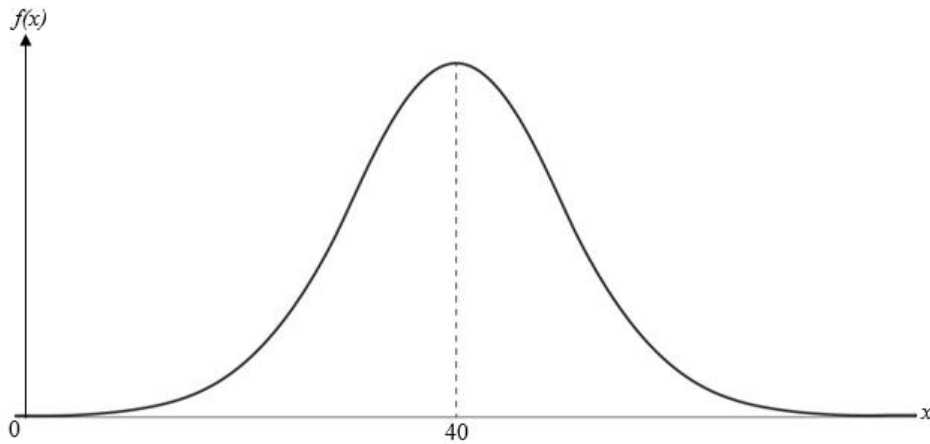
$$x - 150 = 6.405$$

$$x = 156.405$$

$$x = 156.4$$

15. Rajah 15 menunjukkan graf taburan bagi masa yang diperolehi sekumpulan peserta dalam satu permainan mencari harta karun. Masa peserta bertaburan secara normal dengan sisihan piawai 8 minit.

*Diagram 15 shows the distribution graph of the times obtained by a group of participants in a treasure hunt. The times of the participants are normally distributed with a standard deviation of 8 minutes.*



Rajah 15  
Diagram 15

Terdapat 20 orang peserta menggunakan masa antara 32 minit hingga 38 minit. Berapakah masa minimum yang patut diperolehi pemenang dalam permainan mencari harta karun itu?

*There are 20 participants spend the times between 32 minutes to 38 minutes. What is the minimum time that should be obtained by the winner in the treasure hunt?*

[6 markah]

[6 marks]

### Penyelesaian 15

$$\begin{aligned}
 \mu &= 40 \\
 P(30 < X < 38) \\
 &= P\left(\frac{30-40}{8} < Z < \frac{38-40}{8}\right) \\
 &= P(-1 < Z < -0.25) \\
 &= P(Z > 0.25) - P(Z > 1) \\
 &= 0.4013 - 0.1587 \\
 &= 0.2426
 \end{aligned}$$

Jumlah bilangan peserta

*Total number of participants*

$$\begin{aligned}
 &= \frac{20}{0.2426} \\
 &= 82
 \end{aligned}$$

Terdapat seorang pemenang sahaja.

$$P(X \leq x) = \frac{1}{82}$$

$$P\left(Z < \frac{x-40}{8}\right) = 0.0122$$

$$P(Z > 2.25) = 0.0122$$

$$\frac{x-40}{8} = -2.25$$

$$x = 22$$

Masa minimum = 22 minit

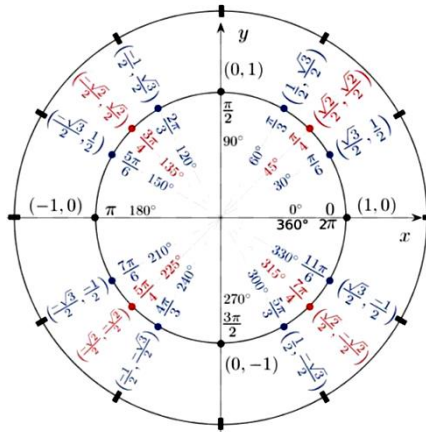
# BAB 6

## FUNGSI TRIGONOMETRI TRIGONOMETRIC FUNCTION

1. (a) Diberi  $\sin 30^\circ = \frac{1}{2}$  dan  $\cos 30^\circ = \frac{\sqrt{3}}{2}$ , cari nilai  $\cot(510^\circ)$  tanpa menggunakan kalkulator.

*Given  $\sin 30^\circ = \frac{1}{2}$  and  $\cos 30^\circ = \frac{\sqrt{3}}{2}$ , find the value of  $\cot(510^\circ)$  without using a calculator.*

- (b) Dengan menggunakan bulatan unit di bawah, cari nilai bagi  $\cos(-480^\circ)$ .  
*Using the unit circle below, find the value of  $\cos(-480^\circ)$ .*



[4 markah]

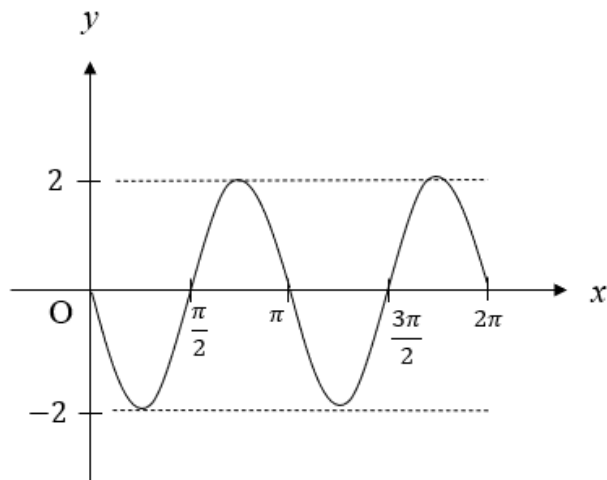
[4 marks]

**Penyelesaian :**

$$\begin{aligned}
 \text{(a) } \cot(510^\circ) &= -\cot(540^\circ - 510^\circ) \\
 &= -\cot 30^\circ \\
 &= -\frac{\cos 30^\circ}{\sin 30^\circ} \\
 &= -\frac{\sqrt{3}}{2} \times \frac{2}{1} \\
 &= -\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } \cos(-480^\circ) &= \cos[-480^\circ - (-360^\circ)] \\
 &= \cos(-120^\circ) \\
 &= -\frac{\sqrt{3}}{2}
 \end{aligned}$$

2. Rajah 2 menunjukkan graf  $y = (p + 1) \sin qx$   
 Diagram 1 shows the graph of  $y = (p + 1) \sin qx$



Rajah 2  
 Diagram 2

- (a) Cari nilai  $p$  dan nilai  $q$   
 Find the value of  $p$  and of  $q$
- (b) Di atas graf tersebut, lakarkan graf  $y = |(p + 1) \sin qx|$  untuk domain  $0 \leq x \leq \pi$   
 On the graph, sketch the graf of  $y = |(p + 1) \sin qx|$  for the domain  $0 \leq x \leq \pi$

[5 markah]  
 [5 marks]

**Penyelesaian :**

- (a)  $p + 1 = -2$   
 $\Rightarrow p = -3, q = 2$
- (b) bahagian graf di bawah dipantulkan di paksi- $x$   
 Dilakarkan dalam domain  $0 \leq x \leq \pi$

3. Cari julat nilai  $x$  di antara  $0^\circ$  dan  $180^\circ$  yang memuaskan ketaksamaan  
*Find the range of values of  $x$  between  $0^\circ$  and  $180^\circ$  which satisfies the inequality*

(a)  $\cos 2x < 0$   
 $\cos 2x < 0$

(b)  $\sin \frac{1}{2}x > 0.5$

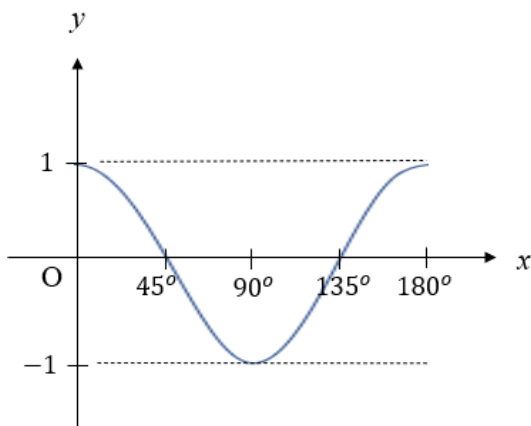
Oleh yang demikian, nyatakan julat nilai  $x$  di antara  $0^\circ$  and  $180^\circ$  yang memuaskan kedua-dua ketaksamaan tersebut.

*Hence state the range of values of  $x$  between  $0^\circ$  and  $180^\circ$  which satisfies both of the inequalities.*

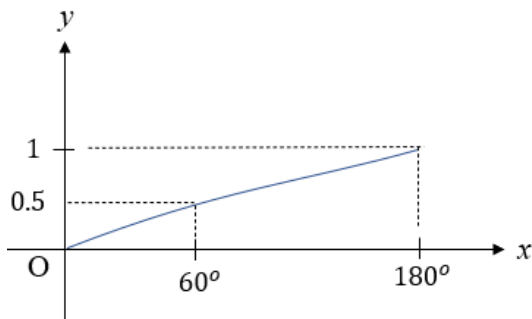
[6 markah]

[6 marks]

**Penyelesaian :**

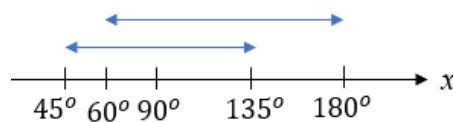


Jika  $\cos 2x < 0 \Rightarrow 45^\circ \leq x \leq 135^\circ$



Jika  $\sin \frac{1}{2}x > 0.5 \Rightarrow 60^\circ \leq x \leq 180^\circ$

Menggabungkan kedua-dua julat nilai  $x$



$\therefore 60^\circ \leq x \leq 135^\circ$



4. Selesaikan persamaan  $2 \tan x = 5 \sin x$  untuk  $0^\circ \leq x \leq 360^\circ$   
*Solve the equation  $2 \tan x = 5 \sin x$  for  $0^\circ \leq x \leq 360^\circ$*

[6 markah]  
 [6 marks]

**Penyelesaian :**

$$2 \left( \frac{\sin x}{\cos x} \right) = 5 \sin x$$

$$2 \sin x = 5 \sin x \cos x$$

$$2 \sin x - 5 \sin x \cos x = 0$$

$$\sin x (2 - 5 \cos x) = 0$$

$$\sin x = 0, 2 - 5 \cos x = 0$$

$$\sin x = 0, \cos x = \frac{2}{5}$$

jika  $\sin x = 0 \Rightarrow x = 0^\circ, 180^\circ, 360^\circ$

jika  $\cos x = \frac{2}{5} \Rightarrow x = 66.42^\circ, 293.58^\circ$

5. Selesaikan persamaan  $\operatorname{cosec}^2 2\theta = 3 \tan \theta - 1$  untuk  $0^\circ \leq \theta \leq 180^\circ$ .  
*Solve the equation  $\operatorname{cosec}^2 2\theta = 3 \tan \theta - 1$  for  $0^\circ \leq \theta \leq 180^\circ$ .*

[4 markah]  
 [4 marks]

**Penyelesaian :**

$$\operatorname{cosec}^2 2\theta = 3 \tan \theta - 1$$

$$1 + \tan^2 2\theta = 3 \tan 2\theta - 1$$

$$\tan^2 2\theta - 3 \tan 2\theta + 2 = 0$$

$$(\tan 2\theta - 2)(\tan 2\theta - 1) = 0$$

$$\tan 2\theta = 2 \quad \text{or} \quad \tan 2\theta = 1$$

$$2\theta = 63^\circ 26', 243^\circ 26' \quad \text{or} \quad 2\theta = 45^\circ, 225^\circ$$

$$\theta = 22^\circ 30', 31^\circ 43', 112^\circ 30', 121^\circ 43'$$

6. Selesaikan persamaan  $\sin \theta = \cos \frac{1}{2} \theta$  untuk  $\frac{\pi}{2} \leq \theta \leq 2\pi$ . Berikan jawapan dalam sebutan  $\pi$ .

