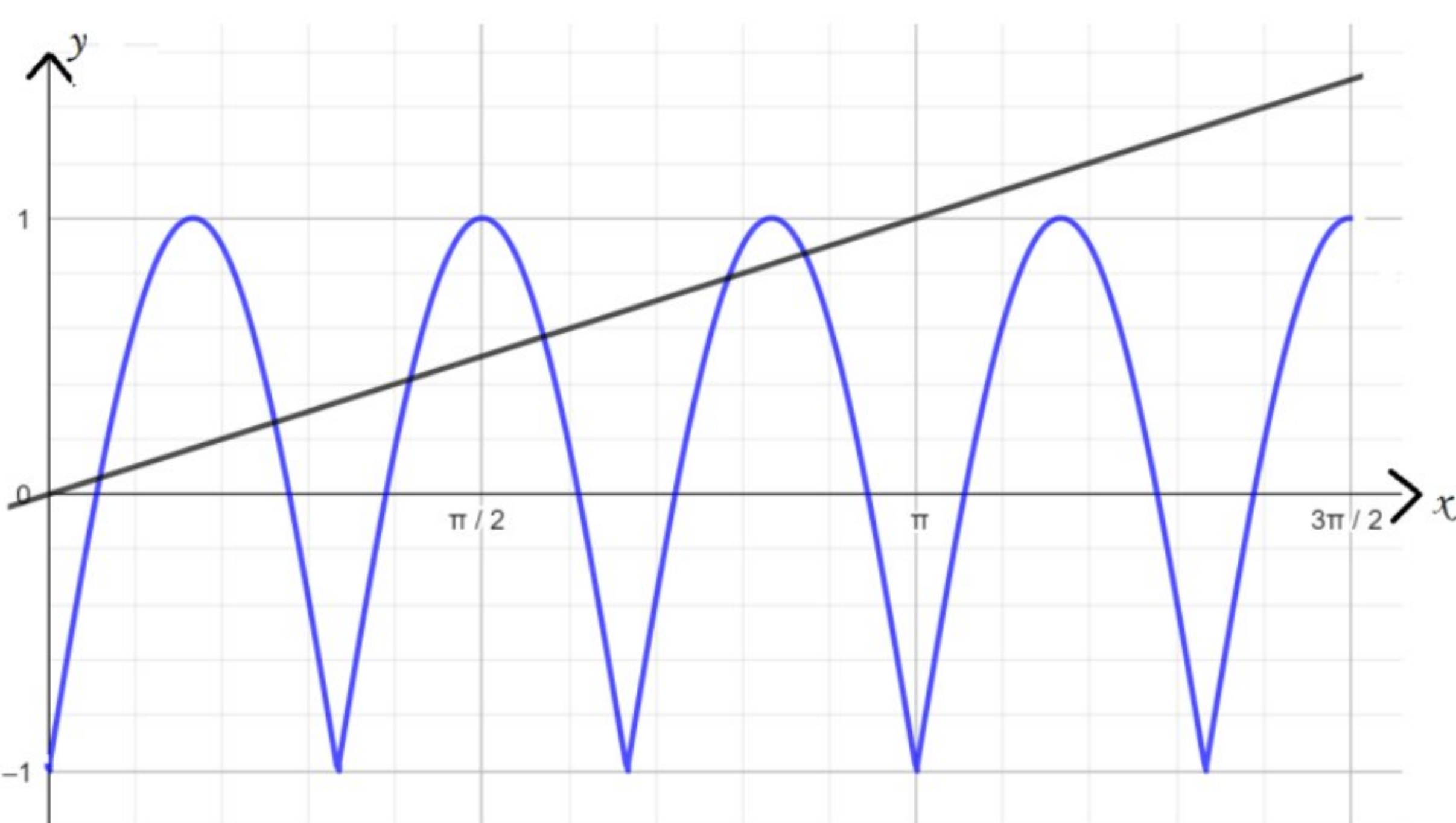




**MAJLIS PENGETUA SEKOLAH MALAYSIA (MPSM)
NEGERI PERAK**

**MODUL KECEMERLANGAN SPM 2023
SET 2**

**MATEMATIK TAMBAHAN
KERTAS 2
SKEMA JAWAPAN**

NO.	BUTIRAN	MARKAH	JUMLAH
BAHAGIAN A			
1	(a)		7
	(b)	$y = \frac{x}{\pi}$ <p>Draw straight line $y = \frac{x}{\pi}$</p> <p>Number of solutions = 6</p>	1 1 1 1
2	(a)	$p = -3$ $q = 8$ $5 = a[0 + (-3)]^2 + 8$ $a = -\frac{1}{3}$	1 1 1

