MODUL TOPIKAL
SOALAN PERCUBAAN SPM 2023

TOPIK TINGKATAN 5

BAB 3

PENGAMIRAN
(INTEGRATION)

SUMBER SOALAN:

SOALAN – SOALAN PERCUBAAN

TERENGGANU
NEGERI SEMBILAN
KELANTAN
SABAH
SBP
MELAKA
SELANGOR SET 1
SELANGOR SET 2
PERAK

SKEMA JAWAPAN

DISUSUN OLEH:

PN. NOORUL HUDA BINTI MOHD HASHIM
(SMK TAMAN TASIK, TAIPING)

PN ZAINAB BINTI ABD RAHMAN
(SMK CONVENT, TAIPING)

SOALAN 1 : SOALAN PERCUBAAN SPM NEGERI TERENGGANU 2023 (KERTAS 1)

12	(*a*) Bezakan $9-x^2$ terhadap x dan samakan dengan -2	K1	**9**
	$P\,(1,8)$	N1	
	(*b*) mencari pintasan -x ; 3 dan 5	K1	
	$\left(\frac{1}{2}\times 8\times 4\right)$ @ $\int_1^3 \left(9-x^2\right)dx$	K1	
	$\left(\frac{1}{2}\times 8\times 4\right)-\int_1^3 \left(9-x^2\right)dx$	K1	
	$\frac{20}{3}$	N1	
	(c) $9y-\frac{y^2}{2}$	K1	
	$\pi\int_k^9 (9-y)\,dy = 16\pi$ dan selesaikan untuk mencari nilai k	K1	
	$k = 3.343$	N1	

SOALAN 2 : SOALAN PERCUBAAN SPM NEGERI SEMBILAN 2023 (KERTAS 1)

6	(a)	$2n$ atau $\left[\frac{x^2}{2}\right]_1^m$	K1
		$2n-\left[\frac{m^2}{2}-\frac{1}{2}\right]=\frac{37}{2}$	K1
		$m=2\sqrt{n-9}$	N1
	(b)(i)	$4(16)$ atau $2(2)$	K1
		$4(16)-2(2)=60$	N1
	(ii)	$\frac{4x^2}{2}+5x+c$	K1
		$2=2(-2)^2+5(-2)+c$ dan selesaikan c	K1
		$f(x)=2x^2+5x+4$	N1
			8 markah

SOALAN 3 : SOALAN PERCUBAAN SPM NEGERI SEMBILAN 2023 (KERTAS 2)

10(a)	$h=-16$	N1
10(b)	$\left[-\frac{16x^{-1}}{-1}\right]$ atau $\left[-\frac{x^3}{3}+\frac{4x^2}{2}\right]$ (untuk bahagian seterusnya)	K1
	$\left(\frac{16}{m}\right)-\left(\frac{16}{2}\right)=-\frac{8}{3}$ atau $\frac{1}{2}(2)(4)$ (m atau sebarang huruf mewakili pemalar)	K1
	$m=3$	N1
	$\left(-\frac{(*3)^3}{3}+\frac{4(*3)^2}{2}\right)-\left(-\frac{(0)^3}{3}+\frac{4(0)^2}{2}\right)$	K1
	$9+\frac{20}{3}$	K1
	$\frac{47}{3}$ // 15.67	N1
10(c)	$\pi\left[\frac{x^5}{5}-\frac{8x^4}{4}+\frac{16x^3}{3}\right]_0^{*3}$	K1
	$\pi\left[\left(\frac{(*3)^5}{5}-\frac{8(*3)^4}{4}+\frac{16(*3)^3}{3}\right)-\left(\frac{(0)^5}{5}-\frac{8(0)^4}{4}+\frac{16(0)^3}{3}\right)\right]$	K1
	$\frac{153}{5}\pi$ // 30.6π	N1
		10 markah

SOALAN 4 : SOALAN PERCUBAAN SPM NEGERI KELANTAN 2023 (KERTAS 1)