*Solve the equation  $\sin \theta = \cos \frac{1}{2} \theta$  for  $\frac{\pi}{2} \leq \theta \leq 2\pi$ . Give your answer in terms of  $\pi$ .*

[4 markah]

[4 marks]

**Penyelesaian :**

$$\sin \theta = \cos \frac{1}{2} \theta$$

$$2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = \cos \frac{\theta}{2}$$

$$2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} - \cos \frac{\theta}{2} = 0$$

$$\cos \frac{\theta}{2} \left( 2 \sin \frac{\theta}{2} - 1 \right) = 0$$

$$\cos \frac{\theta}{2} = 0 \quad \text{or} \quad 2 \sin \frac{\theta}{2} - 1 = 0$$

$$\frac{\theta}{2} = \frac{\pi}{2}, \frac{3}{2}\pi \quad \text{or} \quad \frac{\theta}{2} = \frac{\pi}{6}, \frac{5}{6}\pi$$

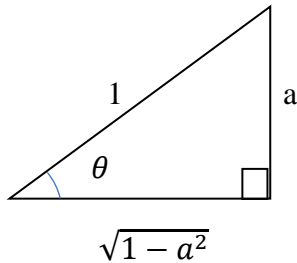
$$\theta = \frac{\pi}{2}, \pi, \frac{5}{3}\pi$$

7. Jika  $\sin \theta = a$  dan  $\sin 2\theta = b$ , ungkapkan  $b$  dalam sebutan  $a$   
*If  $\sin \theta = a$  and  $\sin 2\theta = b$ , express  $b$  in terms of  $a$*

[5 markah]

[5 marks]

**Penyelesaian :**



$$\sin \theta = a$$

$$\sin 2\theta = b$$

$$\Rightarrow 2 \sin \theta \cos \theta = b$$

$$\Rightarrow 2a \cos \theta = b$$

$$\Rightarrow \cos \theta = \frac{b}{2a}$$

$$\Rightarrow \sqrt{1 - a^2} = \frac{b}{2a}$$

$$\Rightarrow b = 2a\sqrt{1 - a^2}$$

8. Buktikan setiap identiti trigonometri yang berikut.  
*Prove each of the following trigonometric identities.*

(a)  $(\cos x + \sin x)^2 - 1 = 2 \sin x \cos x$

$$(\cos x + \sin x)^2 - 1 = 2 \sin x \cos x$$

(b)  $3 + 2 \operatorname{cosec}^2 x = 5 + 2 \cot^2 x$

$$3 + 2 \operatorname{cosec}^2 x = 5 + 2 \cot^2 x$$

(c)  $7 - 6 \sin^2 x = 1 + 6 \cos^2 x$

$$7 - 6 \sin^2 x = 1 + 6 \cos^2 x$$

[6 markah]

[6 marks]

**Penyelesaian :**

$$\begin{aligned} \text{a) } & (\cos x + \sin x)^2 - 1 \\ &= \cos^2 x + 2 \sin x \cos x + \sin^2 x - 1 \\ &= (\cos^2 x + \sin^2 x) + 2 \sin x \cos x - 1 \\ &= 1 + 2 \sin x \cos x - 1 \end{aligned}$$

$$\begin{aligned} \text{b) } & 3 + 2 \operatorname{cosec}^2 x \\ &= 3 + 2 (\cot^2 x + 1) \\ &= 3 + 2 \cot^2 x + 2 \\ &= 5 + 2 \cot^2 x \end{aligned}$$

$$\begin{aligned} \text{c) } & 7 - 6 \sin^2 x \\ &= 7 - 6 (1 - \cos^2 x) \\ &= 7 - 6 + 6 \cos^2 x \\ &= 1 + 6 \cos^2 x \end{aligned}$$

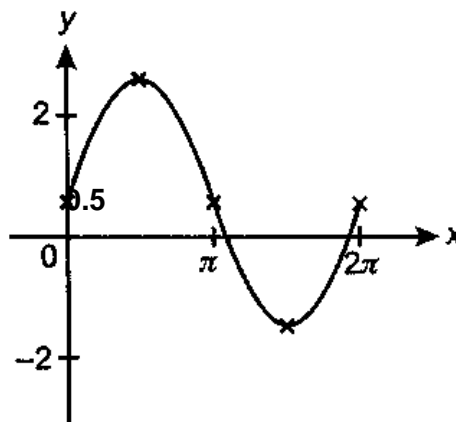
9. (a) Diberi  $\sec \alpha = \sqrt{t^2 + 1}$ , dengan  $t$  ialah pemalar dan  $0^\circ \leq \alpha \leq 90^\circ$ . Ungkapkan dalam sebutan  $t$

Given  $\sec \alpha = \sqrt{t^2 + 1}$  where  $t$  is a constant and  $0^\circ \leq \alpha \leq 90^\circ$ . Express in terms of  $t$

- (i)  $\cos \alpha$ ,  
 $\cos \alpha$ ,  
(ii)  $\sin(180^\circ + \alpha)$ ,  
(iii)  $\cot 2\alpha$ .  
 $\cot 2\alpha$

- (b) Rajah 9 menunjukkan graf  $y = p \cos qx + \frac{1}{2}$  untuk  $0 \leq x \leq 2\pi$ .

Diagram 9 shows the graph  $y = p \cos qx + \frac{1}{2}$  for  $0 \leq x \leq 2\pi$ .



Rajah 9  
Diagram 9

Nyatakan

State

- (i) nilai bagi  $p$  dan  $q$ ,  
the value of  $p$  and  $q$ ,  
(ii) bilangan penyelesaian untuk  $p \cos qx = \frac{3}{2}$ .

number of solutions for  $p \cos qx = \frac{3}{2}$ .

[8 markah]

[8 marks]

**Penyelesaian :**

(a)  $\sec \alpha = \sqrt{t^2 + 1}$

(i)  $\cos \alpha$

$$\sec \alpha = \sqrt{t^2 + 1}$$

$$\frac{1}{\cos \alpha} = \sqrt{t^2 + 1}$$

$$\cos \alpha = \frac{1}{\sqrt{t^2 + 1}}$$

(ii)  $\sin(180^\circ + \alpha)$

$$= -\sin \alpha$$

$$= -\frac{t}{\sqrt{t^2 + 1}}$$

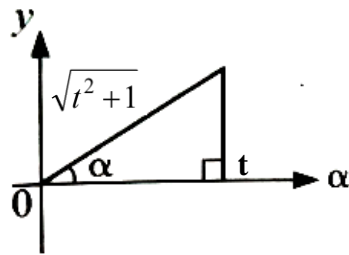
(iii)  $\cot 2\alpha$

$$= \frac{1}{\tan 2\alpha}$$

$$= \frac{1}{\frac{2 \tan \alpha}{1 - \tan^2 \alpha}}$$

$$= \frac{1 - \tan^2 \alpha}{2 \tan \alpha}$$

$$= \frac{1 - t^2}{2t}$$



(b)  $y = p \cos qx + \frac{1}{2}$

(i)  $p = 2, q = 1$

(ii)  $p \cos qx = \frac{3}{2}$

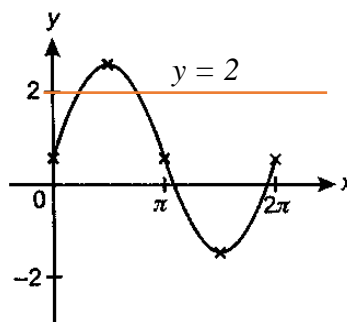
$$y = p \cos qx + \frac{1}{2}$$

$$p \cos qx = y - \frac{1}{2}$$

$$y - \frac{1}{2} = \frac{3}{2}$$

$$y = 2$$

Bilangan penyelesaian = 2



10. (a) Buktikan bahawa  $\tan A (\operatorname{cosec} 2A + \cot 2A) = 1$   
*Prove that  $\tan A (\operatorname{cosec} 2A + \cot 2A) = 1$*

(b) Selesaikan persamaan trigonometri  $4 \tan 2x = 9 \tan x$  untuk  $0^\circ \leq x \leq 360^\circ$   
*Solve the trigonometric equation  $4 \tan 2x = 9 \tan x$  for  $0^\circ \leq x \leq 360^\circ$*

[8 markah]

[8 marks]

**Penyelesaian :**

(a) LHS =  $\tan A (\operatorname{cosec} 2A + \cot 2A)$

$$\begin{aligned} &= \frac{\sin A}{\cos A} \left( \frac{1}{\sin 2A} + \frac{\cos 2A}{\sin 2A} \right) \\ &= \frac{\sin A}{\cos A} \left( \frac{1 + \cos 2A}{\sin 2A} \right) \\ &= \frac{\sin A}{\cos A} \left( \frac{1 + 2\cos^2 A - 1}{2\sin A \cos A} \right) \\ &= \frac{2 \sin A \cos^2 A}{2 \sin A \cos^2 A} \\ &= 1 = \text{RHS (Terbukti)} \end{aligned}$$

(b)  $4 \tan 2x = 9 \tan x$

$$4 \left( \frac{2 \tan x}{1 - \tan^2 x} \right) = 9 \tan x$$

Let  $\tan x = y$

$$\frac{8y}{1 - y^2} = 9y$$

$$8y = 9y - 9y^3$$

$$9y^3 - y = 0$$

$$y(9y^2 - 1) = 0$$

$$y = 0, (9y^2 - 1) = 0$$

$$y^2 = \frac{1}{9}$$

$$y = \frac{1}{3}, y = -\frac{1}{3}$$

Jika  $\tan x = 0 \Rightarrow x = 0^\circ, 180^\circ, 360^\circ$

Jika  $\tan x = 1/3 \Rightarrow x = 18.43^\circ, 198.43^\circ$

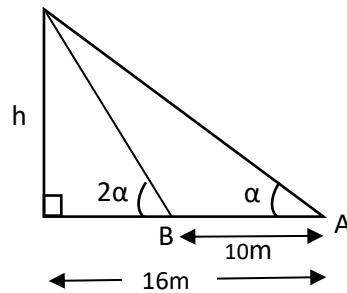
Jika  $\tan x = -1/3 \Rightarrow x = 161.57^\circ, 341.57^\circ$

11. Sudut dongakan titik A dan titik B yang terletak di atas suatu garis lurus dalam satah mengufuk dari puncak sebuah bangunan ialah  $\alpha$  dan  $2\alpha$  masing-masing. Diberi  $AB = 10$  m dan jarak A dari bangunan itu ialah 16 m, cari ketinggian bangunan tersebut.  
*The angle of elevation of the top of a building from two points A and B in a straight line in the horizontal plane through the foot of the tower are  $\alpha$  and  $2\alpha$  respectively. If  $AB = 10$  m and the distance A from the building is 16m, find the height of the building.*

[5 markah]

[5 marks]

**Penyelesaian :**



$$\tan \alpha = \frac{h}{16}$$

$$\tan 2\alpha = \frac{h}{6}$$

$$\frac{2\left(\frac{h}{16}\right)}{1 - \left(\frac{h}{16}\right)^2} = \frac{h}{6}$$

$$h^2 = 64 \text{ atau setara}$$

$$h = 8$$

12. (a) Diberi  $\sin x = p$  dan  $\sin y = q$  dengan keadaan  $x$  dan  $y$  adalah sudut tirus.

Cari dalam sebutan  $p$  dan  $q$

*Given that  $\sin x = p$  and  $\sin y = q$  where  $x$  and  $y$  are acute angles.*

*Find in terms of  $p$  and  $q$*

(i)  $\sin(x + y) \sin(x - y)$

(ii)  $\cos 2x$

- (b) Diberi  $\tan A$  dan  $\tan B$  adalah punca-punca bagi persamaan  $x^2 - px + q = 0$ .

Ungkapkan  $\tan(A + B)$  dalam sebutan  $p$  dan  $q$

*It is given that  $\tan A$  and  $\tan B$  are the roots of the equation  $x^2 - px + q = 0$ .*

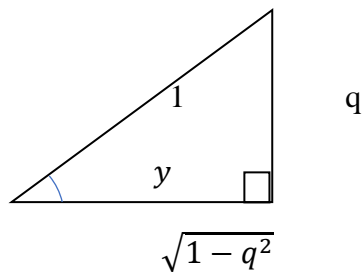
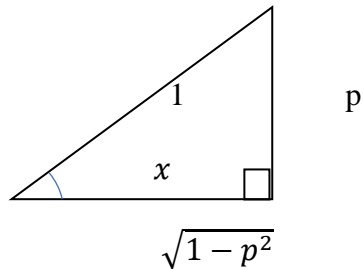
*Express  $\tan(A + B)$  in terms of  $p$  and  $q$*

[8 markah]

[8 marks]

**Penyelesaian :**

(a)



$$\begin{aligned}
 \text{(i) } \sin(x+y) \sin(x-y) &= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \\
 &= (\sin x \cos y)^2 - (\cos x \sin y)^2 \\
 &= (p \sqrt{1-q^2})^2 - (\sqrt{1-p^2} \cdot q)^2 \\
 &= p^2(1-q^2) - (1-p^2)q^2 \\
 &= p^2 - p^2q^2 - q^2 + p^2q^2 \\
 &= p^2 - q^2 \\
 &= (p+q)(p-q)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } \cos 2x &= 2\cos^2 x - 1 \\
 &= 2(\sqrt{1-p^2})^2 - 1 \\
 &= 2(1-p^2) - 1 \\
 &= 2 - 2p^2 - 1 \\
 &= 1 - 2p^2
 \end{aligned}$$

(b)  $x^2 - px + q = 0$ .