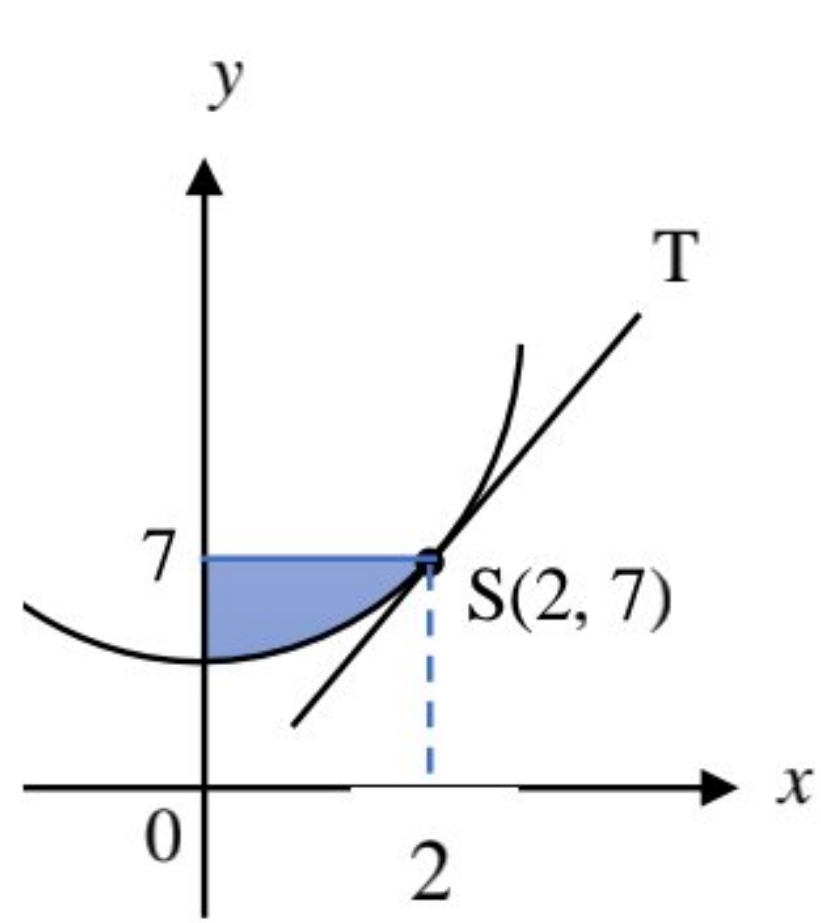
NO.	BUTIRAN	MARKAH	JUMLAH
	(b) $f(x) = -\frac{1}{3}x^2 + 2x + 5$ Selesaikan $-\frac{1}{3}x^2 + 2x + 5 = 0$ $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-15)}}{2(1)}$ 7.899 , -1.899	1 1 1, 1	
3	(a) <i>N ialah titik tengah bagi BC, maka</i> $\left(\frac{h+5}{2}, \frac{0+k}{2}\right) = (4, 3)$ $\frac{h+5}{2} = 4 \qquad \frac{0+k}{2} = 3$ $h+5 = 8 \qquad k = 6$ $h = 3$	1 1,1	
	(b) $\begin{aligned} \text{luas } \Delta CMN &= \frac{1}{2} \begin{vmatrix} 5 & 2 & 4 & 5 \\ 6 & 4 & 3 & 6 \end{vmatrix} \\ &= \frac{1}{2} (20 + 6 + 24) - (12 + 16 + 15) \\ &= \frac{1}{2} 50 - 43 \\ &= \frac{1}{2} (7) \\ &= 3\frac{1}{2} \text{ unit}^2 \end{aligned}$ <i>luas segi empat ABNM</i> $\begin{aligned} &= \frac{1}{2} \begin{vmatrix} -1 & 3 & 4 & 2 & -1 \\ 2 & 0 & 3 & 4 & 2 \end{vmatrix} \\ &= \frac{1}{2} (0 + 9 + 16 + 4) - (6 + 0 + 6 - 4) \\ &= \frac{1}{2} 29 - 8 \\ &= \frac{1}{2} (21) \\ &= 10\frac{1}{2} \text{ unit}^2 \end{aligned}$ <i>maka, nisbah luas ΔCMN kepada luas segi empat ABNM</i>	1 1 1	8

Selamat mengulangkaji dari telegram@soalanpercubaanspm

		$\Delta CMN : ABNM = 3\frac{1}{2} : 10\frac{1}{2}$ $= \frac{7}{2} : \frac{21}{2}$ $= 1 : 3$	1	
NO.	BUTIRAN		MARKAH	JUMLAH
4	(a)	$a = 90\,000, \quad r = 100\% + 4\%$ $= 104\%$ $= 1.04$ <p><i>pada tahun 2029, T_{11}</i></p> $T_{11} = 90\,000(1.04)^{10}$ $= RM133\,221.99$ <p>\therefore terbukti kurang daripada RM135 000</p>	1 1 1	
	(b)	$a = 90\,000, \quad r = 1.04$ $T_n > 280\,000$ $90\,000(1.04)^{n-1} > 280\,000$ $(1.04)^{n-1} > \frac{28}{9}$ $(n-1)\log_{10} 1.04 > \log_{10} \frac{28}{9}$ $(n-1) > \frac{0.4929}{0.017}$ $n > 28.94 + 1$ $n > 29.94$ $n = 30, 31, 32, \dots$ <p>\therefore tahun pertama ramalan keuntungan melebihi RM280 000 ialah pada tahun 2048.</p>	1 1 1 1	8
	(c)	$S_6 = \frac{90\,000[(1.04)^6 - 1]}{1.04 - 1}$ $= RM596\,967.79$	1	
5		$3p + 2q + 2r = -7 \dots\dots\dots (1)$ $5p - 4q - 3r = 6 \dots\dots\dots (2)$ $-2p - 3q - 4r = 5 \dots\dots\dots (3)$ $eq(1) \times 2: \quad 6p + 4q + 4r = -14 \dots\dots\dots (4)$ $eq(2) + eq(4): \quad 11p + r = -8 \dots\dots\dots (5)$ $eq(2) \times 3: \quad 15p - 12q - 9r = 18 \dots\dots\dots (6)$ $eq(3) \times 4: \quad -8p - 12q - 16r = 20 \dots\dots\dots (7)$ $eq(6) - eq(7): \quad 23p + 7r = -2 \dots\dots\dots (8)$ $eq(5) \times 7: \quad 77p + 7r = -56 \dots\dots\dots (9)$ $eq(9) - eq(8): \quad 54p = -54$ $p = -1$	1 1 1	5