8	$16a+12=64a$ $a=\frac{1}{4}\ and\ p=4\ (Both)$ $\pi\int_4^{16}4y\ dy=\pi\left[2y^2\right]_4^{16}$ $\pi\left[2(16)^2-2(4)^2\right]$ 480π	K1 N1 K1 K1 N1	5

SOALAN 5 : SOALAN PERCUBAAN SPM NEGERI KELANTAN 2023 (KERTAS 2)

11(a)	$Q(4,0)$ $\sin 60^0=\frac{h}{4}$ atau $kos60^0=\frac{b}{4}$ atau $h=2\sqrt{3}$ atau $b=2$ $V_1(kon)=\frac{1}{3}\pi\left(2\sqrt{3}\right)^2(2)$ atau $V_2=\pi\left[16x-\frac{x^3}{3}\right]_2^4$ $\pi\left[\left(16(4)-\frac{4^3}{3}\right)-\left(16(2)-\frac{2^3}{3}\right)\right]$ $V_1+V_2=\frac{1}{3}\pi\left(2\sqrt{3}\right)^2(2)+\pi\left[\left(16(4)-\frac{4^3}{3}\right)-\left(16(2)-\frac{2^3}{3}\right)\right]$ $\frac{64\pi}{3}$	P1 K1 K1 K1 K1 N1	
11(b)	$-\frac{1}{5}\times m_T=-1$ dan $m_T=5$ $\frac{dy}{dx}=4x-3=5$ $y=2(2)^2-3(2)+2$ atau $A(2,4)$ $y=5x-6$	K1 K1 K1 N1	10

SOALAN 6: SOALAN PERCUBAAN SPM NEGERI SABAH 2023 (KERTAS 1)

4	a) $m_1 = -\frac{1}{8}$ atau $m_2 = 8$ atau setara Guna $m_1 \times m_2 = -1$ $\left((4(2)^3 - p(2))\left(-\frac{1}{8}\right) = -1\right)$ atau $8 = 4x^3 - px$ $p = 12$	K1 K1 N1	6
	b) Kamirkan y terhadap x $y = \frac{4x^4}{4} - \frac{12x^2}{2} + c$ Ganti (2, 5) ke dalam *kamiran **&** Selesaikan untuk c $5 = (2)^4 - 6(2)^2 + c$ $c = 13$ $y = x^4 - 6x^2 + 13$	K1 K1 N1	

SOALAN 7 : SOALAN PERCUBAAN SPM NEGERI SABAH 2023 (KERTAS 2)

10	a) i) Gantikan $x = 2$ ke dalam $\frac{dy}{dx}$ **&** Samakan $\frac{dy}{dx}$ dengan 0 ——————— $p = -1$ Kamirkan $\int(-x + 2)\,dx$ **&** Gantikan $x = 2$ dan $y = 8$ ke dalam kamiran ——————— $8 = -\frac{(2)^2}{2} + 2(2) + c$ $c = 6$ $y = -\frac{x^2}{2} + 2x + 6$ ii) Kamirkan $\int\left(-\frac{x^2}{2} + 2x + 6\right)dx$ ——————— $2\int_0^2\left(-\frac{x^2}{2} + 2x + 6\right)dx$ $2\left[\left(-\frac{(2)^3}{6} + (2)^2 + 6(2)\right) - \left(-\frac{(0)^3}{6} + (0)^2 + 6(0)\right)\right]$ $\frac{88}{3} \times 30$ 880 Atau Kamirkan $\int\left(-\frac{x^2}{2} + 2x + 6\right)dx$ ——————— $\int_0^4\left(-\frac{x^2}{2} + 2x + 6\right)dx$ $\left(-\frac{(4)^3}{6} + (4)^2 + 6(4)\right) - \left(-\frac{(0)^3}{6} + (0)^2 + 6(0)\right)$ $\frac{88}{3} \times 30$ 880	K1 K1 N1 K1 K1 N1 K1 K1 N1	10