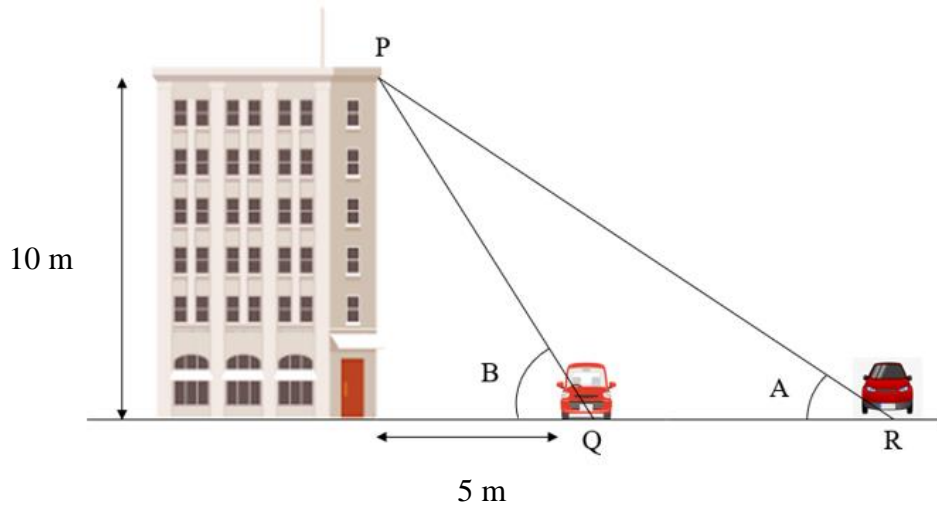
Diberi  $\tan A$  dan  $\tan B$  adalah punca-punca

$\Rightarrow \tan A + \tan B = p$  dan  $\tan A \tan B = q$

$$\begin{aligned}
 \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\
 &= \frac{p}{1-q}
 \end{aligned}$$



13.



Rajah 13(a)  
Diagram 13(a)

- (a) Rajah 13(a) menunjukkan dua buah kereta yang diparkir di sisi sebuah bangunan. Cari jarak antara kereta tersebut tanpa menentukan  $\angle A$  dan  $\angle B$ .

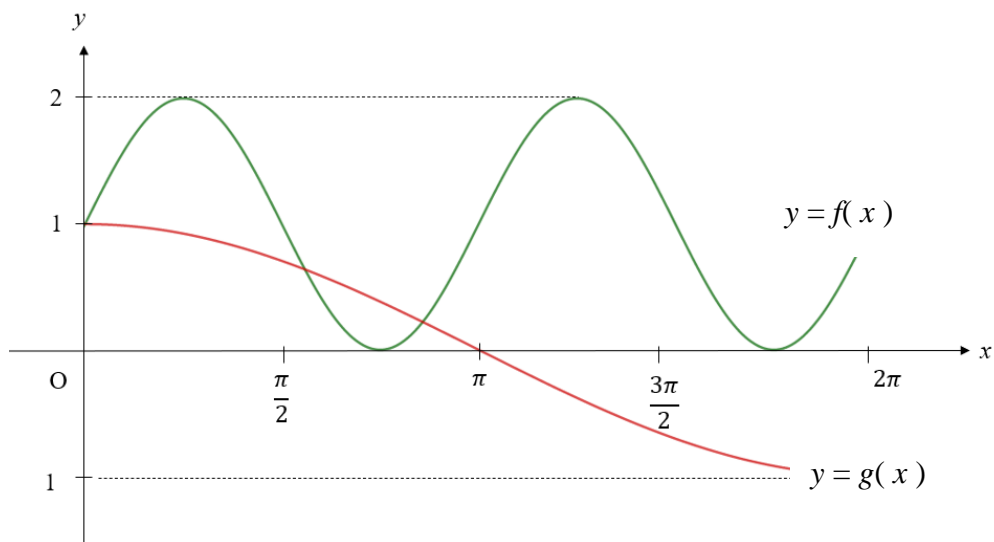
Diberi  $\tan ( A+B ) = -4$

*Diagram 13(a) shows two cars are parked next to a building.*

*Find the distance between both cars without determining the  $\angle A$  and  $\angle B$ .*

*Given that  $\tan ( A + B ) = -4$*

(b)



Rajah 13(b)  
Diagram 13(b)

Berdasarkan kepada rajah 13(b) ,  
Based on diagram 13(b),

- (i) tuliskan fungsi bagi  $y = f(x)$  dan  $y = g(x)$   
write the functions of  $y = f(x)$  and  $y = g(x)$
- (ii) cari julat nilai  $x$  yang supaya kedua-dua fungsi  $f(x)$  dan  $g(x)$  adalah positif  
find the range values of  $x$  for which both of the functions  $f(x)$  and  $g(x)$  are positive.
- [8 markah]  
[8 marks]

**Penyelesaian :**

$$\begin{aligned}
 \text{(a) } \tan B &= \frac{10}{5} = 2 \\
 \tan A &= \frac{10}{5+QR} \\
 \tan(A+B) &= -4 \\
 \Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} &= -4 \\
 \Rightarrow \frac{\frac{10}{5+QR} + 2}{1 - \left(\frac{10}{5+QR}\right)(2)} &= -4 \\
 \Rightarrow \frac{10}{5+QR} + 2 &= -4 \left(1 - \left(\frac{10}{5+QR}\right)(2)\right) \\
 \Rightarrow \frac{10}{5+QR} + 2 &= -4 + \frac{80}{5+QR} \\
 \Rightarrow \frac{80}{5+QR} - \frac{10}{5+QR} &= 6 \\
 \Rightarrow \frac{70}{5+QR} &= 6 \\
 \Rightarrow 70 &= 6(5+QR) \\
 \Rightarrow 70 &= 30 + 6QR \\
 \Rightarrow 6QR &= 40 \\
 \Rightarrow QR &= \frac{40}{6} = \frac{20}{3} = 6.667 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) (i) } y = f(x) &\Rightarrow y = 1 + \sin 2x \\
 y = g(x) &\Rightarrow y = \cos \left(\frac{1}{2}x\right)
 \end{aligned}$$

$$\text{(ii) } 0 \leq x < \pi$$

14. (a) Buktikan bahawa  $\frac{\tan 2x \cos 2x}{\sin x} = 2 \cos x$

Prove that  $\frac{\tan 2x \cos 2x}{\sin x} = 2 \cos x$

- (b) (i) Lakar graf  $y = 5 \cos x - 2$  untuk  $0 \leq x \leq 2\pi$ .  
Sketch the graph of  $y = 5 \cos x - 2$  for  $0 \leq x \leq 2\pi$ .

- (ii) Seterusnya, dengan menggunakan paksi yang sama, lakar sat ugaris lurus yang sesuai untuk mencari bilangan penyelesaian bagi persamaan

$$\frac{\tan 2x \cos 2x}{2 \sin x} + \frac{3}{5} = \frac{x}{2\pi} \text{ untuk } 0 \leq x \leq 2\pi.$$

Nyatakan bilangan penyelesaian itu.

Hence, using the same axes, sketch a suitable straight line to find the number of solutions to the equation

$$\frac{\tan 2x \cos 2x}{2 \sin x} + \frac{3}{5} = \frac{x}{2\pi} \text{ for } 0 \leq x \leq 2\pi.$$

State the number of solutions.

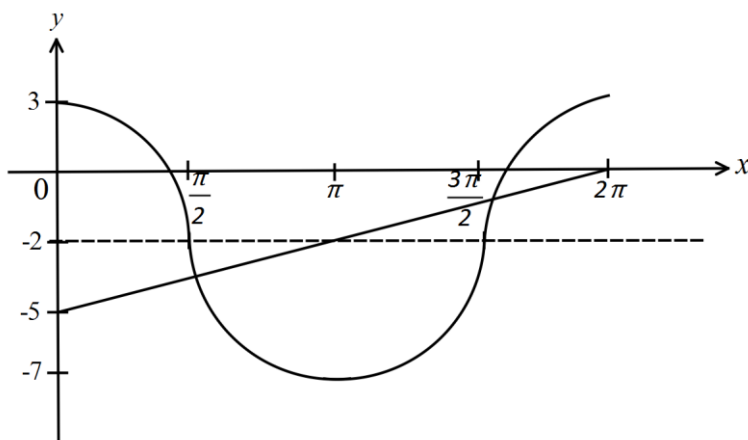
[10 markah]

[10 marks]

**Penyelesaian :**

$$(a) \frac{\frac{\tan 2x \cos 2x}{\sin x}}{\frac{2 \sin x \cos x}{\sin x}} = \frac{\left(\frac{\sin 2x}{\cos 2x}\right) \cos 2x}{\sin x}$$

(b) (i)



$$ii) y = \frac{5x}{2\pi} - 5$$

Correct line for  $y = \frac{5x}{2\pi} - 5$

Number of solutions = 2

15. (a) Cari nilai setiap pemalar a dan b dengan keadaan  
 $\sin x \cos x (5 \tan x + 2 \cot x) = a + b \sin^2 x$   
*Find the value of each of the constants a and b for which*  
 $\sin x \cos x (5 \tan x + 2 \cot x) = a + b \sin^2 x$

- (b) Persamaan yang berikut adalah benar bagi semua nilai A  

$$\frac{1 + \cos A}{1 - \cos A} - \frac{1 - \cos A}{1 + \cos A} = 4 \cot A \operatorname{cosec} A$$
  
*The following equation is correct to all values of A*  

$$\frac{1 + \cos A}{1 - \cos A} - \frac{1 - \cos A}{1 + \cos A} = 4 \cot A \operatorname{cosec} A$$

- (i) Buktikan persamaan tersebut  
*Prove the equation*

- (ii) Seterusnya, cari nilai  $\cot A \operatorname{cosec} A$  jika nilai  $\cos A = 0.6$   
*Hence, find the value of  $\cot A \operatorname{cosec} A$  if the value of  $\cos A = 0.6$*

[8 markah]  
 [8 marks]

**Penyelesaian :**

(a)  $\sin x \cos x (5 \tan x + 2 \cot x) = a + b \sin^2 x$   
 $= \sin x \cos x \left( \frac{5 \sin x}{\cos x} + \frac{2 \cos x}{\sin x} \right)$   
 $= 5 \sin^2 x + 2 \cos^2 x$   
 $= 5 \sin^2 x + 2(1 - \sin^2 x)$   
 $= 5 \sin^2 x + 2 - 2 \sin^2 x$   
 $= 2 + 3 \sin^2 x$   
 $\Rightarrow a = 2, b = 3$

(b) (i)  

$$\frac{1 + \cos A}{1 - \cos A} - \frac{1 - \cos A}{1 + \cos A} = 4 \cot A \operatorname{cosec} A$$

$$\begin{aligned} LHS &= \frac{[(1 + \cos A) + (1 - \cos A)][(1 + \cos A) - (1 - \cos A)]}{1 - \cos^2 A} \\ &= \frac{[(2)(2 \cos A)]}{\sin^2 A} \\ &= \frac{4 \cos A}{\sin^2 A} \\ &= 4 \left( \frac{\cos A}{\sin A} \right) \left( \frac{1}{\sin A} \right) \\ &= 4 \cot A \operatorname{cosec} A \\ &= \text{RHS ( terbukti )} \end{aligned}$$

(ii) jika  $\cos A = 0.6$   

$$\frac{1 + \cos A}{1 - \cos A} - \frac{1 - \cos A}{1 + \cos A} = 4 \cot A \operatorname{cosec} A$$
  

$$4 \cot A \operatorname{cosec} A = \frac{1 + 0.6}{1 - 0.6} - \frac{1 - 0.6}{1 + 0.6}$$
  

$$= 4 - \frac{1}{4} = \frac{15}{4}$$
  

$$\Rightarrow \cot A \operatorname{cosec} A = \frac{15}{16}$$

16. Dua gelombang air yang berpunca dari dua penjuru sebuah kolam diwakili oleh persamaan berikut:

*The water waves that originating from two corners of a pool are described by the following equations:*

$$w_1 = \sin\left(p + \frac{\pi}{3}\right), \quad w_2 = 2 \cos\left(p - \frac{\pi}{6}\right)$$

Dua gelombang tersebut bercantum di tengah kolam itu dan gelombang gabungan diberi oleh  $w = w_1 + w_2$ . Cari  $w$  dalam bentuk  $m \sin(np + \beta)$  dan kemudiannya menyatakan nilai  $m$ ,  $n$  dan  $\beta$ .