			<p><i>gantikan</i> $p = -1$ ke dalam eq(5): $11(-1) + r = -8$ $r = 3$</p> <p><i>gantikan</i> $p = -1$ dan $r = 3$ ke dalam eq(1): $3(-1) + 2q + 2(3) = -7$ $-3 + 2q + 6 = -7$ $2q = -10$ $q = -5$</p> <p>$\therefore p = -1, q = -5$ dan $r = 3$</p>	1		
				1		
NO.	BUTIRAN		MARKAH	JUMLAH		
6	(a)	$\log_x 45 + 4 \log_x 2 - \frac{1}{2} \log_x 81 - \log_x 10$ $= \log_x 45 + \log_x 2^4 - \log_x \sqrt{81} - \log_x 10$ $= \log_x 45 + \log_x 16 - \log_x 9 - \log_x 10$ $= \log_x \left(\frac{45 \times 16}{9 \times 10} \right)$ $= \log_x 8$	1 1 1 1	7		
	(b)	$\log_x 45 + 4 \log_x 2 - \frac{1}{2} \log_x 81 - \log_x 10 = \frac{3}{2}$ $\log_x 8 = \frac{3}{2}$ $8 = x^{\frac{3}{2}}$ $x = 8^{\frac{2}{3}}$ $x = 4$	1 1 1			
7	(a)	i.	$PT = \frac{1}{5} PS$ $PS = 5PT$ $\vec{PS} = 5\vec{PT}$ $\vec{PS} = 5(6\underline{q})$ $= 30\underline{q}$ <p>dalam ΔPQS, $\vec{QS} = \vec{QP} + \vec{PS}$ $= -15\underline{p} + 30\underline{q}$ $= 30\underline{q} - 15\underline{p}$</p>	1 1	8	
		ii.	$TS = \frac{4}{5} PS$ $\vec{TS} = \frac{4}{5} \vec{PS}$ $\vec{TS} = \frac{4}{5} (30\underline{q})$	1		

			$= 24\underline{q}$ <p>dalam ΔRST,</p> $\overrightarrow{TR} = \overrightarrow{TS} + \overrightarrow{SR}$ $= 24\underline{q} + 15\underline{p} - 18\underline{q}$ $= 15\underline{p} + 6\underline{q}$	1	
	(b)		$\overrightarrow{TU} = \frac{2}{3}\overrightarrow{TR}$ $= \frac{2}{3}(15\underline{p} + 6\underline{q})$ $= 10\underline{p} + 4\underline{q}$ $\overrightarrow{US} = \overrightarrow{UT} + \overrightarrow{TS}$ $= -10\underline{p} - 4\underline{q} + 24\underline{q}$ $= 20\underline{q} - 10\underline{p}$ $= 10(2\underline{q} - \underline{p})$ $\overrightarrow{QS} = 30\underline{q} - 15\underline{p}$ $= \frac{3}{2}[10(2\underline{q} - \underline{p})]$ $\overrightarrow{QS} = \frac{3}{2}\overrightarrow{US}, \quad \text{maka } Q, U \text{ dan } S \text{ adalah segaris.}$	1	
NO.	BUTIRAN		MARKAH	JUMLAH	
BAHAGIAN B					
8	(a)	i	$p = 0.45, \quad q = 1 - 0.45 = 0.55, \quad n = 7$ $P(X \geq 3) = 1 - P(X < 3)$ $= 1 - P(X = 0) - P(X = 1) - P(X = 2)$ $= 1 - {}^7C_0(0.45)^0(0.55)^7 - {}^7C_1(0.45)^1(0.55)^6$ $\quad - {}^7C_2(0.45)^2(0.55)^5$ $= 1 - 0.0152 - 0.0872 - 0.2140$ $= 0.6836$	1	
		ii	$\sigma^2 = 866.25$ $npq = 866.25$ $n(0.45)(0.55) = 866.25$ $n = 3500$	1	10
	(b)		$\mu = 65.34 \text{ kg}, \quad \sigma^2 = 56.25 \text{ kg}$ $\sigma = 7.5 \text{ kg}$ $P(48 < X < 72) = P\left(\frac{48 - 65.34}{7.5} < z < \frac{72 - 65.34}{7.5}\right)$ $= P(2.312 < z < 0.888)$	1	