	b) Kamirkan $\pi\int(\sqrt{x+6})^2 dx$ @ cari isipadu kon $\left(V_1 = \pi\left[\frac{x^2}{2}+6x\right] \ @ \ V_2 = \frac{1}{3}\pi(3)^2(k-3)\right)$ Guna had $\int_3^k$ ke dalam V_1 $\left(V_1 = \pi\left[\left(\frac{(k)^2}{2}+6(k)\right)-\left(\frac{(3)^2}{2}+6(3)\right)\right]\right)$ Samakan $*V_1 - *V_2$ ($V_1 > V_2$) dengan $42\frac{1}{2}\pi$ $\pi\left[\left(\frac{(k)^2}{2}+6(k)\right)-\left(\frac{(3)^2}{2}+6(3)\right)\right]-\frac{1}{3}\pi(3)^2(k-3)=42\frac{1}{2}\pi$ $k=8$	K1 K1 K1 N1	

SOALAN 8 : SOALAN PERCUBAAN SPM SBP 2023 (KERTAS 1)

10(a)(i)	$-1\frac{5}{12}$ **N1**	5	
(ii)	Guna $\int_a^b f(x)dx - \int_a^b g(x)dx$ $\int_0^1 3f(x)dx - \int_0^1 kdx = 12$ **K1** Kamirkan $\int_0^1 kdx$ $3\int_0^1 f(x)dx - [kx]_0^1 = 12$ **K1** Guna had $\int_0^1$ ke dalam hasil kamiran $3\left(1\frac{5}{12}\right) - [k(1)-k(0)] = 12$ **K1** $k = -\frac{31}{4}$ **N1**		
(b)	$y = x^3 - 4x^2 + 5x$ **N1**	1	6

SOALAN 9 : SOALAN PERCUBAAN SPM SBP 2023 (KERTAS 2)

7(a) — 4

Kaedah 1

Kamir dengan had yang betul

$\left[\frac{(y-4)^3}{3(1)}\right]_4^6$ **K1**

Guna had yang betul @ Cari luas segi tiga

$\frac{1}{3}\left[(6-4)^3-(4-4)^3\right]$ @ $\frac{1}{2}(22-6)(4)$ **K1**

$\left(\frac{1}{3}\left[(6-4)^3-(4-4)^3\right]\right)+\left(\frac{1}{2}(22-6)(4)\right)$ **K1**

$34\frac{2}{3}$ unit^2 **N1**

Kaedah 2

Kamir dengan had yang betul

$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+4x\right]_0^4$ **K1**

Guna had yang betul @ Cari luas trapezium

$\left[\frac{(4)^{\frac{3}{2}}}{\frac{3}{2}}+4(4)\right]-0$ @ $\frac{1}{2}(4)(6+22)$ **K1**

$\left(\left[\frac{(4)^{\frac{3}{2}}}{\frac{3}{2}}+4(4)\right]-0\right)-\left(\frac{1}{2}(4)(6+22)\right)$ **K1**

$34\frac{2}{3}$ unit^2 **N1**

(b) — 3

Kamirkan $\int \pi\left((y-4)^2\right)^2 dy$

$V=\pi\left(\frac{(y-4)^5}{5(1)}\right)$ **K1**

Guna had $\int_4^6$ ke dalam $*V$

$V=\pi\left[\frac{(6-4)^5}{5}-\frac{(4-4)^5}{5}\right]$ **K1**

$\frac{32}{5}\pi$ **N1**

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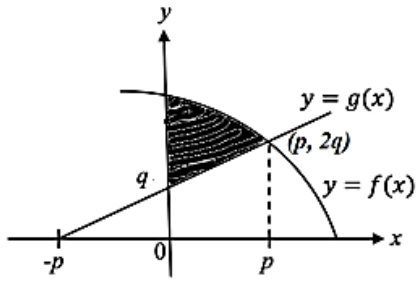
SOALAN 10 : SOALAN PERCUBAAN SPM NEGERI MELAKA 2023 (KERTAS 1)

7.	(a)	$2(7)$ atau $3\left[\frac{x^2}{2}\right]_{-3}^{5}$	1	
		$2(7)-3\left[\frac{5^2}{2}-\frac{(-3)^2}{2}\right]$	1	
		-10	1	
	(b) (i)		1	
	(ii)	$10+\frac{1}{2}p[q+2q]$	1	
		$10+\frac{3pq}{2}$	1	6