*The two waves combine as they propagate to the middle of the pool and combined wave is given by  $w = w_1 + w_2$ . Find  $w$  in the form  $m \sin(np + \beta)$  and hence state the values of  $m$ ,  $n$  and  $\beta$ .*

[8 markah]

[8 marks]

**Penyelesaian :**

$$\begin{aligned} w &= w_1 + w_2 \\ &= \sin\left(p + \frac{\pi}{3}\right) + 2 \cos\left(p - \frac{\pi}{6}\right) \\ &= \sin p \cos \frac{\pi}{3} \cos p \sin \frac{\pi}{3} + 2\left(\cos p \cos \frac{\pi}{6} + \sin p \sin \frac{\pi}{6}\right) \\ &= \sin p \cos \frac{\pi}{3} + \cos p \sin \frac{\pi}{3} + 2 \cos p \cos \frac{\pi}{6} + 2 \sin p \sin \frac{\pi}{6} \\ &= \sin p \left(\cos \frac{\pi}{3} + 2 \sin \frac{\pi}{6}\right) + \cos p \left(\sin \frac{\pi}{3} + 2 \cos \frac{\pi}{6}\right) \\ &= \sin p \left(\frac{1}{2} + 2 \times \frac{1}{2}\right) + \cos p \left(\frac{\sqrt{3}}{2} + 2 \times \frac{\sqrt{3}}{2}\right) \\ &= \sin p \left(\frac{3}{2}\right) + \cos p \left(\frac{3\sqrt{3}}{2}\right) \\ &= 3 \left[ \sin p \left(\frac{1}{2}\right) + \cos p \left(\frac{\sqrt{3}}{2}\right) \right] \\ &= 3 \left( \sin p \cos \frac{\pi}{3} + \cos p \sin \frac{\pi}{3} \right) \\ &= 3 \sin\left(p + \frac{\pi}{3}\right) \end{aligned}$$

Bandingkan dengan  $m \sin(np + \beta)$ :

*Compare with  $m \sin(np + \beta)$ :*

$$m = 3, \quad n = 1, \quad \beta = \frac{\pi}{3}$$

17. (a) Buktikan  $\cos\left(3x - \frac{\pi}{6}\right) - \cos\left(3x + \frac{\pi}{6}\right) = \sin 3x$ .

*Prove  $\cos\left(3x - \frac{\pi}{6}\right) - \cos\left(3x + \frac{\pi}{6}\right) = \sin 3x$ .*

(b) Seterusnya,

*Hence,*

(i) selesaikan persamaan  $\cos\left(\frac{3x}{2} - \frac{\pi}{6}\right) - \cos\left(\frac{3x}{2} + \frac{\pi}{6}\right) + \frac{1}{2} = 0$  for  $0 \leq x \leq 2\pi$  dan beri jawapan anda dalam bentuk pecahan termudah dalam sebutan  $\pi$  radian.

*solve the equation  $\cos\left(\frac{3x}{2} - \frac{\pi}{6}\right) - \cos\left(\frac{3x}{2} + \frac{\pi}{6}\right) + \frac{1}{2} = 0$  for  $0 \leq x \leq 2\pi$  and give your answers in the simplest fraction form in terms of  $\pi$  radian.*

(ii) lakar graf bagi  $y = \cos\left(3x - \frac{\pi}{6}\right) - \cos\left(3x + \frac{\pi}{6}\right) + \frac{1}{2}$  untuk  $0 \leq x \leq \pi$ .

*sketch the graph of  $y = \cos\left(3x - \frac{\pi}{6}\right) - \cos\left(3x + \frac{\pi}{6}\right) + \frac{1}{2}$  for  $0 \leq x \leq \pi$ .*

[10 markah]

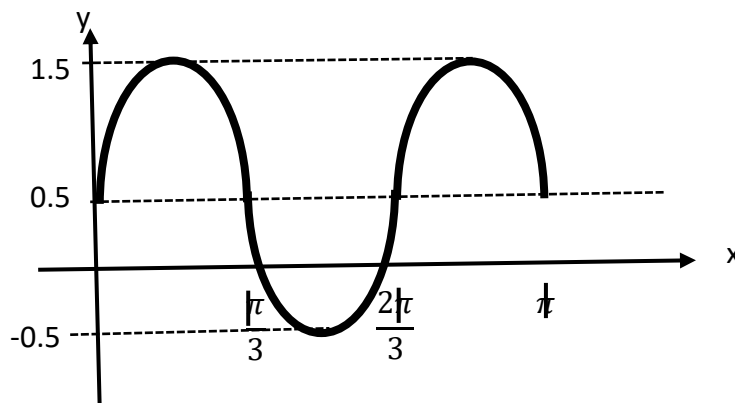
[10 marks]

**Penyelesaian :**

$$\begin{aligned} \text{a) } & \cos\left(3x - \frac{\pi}{6}\right) - \cos\left(3x + \frac{\pi}{6}\right) \\ &= \cos 3x \cos \frac{\pi}{6} + \sin 3x \sin \frac{\pi}{6} - \left[\cos 3x \cos \frac{\pi}{6} - \sin 3x \sin \frac{\pi}{6}\right] \\ &= \cos 3x \cos \frac{\pi}{6} + \sin 3x \sin \frac{\pi}{6} - \cos 3x \cos \frac{\pi}{6} + \sin 3x \sin \frac{\pi}{6} \\ &= 2\left[\sin 3x \sin \frac{\pi}{6}\right] \\ &= 2(\sin 3x)\left(\frac{1}{2}\right) \\ &= \sin 3x \end{aligned}$$

$$\begin{aligned} \text{b) } & \cos\left(\frac{3x}{2} - \frac{\pi}{6}\right) - \cos\left(\frac{3x}{2} + \frac{\pi}{6}\right) + \frac{1}{2} = 0 \\ & \cos\left(\frac{3x}{2} - \frac{\pi}{6}\right) - \cos\left(\frac{3x}{2} + \frac{\pi}{6}\right) = -\frac{1}{2} \\ & \sin \frac{3x}{2} = -\frac{1}{2} \\ & \frac{3x}{2} = \frac{7\pi}{6}, \frac{11\pi}{6} \\ & x = \frac{7\pi}{9}, \frac{11\pi}{9} \end{aligned}$$

c)



1. Sebuah kilang menghasilkan  $x$  dozen buku latihan 100 muka surat dan  $y$  dozen buku latihan 200 muka surat setiap hari. Kos penghasilan satu dozen buku latihan 100 muka surat dan satu dozen buku latihan 200 muka surat masing-masing ialah RM24 dan RM36. Penghasilan buku latihan adalah berdasarkan syarat-syarat yang berikut.

*A factory produce  $x$  dozen exercise book 100 pages and  $y$  dozen exercise book 200 pages every day. Producing costs of one dozen exercise book 100 pages and one dozen exercise book 200 pages are RM24 and RM 36 respectively. Producing costs of exercise book is based on the following constraints.*

- I Kilang itu mesti menghasilkan sekurang-kurangnya 400 dozen buku latihan setiap hari.  
*The factory must produce at least 400 dozen exercise book every day.*
- II Bilangan buku latihan 100 muka surat mesti selebih-lebihnya dua kali bilangan buku latihan 200 muka surat.  
*Number of 100 pages exercise book must be at most twice the number of 200 pages exercise book.*
- III Kos penghasilan harian buku latihan itu tidak boleh melebihi RM25200.  
*Producing cost per day of exercise book cannot more than RM25200.*

Tulis ketaksamaan yang mentakrifkan syarat-syarat di atas. Seterusnya, bina dan lorekkan rantau yang memenuhi syarat-syarat itu.

*Write down inequalities satisfy the above conditions, Hence, construct and shade the area satisfy all the constraints above.*

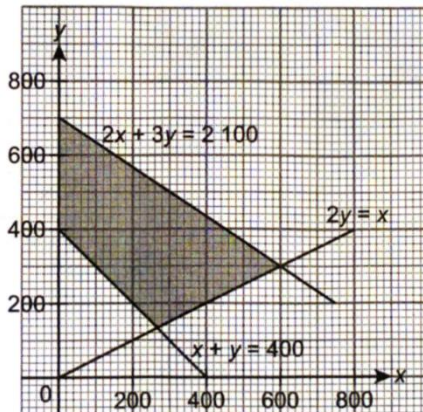
[6 markah]

[6 marks]

**Penyelesaian 1 :**

- (i)  $x + y \geq 400$   
(ii)  $2y \geq x$   
(iii)  $2x + 3y \leq 2100$

Graph---attachment



2. Diet seekor haiwan peliharaan di dalam sebuah ladang terdiri daripada makanan X dan makanan Y. Haiwan itu dibela dengan  $x$  g makanan X dan  $y$  g makanan Y setiap hari. Setiap 100g makanan X mengandungi 3g serabut manakala setiap 100g makanan Y mengandungi 5g serabut. Diet haiwan tersebut adalah tertakluk kepada syarat-syarat yang berikut.

*The diet of an animal consists of food X and food Y. The animal is fed with  $x$  g of food X and  $y$  g of food Y every day. Each 100gram of food X contains 3g of dietary fibre whereas each 100gram of food Y contains 5g of dietary fibre. The diet is subject to the following conditions.*

- I Jisim makanan X mesti selebih-lebihnya 3 kali jisim makanan Y.  
*The amount of food X must be at most 3 times that of food Y.*
- II Haiwan itu memerlukan selebih-lebihnya 200g makanan setiap hari.  
*The animal requires at most 200g of dietary fibre per day.*
- III Haiwan itu memerlukan sekurang-kurangnya 2g serabut setiap hari.  
*The animal requires at least 2g of dietary fibre per day.*

Tulis ketaksamaan yang menghuraikan syarat-syarat di atas. Seterusnya, bina dan lorekkan rantau yang memenuhi syarat-syarat itu.

*Write the inequalities that describe the conditions and hence construct and shade the region of feasible solutions.*

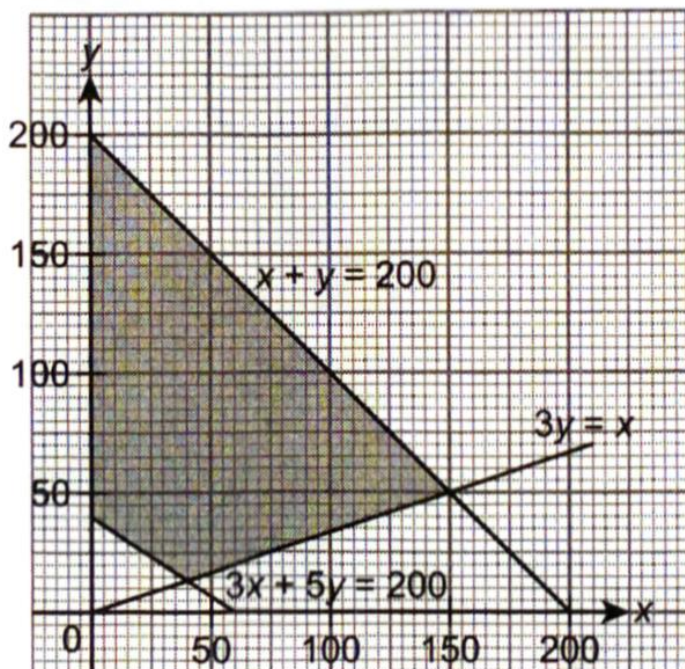
[6 markah]

[6 marks]

### Penyelesaian 2 :

- (i)  $3y \geq x$   
 (ii)  $x + y \leq 200$   
 (iii)  $3x + 5y \geq 200$





3. Sebuah kilang menghasilkan dua jenis peralatan sukan, papan seluncur dan kayu hoki. Kilang ini dapat menghasilkan  $x$  buah papan seluncur dan  $y$  batang kayu hoki dalam sehari. Pengurus kilang itu menetapkan bahawa kos pembuatan dua jenis peralatan sukan tersebut tidak boleh melebihi RM7200 dalam sehari. Penghasilan peralatan sukan berkenaan adalah berdasarkan kekangan yang berikut:

*A factory produces two types of sports equipment, skating board and hockey stick. The factory is able to produce  $x$  skating boards and  $y$  hockey sticks in a day. The manager of the factory has decided that the production cost for the two types of sports equipment should not exceed RM7200 per day. The production of these sports equipment is based on the following constraints:*

- I Kos menghasilkan sebuah papan seluncur ialah RM90 dan sebatang kayu hoki ialah RM80.

*The cost of making a skating board is RM90 and the cost of making a hockey stick is RM80*

- II Jumlah minimum bilangan papan seluncur dan kayu hoki ialah 40.

*The minimum total number of skating board and hockey stick is 40.*

- III Bilangan kayu hoki mesti sekurang-kurangnya 75% daripada bilangan papan seluncur.