		$= 1 - P(z < -0.312) - P(z > 0.888)$ $= 1 - 0.0103 - 0.1872$ $= 0.8025$ $0.8025 \times n = 321$ $n = \frac{321}{0.8025}$ $= 400 \text{ orang dewasa}$	1	
			1	
			1	
NO.	BUTIRAN		MARKAH	JUMLAH
9	(a)	$y = \frac{1}{2}x^2 + 5$ $\frac{dy}{dx} = 2 \left(\frac{1}{2}\right) x^{2-1}$ $\frac{dy}{dx} = x$ <p>pada $P(2,7)$, $\frac{dy}{dx} = 2$</p> <p><i>persamaan tangen ST,</i></p> $y - 7 = 2(x - 2)$ $y = 2x - 4 + 7$ $y = 2x + 3$	1	
	(b)	<p><i>luas kawasan berlorek</i></p> $= (7 \times 2) - \int_0^2 \left(\frac{1}{2}x^2 + 5\right) dx$ $= 14 - \left[\frac{x^3}{6} + 5x\right]_0^2$ $= 14 - \left[\left(\frac{2^3}{6} + 5(2)\right) - 0\right]$ $= 14 - \frac{34}{3}$ $= 2\frac{2}{3} \text{ cm}^2$	1	10
			1	
	(c)	<p><i>pintasan - y = 5, maka $\int_5^7 x^2 dy$</i></p> $y = \frac{1}{2}x^2 + 5$ $x^2 = 2y - 10$ $V = \pi \int_5^7 x^2 dy$	1	

$$\begin{aligned}
 &= \pi \int_5^7 (2y - 10) dy \\
 &= \pi [y^2 - 10y]_5^7 \\
 &= \pi([(7)^2 - 10(7)] - [(5)^2 - 10(5)]) \\
 &= \pi[-21 - (-25)] \\
 &= 4\pi \text{ cm}^3
 \end{aligned}$$

1
1
1

NO.