*The number of hockey sticks must be at least 75% of the number of skating boards.*

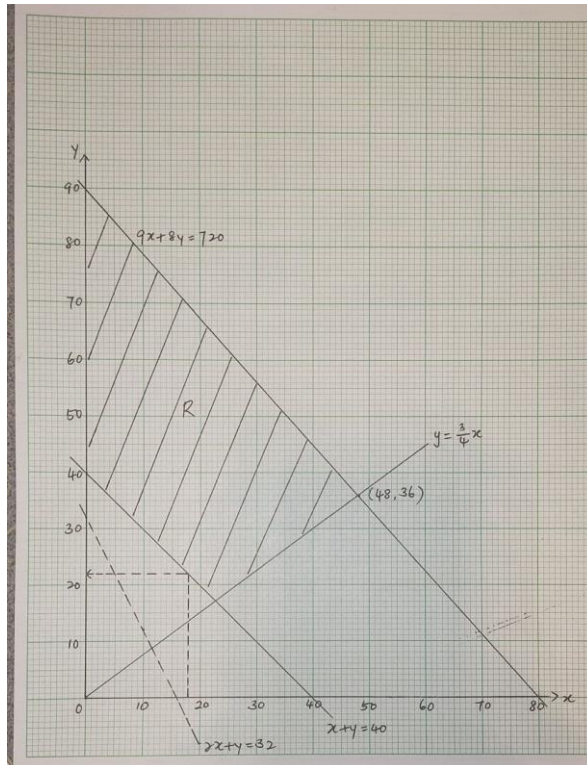
- (a) Tulis tiga ketaksamaan, selain daripada  $x \geq 0$  dan  $y \geq 0$  , yang memenuhi semua kekangan di atas.  
*Write three inequalities, other than  $x \geq 0$  dan  $y \geq 0$  , which satisfy all the above constraints*
- (b) Dengan menggunakan skala 2cm kepada 10 peralatan sukan pada kedua-dua paksi, bina dan lorek rantau R yang memenuhi semua kekangan di atas.  
*Using a scale of 2cm to 10 sports equipment on both axes, construct and shade the region R which satisfies all the above constraints.*
- (c) Dengan menggunakan graf yang dibina di 2(b), cari  
*Using the graph constructed in 2(b), find*
- (i) Bilangan minimum kayu hoki yang dihasilkan jika 18 buah papan seluncur dihasilkan.  
*The minimum number of hockey sticks produced if 18 skating boards are produced.*
- (ii) Jumlah keuntungan maksimum yang diperolehi jika keuntungan sebuah papan seluncur ialah RM32 dan keuntungan sebatang kayu hoki ialah RM16.  
*The maximum total profit gained if the profit of a skating board is RM32 and the profit of a hockey stick is RM16.*

[10markah  
 [10 marks]

**Penyelesaian 3 :**

- (a) I  $90x + 80y \leq 7200$  atau  $9x + 8y \leq 720$   
 II  $x + y \geq 40$   
 III  $y \geq \frac{3}{4}x$

(b)



(c)

(i) bila  $x = 18$  , nilai minimum  $y = 22$

Bilangan minimum kayu hoki yang dihasilkan = 22

(ii)  $c = 32x + 16y$  ,  $32x + 16y = 512$  ,  $2x + y = 32$

Titik maksimum = (48 , 36)

Jumlah keuntungan maksimum yang diperoleh

$$= 32(48) + 16(36)$$

$$= RM2112$$

4. Seorang pengurus daripada sebuah restoran francais yang terkenal ingin mengupah  $x$  orang pekerja sepenuh masa dan  $y$  orang pekerja sambilan. Elaun sebulan yang ditawarkan kepada seorang pekerja sepenuh masa ialah RM800 dan RM300 untuk seorang pekerja sambilan. Pengambilan pekerja restoran itu adalah berdasarkan kekangan berikut:

*A manager from a famous franchised restaurant intend to employ  $x$  full-time workers and  $y$  part-time workers. The monthly allowance offered to each full-time worker is RM800 and RM300 to each part-time worker. The employment of the workers is based on the following constraints:*

- I Jumlah elaun yang ditawarkan sebulan tidak harus kurang daripada RM2400.  
*The total monthly allowance offered should not be less than RM2400.*
- II Bilangan pekerja sambilan melebihi separuh daripada bilangan pekerja sepenuh masa sebanyak 4 atau kurang.  
*The number of part-time workers exceeds half the number of full-time workers by 4 or less.*
- III Nisbah bilangan pekerja sambilan kepada bilangan pekerja sepenuh masa adalah sekurang-kurangnya 3 : 2  
*The ratio of the number of part-time workers to the number of full-time workers is at least 3 : 2*

- (a) Tulis tiga ketaksamaan, selain daripada  $x \geq 0$  dan  $y \geq 0$ , yang memenuhi semua kekangan di atas.

*Write three inequalities, other than  $x \geq 0$  dan  $y \geq 0$ , which satisfy all the above constraints* [3 markah/3 marks]

- (b) Dengan menggunakan skala 2cm kepada 1 orang pekerja pada kedua-dua paksi, bina dan lorek rantau R yang memenuhi semua kekangan di atas.

*Using a scale of 2cm to 1 worker on both axes, construct and shade the region R which satisfies all the above constraints.*

[3 markah/3 marks]

- (c) Daripada graf di 3(b), cari julat jumlah bilangan pekerja yang boleh diambil oleh pengurus restoran itu.

*From the graph in 3(b), find the range of the total number of workers that can be employed by the manager of the restaurant.*

[4 markah/4 marks]

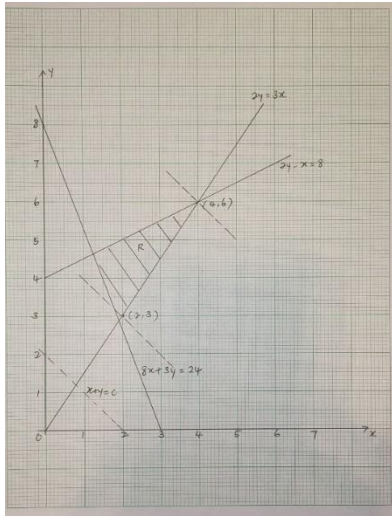
#### Penyelesaian 4 :

(a) I  $800x + 300y \geq 2400$  atau  $8x + 3y \geq 24$

II  $y - \frac{1}{2}x \leq 4$  atau  $2y - x \leq 8$

III  $\frac{y}{x} \geq \frac{3}{2}$  atau  $2y \geq 3x$

(b)



(c)

(i)  $c = x + y$

Titik maksimum = (4 , 6) atau titik minimum = (2 , 3)

Bilangan maksimum pekerja yang boleh diambil =  $4+6 = 10$

Bilangan minimum pekerja yang boleh diambil =  $2+3 = 5$

Julat bilangan pekerja yang boleh diambil  $5 \leq c \leq 10$

5. Sebuah syarikat bas ekspres di Pulau Pinang menyediakan  $x$  perjalanan bas ke Melaka dan  $y$  perjalanan bas ke Ipoh setiap hari. Perkhidmatan bas yang disediakan adalah berdasarkan kekangan berikut:

*An express bus company in Pulau Pinang provides  $x$  bus trips to Melaka and  $y$  bus trips to Ipoh per day. The bus service provided is based on the following constraints:*

- I Jumlah bilangan perjalanan bas yang disediakan tidak lebih daripada 15.  
*The total number of bus trips provided is not more than 15.*
- II Bilangan perjalanan ke Melaka tidak lebih empat kali bilangan perjalanan ke Ipoh.  
*The number of trips to Melaka is not more than four times the number of trips to Ipoh.*

III Tambang satu perjalanan bas ke Melaka ialah RM80 dan tambang satu perjalanan bas ke Ipoh ialah RM40. Jumlah tambang yang dipungut setiap hari adalah tidak kurang daripada RM480.

*The bus fare for a trip to Melaka is RM80 and the fare for a trip to Ipoh is RM40. The total fare collected per day is not less than RM480.*

(a) Tulis tiga ketaksamaan, selain daripada  $x \geq 0$  dan  $y \geq 0$  , yang memenuhi semua kekangan di atas.

*Write three inequalities, other than  $x \geq 0$  dan  $y \geq 0$  , which satisfy all the above constraints* [3 markah/3 marks]

(b) Dengan menggunakan skala 2cm kepada 2 perjalanan bas pada kedua-dua paksi, bina dan lorek rantau R yang memenuhi semua kekangan di atas.

*Using a scale of 2cm to 2 bus trips on both axes, construct and shade the region R which satisfies all the above constraints.* [3 markah/3 marks]

(c) Dengan menggunakan graf di 4(b), cari

*By using the graph in 4(b), find*

(i) Bilangan minimum perjalanan bas ke Ipoh jika 4 perjalanan bas ke Melaka disediakan pada setiap hari.

*The minimum number of trips to Ipoh if 4 trips are provided to Melaka per day.*

(ii) Bilangan maksimum penumpang yang dapat dibawa jika setiap bas ke Melaka dapat membawa 40 orang penumpang dan setiap bas ke Ipoh dapat membawa 20 orang penumpang.

*The maximum number of passengers that the buses could fetch if each bus to Melaka can bring 40 passengers and each bus to Ipoh can bring 20 passengers.*

[4 markah/4 marks]

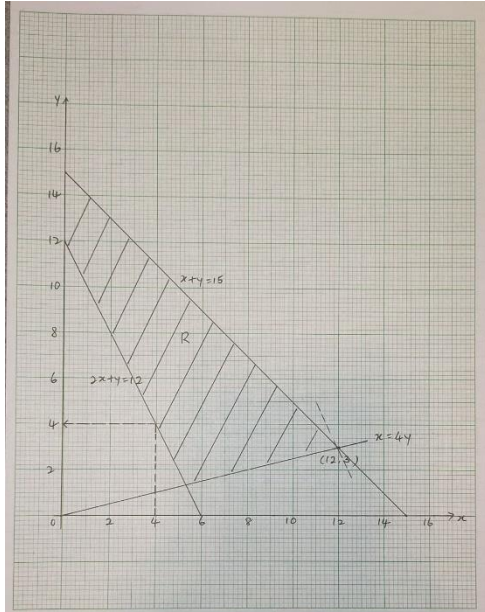
### Penyelesaian 5 :

(a) I  $x + y \leq 15$

II  $x \leq 4y$

III  $80x + 40y \geq RM480$  atau  $2x + y \geq 12$

(b)



(c)

(i) bila  $x = 4$  , nilai minimum  $y = 4$

(ii)  $c = 40x + 20y$

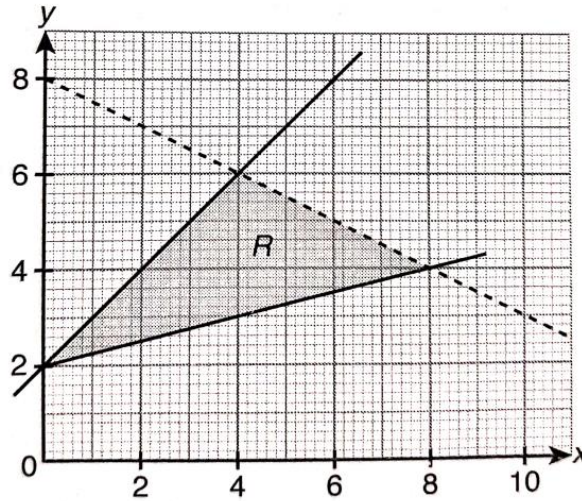
Titik maksimum = (12 , 3)

Bilangan maksimum penumpang yang dapat dibawa

$$= 40(12) + 20(3)$$

$$= 540$$

6. Rajah menunjukkan kawasan berlorek yang memuaskan 3 ketaksamaan.  $x$  dan  $y$  ialah nombor bulat.  
 Cari  
 Diagram show shaded region which satisfy 3 inequalities.  $x$  and  $y$  are whole number.  
 Find



- (a) nilai maksimum dan nilai minimum  $x$  dalam kawasan berlorek  
*maximum value and minimum value of  $x$  in the shaded region*
- (b) nilai maksimum dan nilai minimum  $y$  dalam kawasan berlorek  
*maximum value and minimum value of  $y$  in the shaded region*
- (c) nilai-nilai  $y$  apabila  $x = 3$   
*values of  $y$  when  $x = 3$*

[6 markah]  
 [6 marks]

**Penyelesaian 6 :**

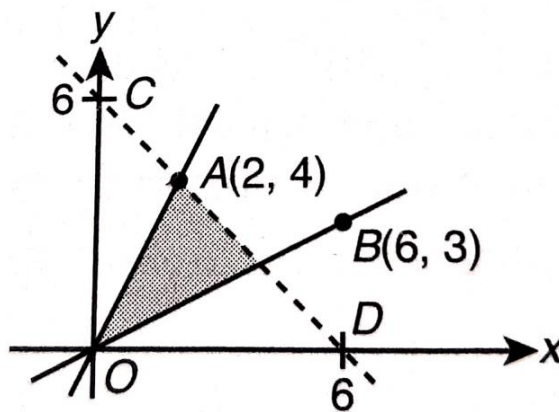
- (a) Nilai maksimum  $x = 7$   
 Nilai minimum  $x = 0$
- (b) Nilai maksimum  $y = 5$   
 Nilai minimum  $y = 2$
- (c)  $y = 3, 4$  dan  $5$



7. (a) Tulis ketaksamaan linear untuk kes-kes berikut:  
*Write inequalities for the cases below:*
- (i) Nisbah  $x$  kepada  $y$  tidak lebih daripada  $3 : 4$   
*Ratio  $x$  to  $y$  not more than  $3:4$*
- (ii)  $x + y$  kurang daripada  $2x + 3y$  sebanyak  $4$   
 *$x+y$  less than  $2x+3y$  by  $4$*

[2 markah]  
 [2 marks]

- (b) Cari ketaksamaan linear bagi kawasan berlorek.  
*Find linear inequalities for shaded region*



[3 markah]  
 [3 marks]

**Penyelesaian 7 :**

- (a) (i)  $4x \leq 3y$   
 (ii)  $x + 2y \geq 4$
- (b)  $y \leq 2x$   
 $y \geq \frac{1}{2}x$   
 $x + y \leq 6$

8. Sebuah agen pelancongan menganjurkan satu lawatan cuti ke Pulau Langkawi. Setiap bilik penginapan adalah perkongsian berkembar dan dihuni oleh 2 orang yang sama jantina sebilik. Terdapat  $x$  bilik untuk lelaki dan  $y$  bilik untuk perempuan. Lawatan itu ditetapkan dengan kekangan berikut:

*A travel agent organizes a holiday trip to Langkawi Island. Each guest room is a twin sharing and is occupied by 2 people of the same sex per room. There are  $x$  rooms for boys and  $y$  rooms for girls. The visit was scheduled with the following constraints:*

- I. Jumlah bilangan bilik sekurang-kurangnya 100.