BUTIRAN

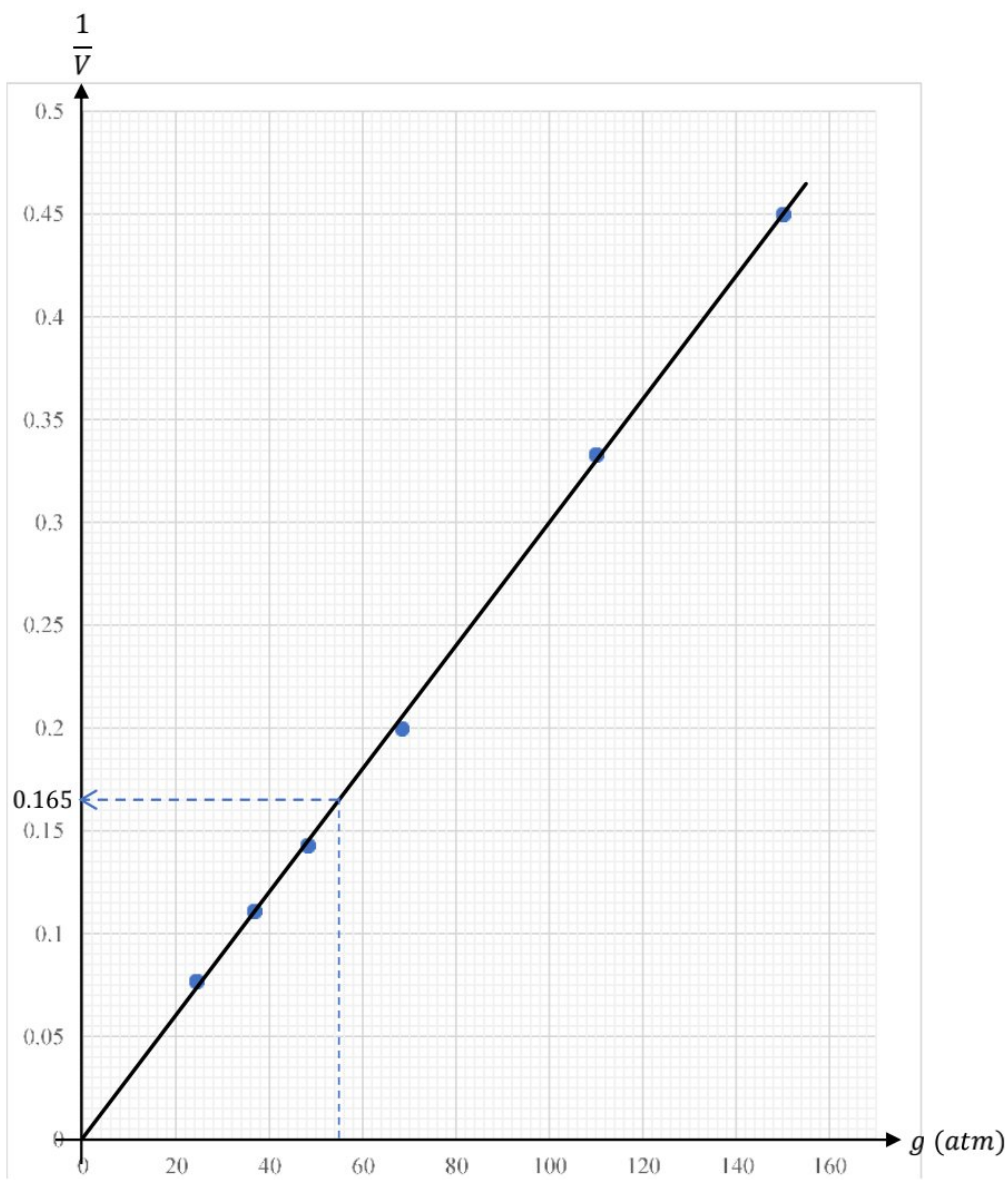
MARKAH

JUMLAH

10

(a)

g (atm)	150	110	68.4	48.2	36.7	24.3
1/V	0.450	0.333	0.200	0.143	0.111	0.077



1
1
1

10

(b)

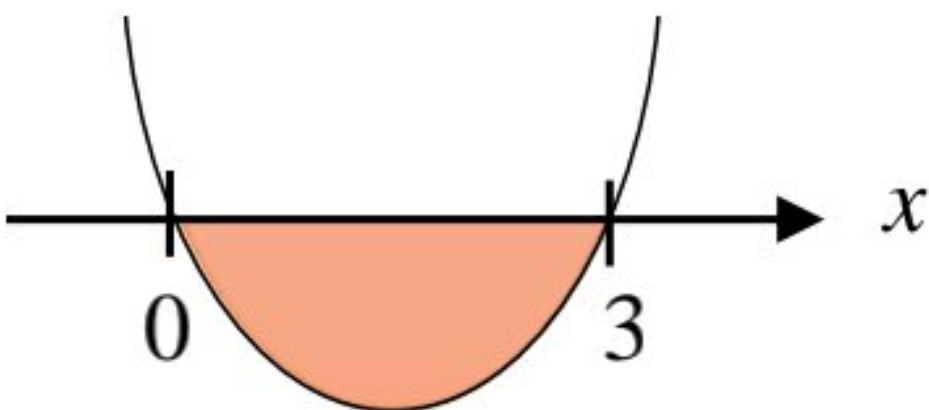
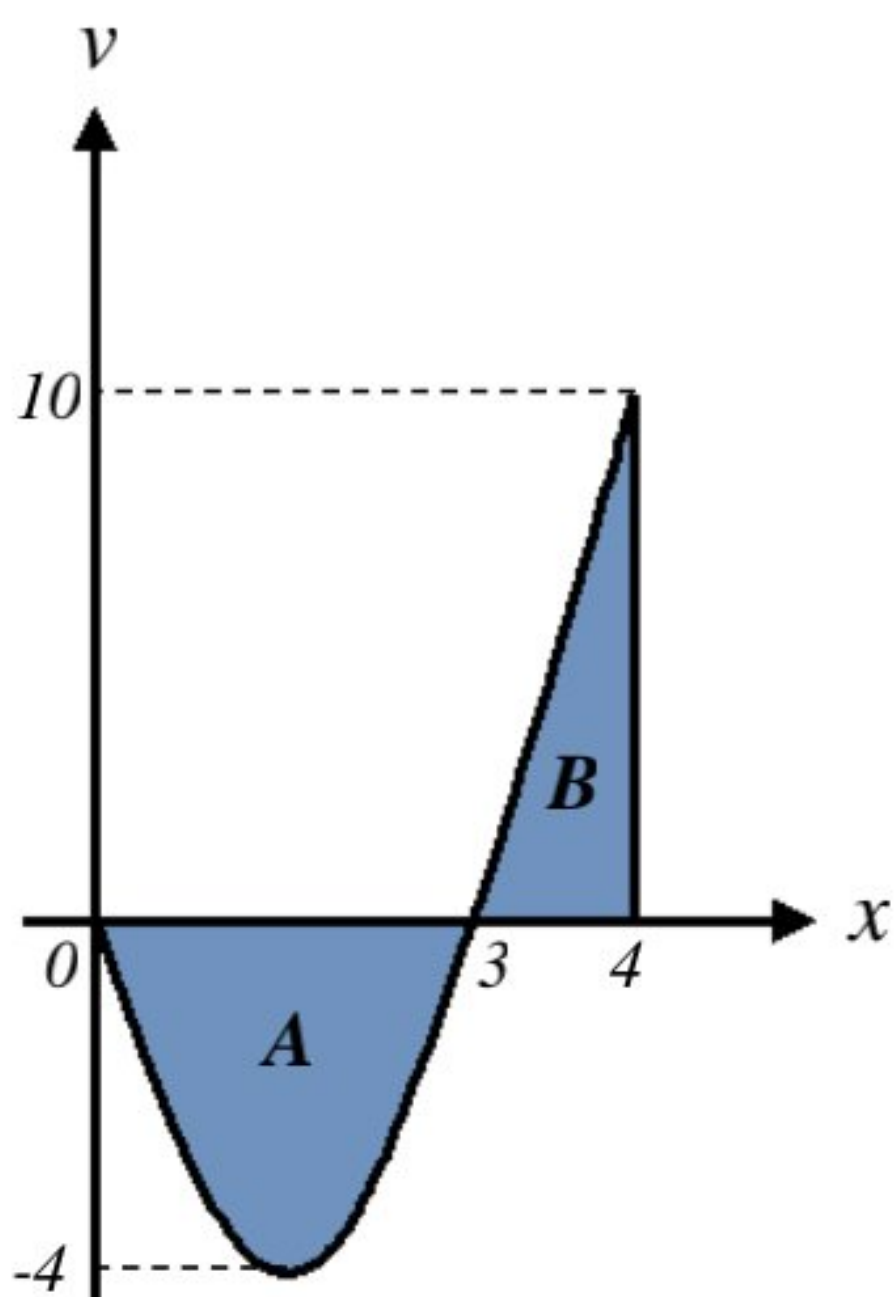
i

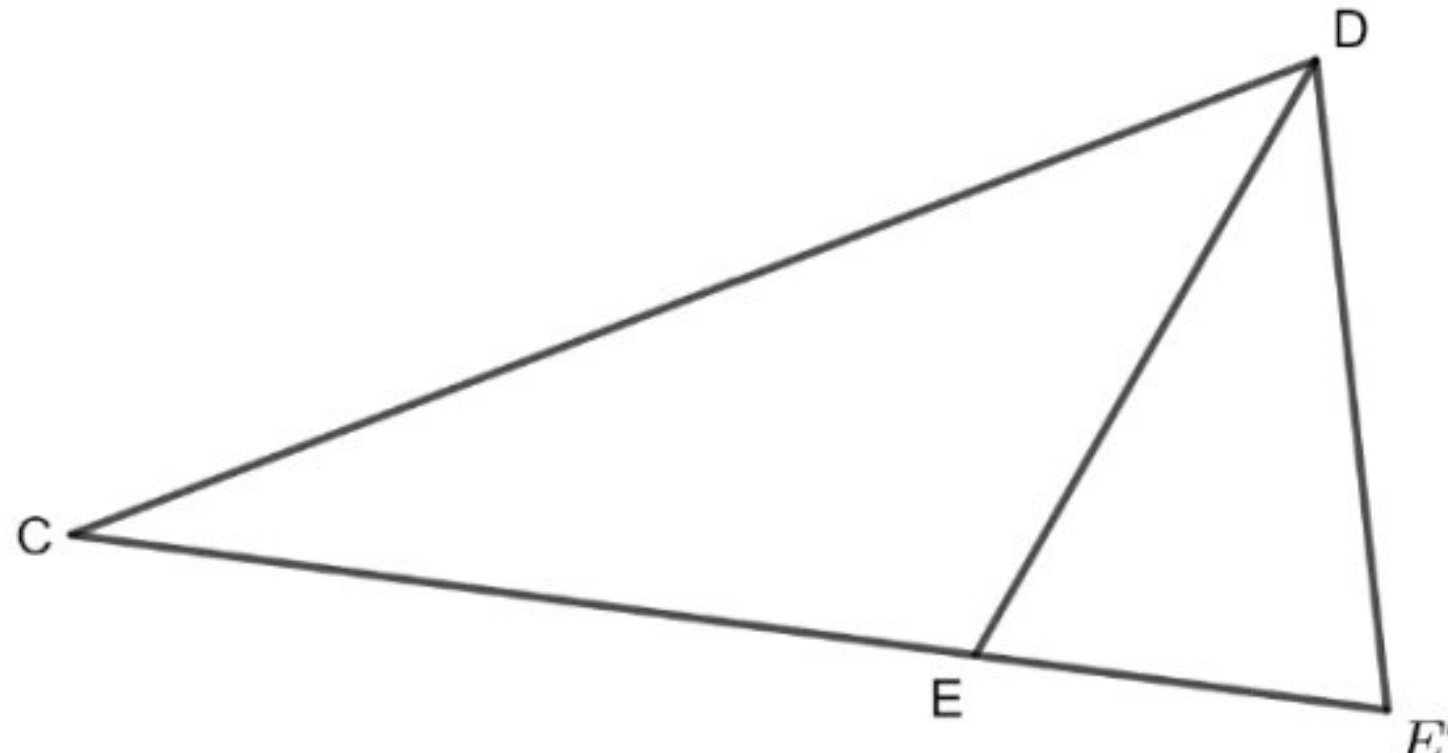
$$\begin{aligned}
 gV &= k \\
 \frac{1}{gV} &= \frac{1}{k} \\
 \frac{1}{V} &= \left(\frac{1}{k}\right)g
 \end{aligned}$$