*The total number of rooms is at least 100*

- II. Bilangan bilik untuk perempuan tidak melebihi 2 kali ganda bilangan bilik lelaki.

*The number of rooms for girls does not exceed 2 times the number of rooms for boys.*

- III. Diberi  $y + 3x$  selebih-lebihnya ialah 240.

*Given the maximum of  $y + 3x$  is 240*

- (a) Tuliskan 3 ketaksamaan, selain daripada  $x \geq 0$  dan  $y \geq 0$ , yang memuaskan semua kekangan di atas.

*Write 3 inequalities, other than  $x \geq 0$  and  $y \geq 0$ , which satisfy all the above constraints.*

[3 markah]

[3 marks]

- (b) Menggunakan skala 2 cm untuk 20 bilik pada setiap paksi, bina dan lorek kawasan  $R$  yang memuaskan semua kekangan di atas.

*Using a scale of 2 cm to 20 rooms on both axes, construct and shade the Region  $R$  which satisfies all the above constraints.*

[3 markah]

[3 marks]

- (c) Gunakan graf di (b), cari

*Using the graph in (b), find*

- (i) bilangan maksimum dan minimum bilik untuk lelaki apabila bilangan bilik untuk perempuan ialah 45

*the maximum and minimum number of rooms for boys if the number of rooms for girls is 45*

- (ii) caj maksimum yang dibayar untuk semua bilik jika setiap bilik dikenakan RM120 dan RM140 masing-masing untuk bilik lelaki dan bilik perempuan

*the maximum charge paid for all rooms if each room is charged RM120 and RM140 respectively for male and female rooms*

[4 markah]

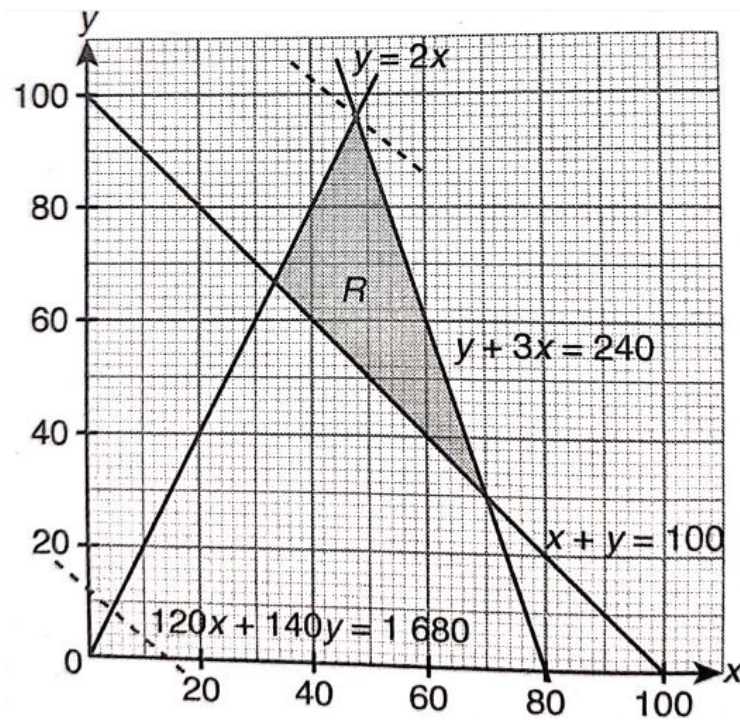
**Penyelesaian 8 :**

(a)  $x + y \geq 100$

$y \leq 2x$

$y + 3x \leq 240$

(b)



(c) (i) Maksimum  $x = 65$   
Minimum  $x = 55$

(ii) Carj maksimum  
=  $120(48) + 140(96)$   
= RM 19200

10. Satu pusat tuisyen menawarkan mata pelajaran Biologi,  $B$ , dan Fizik,  $F$ , untuk murid Tingkatan 4. Bilangan murid untuk  $B$  dan  $F$  ialah  $x$  dan  $y$  masing-masing. Pengambilan murid adalah berdasarkan syarat-syarat berikut.  
*A tuition center offers the subjects of Biology,  $B$ , and Physics,  $F$ , for Form 4 students. The number of students for  $B$  and  $F$  are  $x$  and  $y$  respectively. Student requirement is based on the following conditions.*
- I. Jumlah murid selebih-lebihnya 80 orang  
*The total number of students is at most 80 people*
  - II. Bilangan murid yang mengambil mata pelajaran  $B$  selebih-lebihnya 3 kali ganda bilangan murid yang mengambil mata pelajaran  $F$   
*The number of students taking subject  $B$  is at most 3 times the number of students taking subject  $F$ .*
  - III. Bilangan murid yang mengambil mata pelajaran  $B$  tidak boleh melebihi bilangan murid yang mengambil mata pelajaran  $F$  sebanyak 10  
*The number of students taking subject  $B$  must not exceed the number of students taking subject  $F$  by 10*
- (a) Tulis 3 ketaksamaan. selain daripada  $x \geq 0$  dan  $y \geq 0$ , yang memuaskan semua kekangan di atas.  
*Write 3 inequalities. other than  $x \geq 0$  and  $y \geq 0$ , which satisfies all the above constraints.*
- [3 markah]  
 [3 marks]
- (b) Menggunakan skala 2 cm mewakili 10 orang murid pada setiap paksi, bina dan lorek kawasan  $R$  yang memuaskan semua syarat di atas.  
*Using a scale of 2 cm representing 10 students on each axis, construct and shade an area  $R$  that satisfies all the above conditions.*
- [3 markah]  
 [3 marks]
- (c) Berdasarkan graf yang dilukis di (b), cari  
*Based on the graph drawn in (b), find*
- (i) julat bilangan murid yang mengambil mata pelajaran  $F$  jika bilangan murid mengambil mata pelajaran  $B$  ialah 20  
*the range of the number of students taking subject  $F$  if the number of students taking subject  $B$  is 20*
  - (ii) yuran kutipan maksimum setiap bulan jika yuran setiap bulan untuk mata pelajaran  $B$  dan  $F$  ialah RM 40 dan RM 20 masing-masing.  
*the maximum collection fee per month if the monthly fee for subjects  $B$  and  $F$  is RM 40 and RM 20 respectively*
- [4 markah]  
 [4 marks]

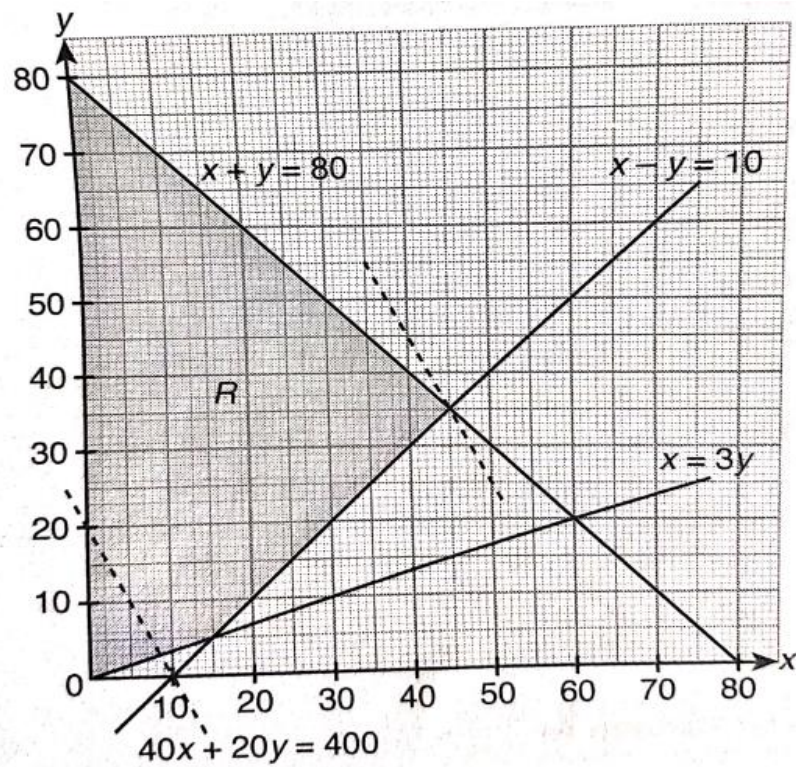
**Penyelesaian 10 :**

(a)  $x + y \leq 80$

$x \leq 3y$

$x - y \leq 10$

(b)



(c) (i)  $10 \leq y \leq 60$

(ii) titik maksimum (45, 35)

$40(45) + 20(35)$

11. Pn. Rosmah ditugaskan untuk membeli buku rujukan Fizik dan Kimia bagi perpustakaan sekolah tertakluk kepada syarat-syarat berikut:  
*Pn. Rosmah was assigned to purchase Physics and Chemistry reference books for the school library subject to the following conditions:*

- I. Jumlah bilangan buku rujukan tidak melebihi 80 buah  
*The total number of reference books does not exceed 80*
- II. Bilangan buku Kimia tidak boleh melebihi 2 kali ganda buku Fizik  
*The number of Chemistry books should not exceed 2 times the number of Physics books*
- III. Bilangan buku Kimia melebihi buku Fizik sekurang-kurangnya 10 buku  
*The number of Chemistry books exceeds Physics books by at least 10 books*

- (a) Tulis 3 ketaksamaan. selain daripada  $x \geq 0$  dan  $y \geq 0$ , yang memuaskan semua kekangan di atas.

*Write 3 inequalities. other than  $x \geq 0$  and  $y \geq 0$ , which satisfies all the above constraints.*

[3 markah]

[3 marks]

- (b) Menggunakan skala 2 cm mewakili 10 buah buku pada setiap paksi, bina dan lorek kawasan  $R$  yang memuaskan semua syarat di atas.

*Using a scale of 2 cm representing 10 books on each axis, construct and shade an area  $R$  that satisfies all the above conditions.*

[3 markah]

[3 marks]

- (c) Berdasarkan graf yang dilukis di (b), cari

*Based on the graph drawn in (b), find*

- (i) bilangan maksimum buku Kimia yang boleh dibeli  
*the maximum number of Chemistry books that can be purchased*

- (ii) kos minimum untuk membeli semua buku jika kos sebuah buku Fizik dan Kimia ialah RM 4 dan RM 2 masing-masing

*the minimum cost to buy all the books if the cost of a Physics and Chemistry book is RM 4 and RM 2 respectively*

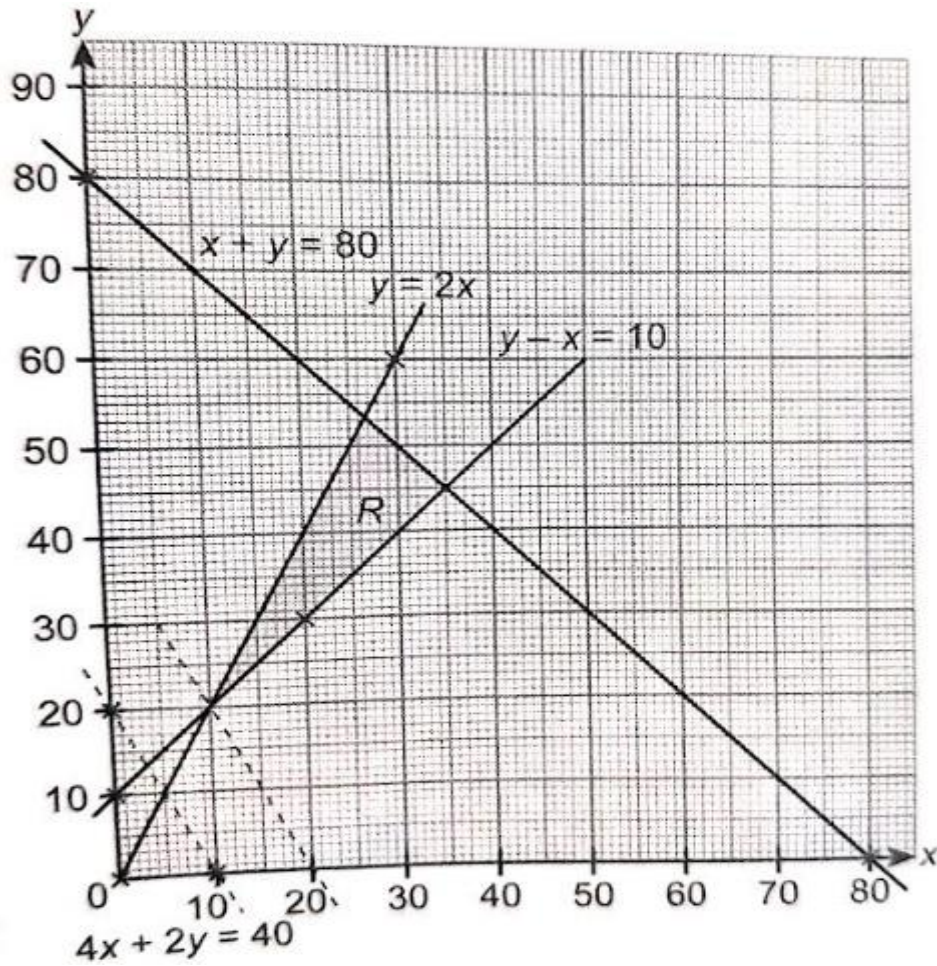
[4 markah]