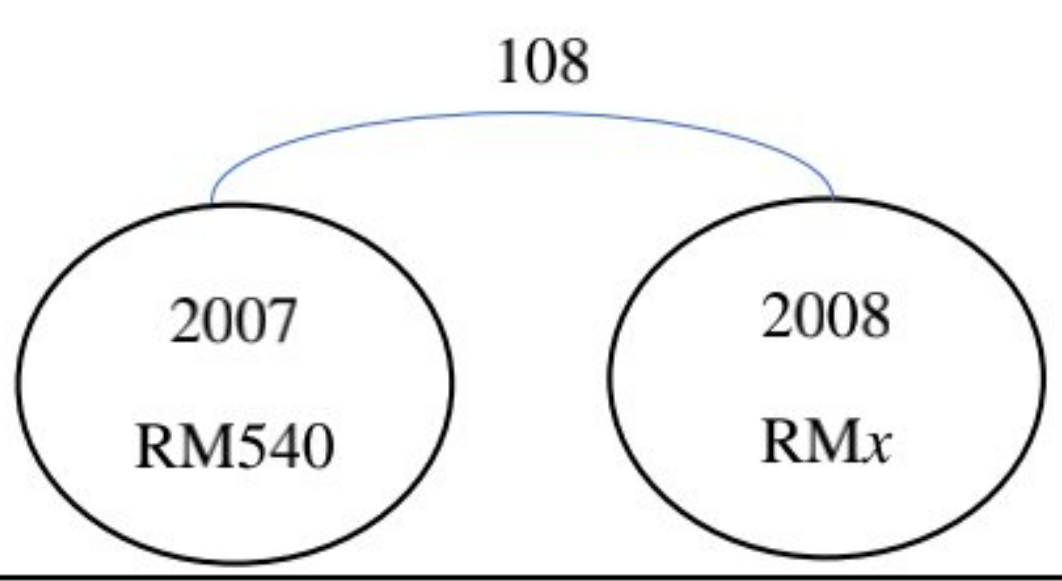
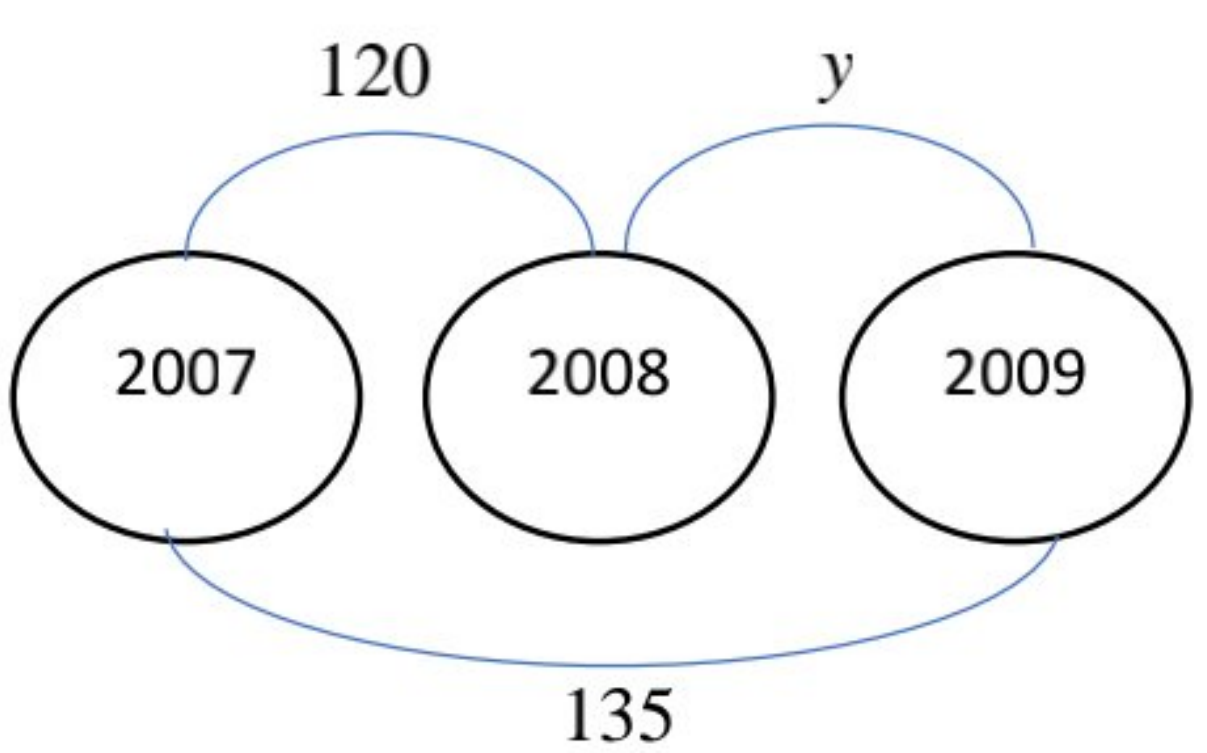
$$\begin{aligned}
 \text{kecerunan} &= \frac{1}{k} \\
 \frac{0.45 - 0.333}{150 - 110} &= \frac{1}{k} \\
 \frac{1}{k} &= 0.002925 \\
 k &= 341.88
 \end{aligned}$$