[4 marks]

**Penyelesaian 11 :**

(a)  $x + y \leq 80$   
 $y \leq 2x$   
 $y - x \geq 10$

(b)



(c) (i) 53

(ii) titik minimum (10, 20)  
 $4(10) + 2(20)$   
RM 80

# BAB 8

## KINEMATIK GERAKAN LINEAR KINEMATICS OF LINEAR MOTION

1. Suatu zarah bergerak di sepanjang garis lurus itu dan melalui titik tetap  $O$ . Halajunya,  $v \text{ m s}^{-1}$ , diberi oleh  $v = 12 + 5t - 3t^2$ , dengan keadaan  $t$  ialah masa, dalam saat, selepas meninggalkan titik  $O$ . Zarah itu berhenti seketika di titik  $B$ .

[Anggapkan gerakan ke arah kanan sebagai positif.]

*A particle moves along the straight line and passes through a fixed point  $O$ . Its velocity,  $v \text{ m s}^{-1}$ , is given by  $v = 12 + 5t - 3t^2$ , where  $t$  is the time, in seconds, after leaving the point  $O$ . The particle stops instantaneously at point  $B$ .*

[Assume motion to the right to be positive.]

Cari

Find

- (a) halaju awal, dalam  $\text{m s}^{-1}$ , zarah itu  
*the initial velocity, in  $\text{m s}^{-1}$ , of the particle*
- (b) pecutan awal, dalam  $\text{m s}^{-2}$ , zarah itu  
*the initial acceleration, in  $\text{m s}^{-2}$ , of the particle*
- (c) halaju maksimum, dalam  $\text{m s}^{-1}$ , zarah itu  
*The maximum velocity, in  $\text{m s}^{-1}$ , of the particle*

[6 markah]

[6 marks]

### Penyelesaian 1 :

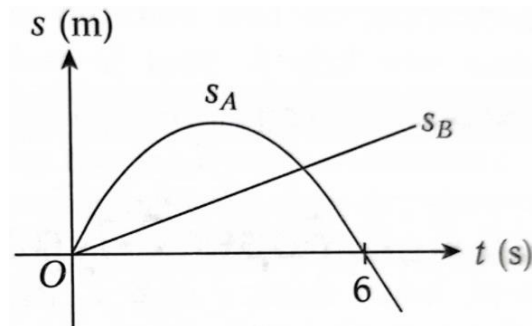
(a) Apabila / When  $t = 0$ ,  
 $v = 12 + 5(0) - 3(0)^2$   
 $= 12$

(b)  $a = \frac{dv}{dt}$   
 $a = 5 - 6t$   
Apabila / When  $t = 0$ ,  
 $a = 5 - 6(0)$   
 $a = 5$

(c) Bagi halaju maksimum / For maximum velocity,  $\frac{dv}{dt} = 0$   
 $5 - 6t = 0$   
 $t = \frac{5}{6}$   
 $v = 12 + 5\left(\frac{5}{6}\right) - 3\left(\frac{5}{6}\right)^2$   
 $= 12 + \frac{25}{6} - \frac{25}{12}$   
 $= 14\frac{1}{12}$



2. Rajah di bawah menunjukkan sesaran bagi dua zarah, A dan B, yang bergerak serentak ke kanan satu titik tetap  $O$ , di sepanjang suatu garis lurus.  
*The diagram shows the displacements of two particles, A and B, moving simultaneously to the right of a fixed point  $O$ , along a straight line.*



Diberi sesaran zarah A dan zarah B masing-masing ialah  $S_A = pt - t^2$  and  $S_B = 2t$  dengan keadaan  $t$  ialah masa dalam saat selepas meninggalkan  $O$ . Cari  
*Given the displacements of the particles A and B are  $S_A = pt - t^2$  dan  $S_B = 2t$  respectively, where  $t$  is the time in seconds after leaving  $O$ . Find*

- (a) nilai  $p$ .  
*the value of  $p$*
- (b) masa apabila zarah A dan zarah B bertemu selepas meninggalkan  $O$ .  
*the time when the particles A and B meet after leaving  $O$ .*
- (c) sesaran maksimum zarah A dari titik  $O$ .  
*the maximum displacement of the particle A from point  $O$ .*

[5 markah]  
 [5 marks]

**Penyelesaian 2 :**

- (a)  $t = 6, S_A = 0,$   
 $6p - 36 = 0$   
 $p = 6$
- (b)  $S_A = S_B$   
 $6t - 4t^2 = 2t$   
 $t^2 - 4t = 0$   
 $t(t - 4) = 0$   
 $t = 4$
- (c)  $t = 3, S_A = 6(3) - (3)^2$   
 $S_A = 9m$

3. Satu zarah bergerak sepanjang satu garis lurus,  $a = (2t - 3) \text{ m s}^{-2}$  ialah pecutan pada  $t$  saat selepas melalui satu titik tetap  $O$ . Diberi halaju awalnya ialah  $-4 \text{ m s}^{-1}$ . Carikan sesaran apabila zarah berhenti seketika.

[Anggapkan gerakan ke arah kanan sebagai positif.]

*A particle moves along a straight line,  $a = (2t - 3) \text{ m s}^{-2}$  is its acceleration at  $t$  seconds after passing through a fixed point  $O$ . Given that its initial velocity is  $-4 \text{ m s}^{-1}$ . Find its displacement when the particle comes to instantaneous rest.*

[Assume motion to the right to be positive.]

[5 markah]

[5 marks]

### Penyelesaian 3 :

$$v = \int 2t - 3 \, dt$$

$$v = t^2 - 3t + c$$

Apabila / when  $t = 0, v = -4,$

$$-4 = 0^2 - 3(0) + c$$

$$\therefore v = t^2 - 3t - 4$$

Apabila / when  $v = 0, t^2 - 3t - 4 = 0$

$$(t - 4)(t + 1) = 0$$

$$t = 4$$

$$s = \int t^2 - 3t - 4 \, dt$$

$$= \frac{t^3}{3} - \frac{3t^2}{2} - 4t + c$$

Apabila / when  $t = 0, s = 0, c = 0$

$$\therefore s = \frac{t^3}{3} - \frac{3t^2}{2} - 4t$$

$$s = \frac{(4)^3}{3} - \frac{3(4)^2}{2} - 4(4)$$

$$= -18\frac{2}{3} \text{ m}$$

4. Suatu zarah bergerak di sepanjang garis lurus dan melalui satu titik tetap  $O$ . Halajunya,  $v \text{ m s}^{-1}$ , diberi oleh  $v = -t^2 + 2t + 8$ , dengan keadaan  $t$  ialah masa, dalam saat, selepas melalui  $O$ .  
*A particle moves along the straight line which passes through a fixed point  $O$ . Its velocity,  $v \text{ m s}^{-1}$ , is given by  $v = -t^2 + 2t + 8$ , where  $t$  is the time, in seconds, after passing through  $O$ .*

[Anggapkan gerakan ke arah kanan sebagai positif.]  
 [Assume the motion to the right to be positive.]

Cari  
 Find

- (a) halaju awal, dalam  $\text{m s}^{-1}$ , bagi zarah itu  
*the initial velocity, in  $\text{m s}^{-1}$ , of the particle*
- (b) nilai  $t$ , dalam saat, apabila zarah itu berhenti seketika  
*the value of  $t$ , in seconds, when the particle stops instantaneously*
- (c) halaju maksimum, dalam  $\text{m s}^{-1}$ , bagi zarah itu  
*The maximum velocity, in  $\text{m s}^{-1}$ , of the particle*

[6 markah]  
 [6 marks]

**Penyelesaian 4 :**

- (a) Apabila / When  $t = 0$ ,  
 $v = -(0)^2 + 2(0) + 8$   
 $= 8$
- (b) Apabila / When  $v = 0$ ,  
 $0 = -t^2 + 2t + 8$   
 $0 = (t + 2)(t - 4)$   
 $t = -2 \text{ or } 4$   
 $t = -2$  is not accepted.  
 $t = 4$  [1 m]
- (c) Bagi halaju maksimum / At maximum velocity,  $\frac{dv}{dt} = 0$   
 $-2t + 2 = 0$   
 $t = 1$   
 $v_{\max} = -(1)^2 + 2(1) + 8$   
 $= 9$

5. Satu zarah bergerak di sepanjang suatu garis lurus dan melalui satu titik tetap  $O$  dengan halaju  $10 \text{ m s}^{-1}$ . Pecutannya,  $a \text{ m s}^{-2}$ , diberi oleh  $a = 2t - 6$ , dengan keadaan  $t$  ialah masa, dalam saat, selepas melalui  $O$ .

[Anggapkan gerakan ke arah kanan sebagai positif]

*A particle moves along a straight line and passes through a fixed point  $O$  with a velocity of  $10 \text{ m s}^{-1}$ . The acceleration,  $a \text{ m s}^{-2}$ , is given by  $a = 2t - 6$ , where  $t$  is the time, in seconds, after passing through  $O$ . [Assume motion to the right is positive]*

Cari

Find

- (a) pecutan awal, dalam  $\text{m s}^{-2}$ , zarah itu  
*the initial acceleration, in  $\text{m s}^{-2}$ , of the particle*
- (b) halaju minimum, dalam  $\text{m s}^{-1}$ , zarah itu.  
*the minimum velocity, in  $\text{m s}^{-1}$ , of the particle*
- (c) jumlah jarak, dalam  $\text{m}$ , yang dilalui oleh zarah itu dalam 7 saat yang pertama.  
*the total distance, in  $\text{m}$ , travelled by the particle in the first 7 seconds.*

[8 markah]

[8 marks]

**Penyelesaian 5 :**

(a) Apabila / When  $t = 0$ ,  
 $a = 2(0) - 6$   
 $a = -6$

(b)  $v = \int a \, dt$   
 $v = \int (2t - 6) \, dt$   
 $v = t^2 - 6t + c$   
 Apabila / When  $t = 0$ ,  $v = 10$ ,  
 $10 = (0)^2 - 6(0) + c$   
 $c = 10$   
 $v = t^2 - 6t + 10$   
 Halaju minimum / Minimum velocity,  $a = 0$   
 $2t - 6 = 0$   
 $t = 3$   
 $v = 3^2 - 6(3) + 10$   
 $v = 9 - 18 + 10$   
 $v = 1$

(c)  $s = \int_0^7 (t^2 - 6t + 10) \, dt$   
 $= \left[ \frac{1}{3}t^3 - 3t^2 + 10t \right]_0^7$   
 $= \left[ \frac{1}{3}(7)^3 - 3(7)^2 + 10(7) \right] - 0$   
 $= 37\frac{1}{3}$

6. Suatu zarah bergerak di sepanjang satu garis lurus dan melalui satu titik tetap  $O$ . Sesarannya,  $s$  cm, dari titik  $O$  diberi oleh  $s = -t^2 + 5t$ .

*A particle moves along a straight line and passes through a fixed point  $O$ . Its displacement,  $s$  cm, from point  $O$  is given by  $s = -t^2 + 5t$ .*

*[Anggapkan gerakan ke arah kanan sebagai positif]*

*[Assume the motion to the right is positive]*

- (a) Cari jarak, dalam cm, yang dilalui oleh zarah itu dalam saat keempat.  
*Find the distance, in cm, travelled by the particle in the fourth second.*
- (b) Cari masa, dalam saat, apabila zarah itu kembali ke  $O$ .  
*Find the time, in seconds, when the particle returns to  $O$ .*
- (c) Lakar graf sesaran melawan masa bagi pergerakan zarah itu untuk  $0 \leq t \leq 5$ .  
Seterusnya, nyatakan masa, dalam saat, ketika zarah itu bertukar arah gerakan.  
*Sketch the displacement-time graph for the motion of the particle for  $0 \leq t \leq 5$ . Hence, state the time, in seconds, when the particle changes its direction of motion.*

[8 markah]

[8 marks]

**Penyelesaian 6 :**

(a) Jarak yang dilalui dalam saat ke-4  
*Distance travelled in the fourth second*  

$$= |s_4 - s_3|$$

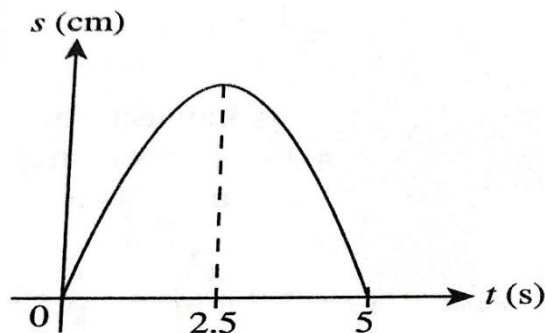
$$= |-(4)^2 + 5(4) - [-(3)^2 + 5(3)]|$$

$$= |4 - 6|$$

$$= 2$$

(b)  $t(t - 5) = 0$  or  $-t(t - 5) = 0$   
 $t = 5$

(c)



Lengkung di lukis dengan betul

*Curve drawn correctly*

Label 0 dan 5 pada  $x$ -axis dengan betul

*Label 0 and 5 correctly on  $x$ -axis.*

$t = 2.5$

7. Satu zarah bergerak di sepanjang suatu garis lurus dengan keadaan pecutannya  $a \text{ m s}^{-2}$ , diberi oleh  $a = -2t + 8$ , dengan keadaan  $t$  ialah masa, dalam saat, selepas melalui titik tetap  $O$ . Halaju awal zarah itu ialah  $-7\text{ms}^{-1}$ .  
*A particle moves along a straight line such that its acceleration,  $a \text{ m s}^{-2}$ , is given by  $a = -2t + 8$ , where  $t$  is the time, in seconds, after passing through a fixed point  $O$ . The initial velocity of the particle is  $-7\text{ms}^{-1}$ .*

[Anggapkan gerakan ke arah kanan sebagai positif.]  
 [Assume motion to the right is positive]

Cari  
 Find

- (a) halaju maximum, dalam  $\text{m s}^{-1}$ , zarah itu.  
*the maximum velocity, in  $\text{m s}^{-1}$ , of the particle.*
- (b) halaju, dalam  $\text{m s}^{-1}$ , zarah itu, apabila  $t = 7$   
*the instantaneous velocity, in  $\text{ms}^{-1}$ , of the particle when  $t = 7$*
- (c) jarak, dalam  $\text{m}$ , yang dilalui oleh zarah itu pada saat kelima.  
*the distance, in  $\text{m}$ , travelled by the particle during the fifth second.*

[8 markah]  
 [8 marks]

### Penyelesaian 7 :

- (a)  $v = \int(-2t + 8) dt$   
 $v = -t^2 + 8t + c$   
 Apabila / When  $t = 0$ ,  $v = -7$ ,  
 $-7 = (0)^2 + 8(0) + c$   
 $c = -7$   
 $v = -t^2 + 8t - 7$   
 Halaju maximum / Maximum velocity,  $a = 0$   
 $-2t + 8 = 0$   
 $t = 4$   
 $v = -(4)^2 + 8(4) - 7$   
 $v = 9$
- (b) Apabila / When  $t = 3$ ,  
 $v = -(3)^2 + 8(3) - 7$   
 $= 8$  [1 m]
- (c) Jarak/ Distance travelled  
 $= \int_4^5 -t^2 + 8t - 7 dt$   
 $= \left[-\frac{1}{3}(5)^3 + 4(5)^2 - 7(5)\right] - \left[-\frac{1}{3}(4)^3 + 4(4)^2 - 7(4)\right]$   
 $= 8\frac{2}{3}$

8. Satu zarah bergerak di sepanjang suatu garis lurus dan melalui satu titik tetap  $O$  dengan halaju  $36 \text{ m s}^{-1}$ . Pecutannya,  $a \text{ m s}^{-2}$ ,  $t \text{ s}$  selepas melalui  $O$  diberi oleh  $a = 6(t - 4)$ .  
[Anggapkan gerakan ke arah kanan sebagai positif]

*A particle moves along a straight line and passes through a fixed point  $O$  at a velocity of  $36 \text{ m s}^{-1}$ . Its acceleration,  $a \text{ m s}^{-2}$ ,  $t \text{ s}$  after passing through  $O$  is given by  $a = 6(t - 4)$ . [Assume the motion to the right is positive]*

(a) Cari halajunya,  $v \text{ m s}^{-1}$ , dalam sebutan  $t$ .  
*Find its velocity,  $v \text{ m s}^{-1}$ , in terms of  $t$ .*

(b) Cari nilai-nilai  $t$  ketika zarah itu bertukar arah gerakannya.  
*Find the values of  $t$  when the particle reverses its direction of motion.*

[5 markah]  
[5 marks]

**Penyelesaian 8:**

$$\begin{aligned} \text{(a) } a &= 6(t - 4) \\ a &= 6t - 24 \\ v &= \int a \, dt \\ v &= \int (6t - 24) \, dt \\ v &= 3t^2 - 24t + c \end{aligned}$$

Apabila / When  $t = 0$ ,  $v = 36$ ,

$$\begin{aligned} 36 &= 3(0)^2 - 24(0) + c \\ c &= 36 \\ v &= 3t^2 - 24t + 36 \end{aligned}$$

$$\begin{aligned} \text{(b) } 3t^2 - 24t + 36 &= 0 \\ t^2 - 8t + 12 &= 0 \\ (t - 6)(t - 2) &= 0 \\ t - 6 = 0 \quad \text{atau/ or } t - 2 &= 0 \\ t = 6 \quad \text{atau/ or } t &= 2 \end{aligned}$$

9. Suatu zarah bergerak di sepanjang suatu garis lurus dan melalui satu titik tetap  $O$ . Sesarannya,  $s$  m, dari  $O$  diberi oleh  $s = 6t^2 - 5t - t^3$  dengan keadaan  $t$  ialah masa, dalam saat, selepas melalui  $O$ .  
*A particle moves along a straight line and passes through a fixed point  $O$ . Its displacement,  $s$  m, from  $O$  is given by  $s = 6t^2 - 5t - t^3$  where  $t$  is the time, in seconds, after passing through  $O$ .*

Cari  
 Find

- (a) masa yang mungkin apabila zarah itu melalui  $O$  again,  
*the possible times when the particle passes through  $O$  again,*
- (b) masa yang diambil untuk mencapai halaju maksimum dan halaju maksimum zarah itu,  
*the time taken to reach the maximum velocity and the maximum velocity of the particle,*

[8 markah]

[8 marks]

**Penyelesaian 9 :**

$$\begin{aligned} \text{(a) } s &= 6t^2 - 5t - t^3 \\ 0 &= 6t^2 - 5t - t^3 \\ t^3 - 6t^2 + 5t &= 0 \\ t(t^2 - 6t + 5) &= 0 \\ t(t - 1)(t - 5) &= 0 \\ \therefore t &= 0, 1, 5 \end{aligned}$$

Masa yang mungkin / Possible times are  $t = 1$  and  $t = 5$ .

$$\begin{aligned} \text{(b) } s &= 6t^2 - 5t - t^3 \\ v &= \frac{ds}{dt} = 12t - 5 - 3t^2 \\ a &= \frac{dv}{dt} = 12 - 6t \\ 12 - 6t &= 0 \\ t &= 2s \\ \text{Semasa / When } t &= 2, \quad v = 12(2) - 5 - 3(2)^2 \\ v &= 7ms^{-1} \end{aligned}$$



10. Satu zarah bergerak di sepanjang suatu garis lurus dengan halaju awal  $-15 \text{ ms}^{-1}$ . Pecutannya,  $a \text{ ms}^{-2}$ , diberi oleh  $a = 8 - 2t$ , dengan keadaan  $t$  ialah masa, dalam saat, selepas melalui titik tetap  $O$ .

[Anggapkan gerak ke arah kanan sebagai positif]

*A particle moves along a straight line with an initial velocity of  $-15 \text{ ms}^{-1}$ . Its acceleration,  $a \text{ ms}^{-2}$  is given by  $a = 8 - 2t$ , where  $t$  is the time, in seconds, after passing through a fixed point  $O$ .*

[Assume motion to the right is positive]

Cari

Find

- (a) Halaju maksimum, dalam  $\text{m s}^{-1}$ , bagi zarah itu.  
*The maximum velocity, in  $\text{m s}^{-1}$ , of the particle.*
- (b) Masa, dalam saat, apabila zarah menukar arah gerakan.  
*The time, in seconds, when the particle changes its direction.*

[7 markah]

[7 marks]

### Penyelesaian 10 :

- (a)  $v = \int a \, dt$   
 $v = \int (8 - 2t) \, dt$   
 $v = 8t - t^2 + c$   
 $t = 0, V = -15$   
 $-15 = C$   
 $\therefore V = 8t - t^2 - 15$   
 $a = 0, 8 - 2t = 0$   
 $t = 4$   
 $t = 4, V_{\text{mak}} = 8(4) - 4^2 - 15$   
 $= 32 - 16 - 15$   
 $= 1 \text{ ms}^{-1}$
- (b) Zarah menukar arah gerakan  
*The particle changes its direction*  
 $V = 0, 8t - t^2 - 15 = 0$   
 $t^2 - 8t + 15 = 0$   
 $(t - 3)(t - 5) = 0$   
 $t = 3 \text{ or } t = 5$

11. Satu zarah bergerak di sepanjang suatu garis lurus dan melalui satu titik tetap  $O$ . Halajunya,  $v$  m s<sup>-1</sup>, diberi oleh  $v = 3t^2 - 8t + 3$ , dengan keadaan  $t$  ialah masa dalam saat selepas melalui  $O$ .

*A particle travels in a straight line and passes through a fixed point  $O$ . Its velocity,  $v$  m s<sup>-1</sup>, is given by  $v = 3t^2 - 8t + 3$ , where  $t$  is the time in seconds after leaving  $O$ .*

Kirakan / Calculate

(a) halaju, dalam m s<sup>-1</sup>, zarah itu apabila pecutannya ialah 4 m s<sup>-2</sup>,  
*the velocity, in m s<sup>-1</sup>, of the particle when the acceleration is 4 m s<sup>-2</sup>,*

(b) nilai-nilai  $t$  apabila zarah itu melalui  $O$  semula.  
*the values of  $t$  at which the particle passes through  $O$  again.*

[6 markah]

[6 marks]

**Penyelesaian 11 :**

(a)  $a = 4$

$$6t - 8 = 4$$

$$t = 2$$

$$v = 3(2)^2 - 8(2) + 3 \\ = -1 \text{ m s}^{-1}$$

(b)  $s = 0$

$$t^3 - 4t^2 + 3t = 0$$

$$t(t - 1)(t - 3) = 0$$

$$t = 0, t = 1, t = 3$$

Zarah melalui  $O$  semula pada,

*The particle passes through  $O$  again at,*

*$t = 1$  dan/and  $t = 3$ .*

12. Suatu zarah bergerak di sepanjang suatu garis lurus. Sesaran,  $s$  meter, dari satu titik tetap  $O$  diberi  $s = \frac{1}{3}t^3 - 3t^2 + 5t$  dengan keadaan  $t$  ialah masa, dalam saat, selepas melalui  $O$ .

*A particle moves along a straight line. The displacement,  $s$  metres, from a fixed point  $O$  is given by  $s = \frac{1}{3}t^3 - 3t^2 + 5t$ , where  $t$  is the time, in seconds, after passing through  $O$ .*

[Anggapkan gerakan ke arah kanan sebagai positif]  
[Assume the motion to the right is positive]

Cari  
Find

- (a) pecutan awal, dalam  $\text{m s}^{-2}$ , zarah itu  
*the initial acceleration, in  $\text{m s}^{-2}$ , of the particle.*
- (b) pecutan, dalam  $\text{m s}^{-2}$ , zarah itu bertukar arah gerakannya  
*the acceleration, in  $\text{m s}^{-2}$ , of the particle for the moment the particle reverses its direction*

[7 markah]  
[7 marks]

**Penyelesaian 12 :**

(a)  $s = \frac{1}{3}t^3 - 3t^2 + 5t$

$$v = t^2 - 6t + 5$$

$$a = 2t - 6$$

Apabila/When  $t = 0$ ,

$$a = -6$$

(b) Apabila/When  $t = 0$ ,

$$t^2 - 6t + 5 = 0$$

$$(t - 5)(t - 1) = 0$$

$$t = 5 \text{ atau/ or } t = 1$$

$$t = 5, a = 2(5) - 6 = 4\text{ms}^{-2}$$

$$t = 1, a = 2(1) - 6 = -4\text{ms}^{-2}$$