1
1
1

	ii	Daripada graf, apabila $g = 55 \text{ atm}$, $\frac{1}{V} = 0.165$, $V = 6.06 \ell$	1 1	
NO.	BUTIRAN		MARKAH	JUMLAH
11	(a)	$\cos \frac{\theta}{2} = \frac{7}{12}$ $\theta = 1.896 \text{ rad}$	1 1	10
	(b)	Perentas $CD = 2\sqrt{(12^2 - 7^2)}$ Lengkung $CD = 12(1.896)$ Perimeter segment $= 12(1.896) + 2\sqrt{(12^2 - 7^2)}$ $= 42.2456$	1 1 1 1	
	(c)	Luas $\triangle COD = \frac{1}{2}(12)^2 \sin(108.63^\circ)$ Luas sektor $OAB = \frac{1}{2}(7)^2 (1.896)$ Luas Kawasan berlerek $= \frac{1}{2}(12)^2 \sin(108.63^\circ) - \frac{1}{2}(7)^2 (1.896)$ $= 21.7753$	1 1 1 1	

NO.	BUTIRAN		MARKAH	JUMLAH
		BAHAGIAN C		
12	(a)	$v = pt^2 + qt$ $a = \frac{dv}{dt} = 2pt + q$ <p>diberi $v = -4$ apabila $t = 1$, $p(1)^2 + q(1) = -4$ $p + q = -4 \dots\dots\dots (1)$</p> <p>diberi $a = 2$ apabila $t = 2$ $2p(2) + q = 2$ $4p + q = 2 \dots\dots\dots (2)$</p> <p>(2) - (1): $3p = 6$ $p = 2$</p> <p>$2 + q = -4$ $q = -6$</p>	1 1 1	
	(b)	$v = 2t^2 - 6t$ <p>zarah bergerak ke kiri, $v < 0$ $2t^2 - 6t < 0$ $2t(t - 3) < 0$</p> <p>katakan $2t(t - 3) = 0$ $t = 0$ dan $t = 3$</p> <p>maka julat nilai t ialah $0 < t < 3$</p>	 1 1 1	10
	(c)	<p>Lakaran graf halaju-masa,</p>  $\text{luas A} = \int_0^3 2t^2 - 6t \, dt$ $= \left[\frac{2}{3}t^3 - 3t^2 \right]_0^3$ $= \left \left(\frac{2}{3}(3)^3 - 3(3)^2 \right) - 0 \right $ $= -9 $ $= 9$ $\text{luas B} = \int_3^4 2t^2 - 6t \, dt$ $= \left[\frac{2}{3}t^3 - 3t^2 \right]_3^4$ $= \left \left[\frac{2}{3}(4)^3 - 3(4)^2 \right] - (-9) \right $	1 1 1	

			$= \left -\frac{16}{3} + 6 \right $ $= 3\frac{2}{3}$ <p><i>jumlah jarak dilalui 4 saat pertama = A + B</i></p> $= 9 + 3\frac{2}{3}$ $= 12\frac{2}{3}m$	1	
				1	
13	(a)	i	$\cos A = \frac{8^2 + 12^2 - 6^2}{2(8)(12)}$ $\angle BAC = 26.38^\circ$	1	
		ii	$\frac{\sin \angle ACB}{8} = \frac{\sin 26.38^\circ}{6}$ $\angle ACB = 36.33^\circ$	1	
		iii	$\angle CED = 118.67^\circ$ $\frac{DE}{\sin 36.33^\circ} = \frac{15}{\sin 118.67^\circ}$ $DE = 10.128$	1	
	(b)	i		1	10
		ii	$\angle CDE' = 180^\circ - 36.33^\circ - 61.33^\circ$ $\text{Luas } \triangle C'D'E' = \frac{1}{2}(10.128)(15) \sin(82.34^\circ)$ $= 75.282$	1	
				1	

NO.	BUTIRAN		MARKAH	JUMLAH
14	(a)	$p = \frac{2.40}{3.20} \times 100 @ \frac{q}{3.00} \times 100 = 130 @ \frac{4.60}{r} \times 10 = 115$ $p = 75$ $q = 3.90$ $r = 4.00$	1 2	
	(b)	$\bar{I} = 180$ $\frac{120(2) + 75(4) + 130(3) + 115(s)}{2 + 4 + 3 + s} = 108$ $\frac{930 + 115s}{9 + s} = 108$ $930 + 115s = 972 + 108s$ $7s = 42$ $s = 6$	1 1 1	10
	(c)	 $\frac{x}{540} \times 100 = 108$ $x = RM583.20$	1 1	
	(d)	 $\frac{I_{2009/2008} \times 120}{100} = 135$ $I_{\frac{2009}{2008}} = \frac{135 \times 100}{120}$ $= 112.5$	1 1	
15	(a)	<p>Katakan x ialah bilangan sepasang anting-anting dan y ialah bilangan rantai.</p> $I: \frac{1}{2}x + y \leq 10$ $II: x + y \leq 15$	1 1	10

NO.	BUTIRAN	MARKAH	JUMLAH
(b)		<p>1 1 1 1</p>	
(c)	<p><i>fungsi objektif, $15x + 20y = k$</i></p> <p><i>daripada graf, titik optimum adalah pada (10, 5). gantikan $x = 10$ dan $y = 5$ ke dalam fungsi objektif.</i></p> <p><i>keuntungan maksimum = $15(10) + 20(5)$ = 250</i></p> <p><i>∴ Diona beroleh keuntungan maksimum sebanyak RM250 jika dia menjual 10 pasang anting – anting dan 5 utas rantai</i></p>	<p>1 1 1 1</p>	