SOALAN 11 : SOALAN PERCUBAAN SPM NEGERI MELAKA 2023 (KERTAS 2)

10 (a)	$m=\frac{dy}{dx}=2x$ $m=2(1)=2$	1	
	$y-4=2(x-1)$	1	
	$y=2x+2$	1	
(b)	Luas lengkung, $A_1=\int_0^1(x^2+3)\,dx=\left[\frac{x^3}{3}+3x\right]_0^1$	1	
	Luas trapezium, $A_2=\frac{1}{2}(2+4)(1)$	1	**10**
	Luas = $A_1 - A_1$ atau setara $=\left[\frac{x^3}{3}+3x\right]_0^1-3$	1	
	$=\frac{1}{3}$	1	
(c)	$=\frac{1}{2}\pi\left[\frac{y^2}{2}-3y\right]_3^5$	1	
	$=\frac{1}{2}\pi\left[\frac{5^2}{2}-3(5)\right]-\frac{1}{2}\pi\left[\frac{3^2}{2}-3(3)\right]$	1	
	$=\pi$	1	

SOALAN 12: SOALAN PERCUBAAN SPM NEGERI SELANGOR SET 1 2023 (KERTAS 1)

					4
9	(a)		Bezakan menggunakan petua hasil bahagi $\frac{dy}{dx} = \frac{3(4x-1)-4(3x+2)}{(4x-1)^2}$ atau $p = 3(4x-1) - 4(3x+2)$ -11	K1 N1	
	(b)	(i)	$-\frac{k}{5}$	N1	
		(ii)	$k - \left[\frac{kx^2}{2}\right]_1^3 = 3$ -1	K1 N1	
	(c)		$\left[\frac{6k^3}{3}\right]_0^3 = 54$	K1	
			$\frac{6(k)^3}{3} - 0 = 54$ $k = 3$	K1 N1	
					8

SOALAN 13 : SOALAN PERCUBAAN SPM NEGERI SELANGOR SET 1 2023 (KERTAS 2)

11	(a)		(4,0) Kamirkan $\int x^2 + 2 \ @ \int 4 - x$ $A_1 = \frac{x^3}{3} + 2x \qquad A_2 = 4x - \frac{x^2}{2}$ Guna had $\int_0^{*1}$ ke ${}^*A_1$ $A_2 = \frac{1}{2}(3)(3) \ @ \int_1^4 4x - \frac{x^2}{2}$ ${}^*A_1 + {}^*A_2$ $\frac{41}{6}$	P1 K1 K1 K1 K1 N1	
	(b)		$B(0,2)$ Kamirkan x^2 terhadap y	P1 K1	
			Guna had $\int_{*2}^3$ ke dalam hasil kamiran $\pi * \left[\frac{y^2}{2} - 2y\right]$ $\frac{1}{2}\pi$	K1 N1	
					10

SOALAN 14 : SOALAN PERCUBAAN SPM NEGERI SELANGOR SET 2 2023 (KERTAS 1)

10	(a)		$8 = a(5^2)$ $a = \frac{8}{25}$	K1 N1
	(b)		Alternatif A $\int_0^8 \frac{5\sqrt{y}}{\sqrt{8}}dy = \left[\frac{5y^{\frac{3}{2}}}{\sqrt{8}\left(\frac{3}{2}\right)}\right]_0^8$ $\frac{1}{2}(12+8)(8) - 2\times\int_0^8 \frac{5\sqrt{y}}{\sqrt{8}}dy$ $= 80 - 2\times\left[\frac{5(8)^{\frac{3}{2}}}{\sqrt{8}\left(\frac{3}{2}\right)} - \frac{5(0)^{\frac{3}{2}}}{\sqrt{8}\left(\frac{3}{2}\right)}\right]$ $= 26\frac{2}{3}$ Alternatif B $\int_0^5 \frac{8x^2}{25}dx = \left[\frac{8x^3}{25(3)}\right]_0^5$ $2\times\left[\int_0^5 \frac{8x^2}{25}dx - \frac{1}{2}(1)(4) + \frac{1}{2}(1)(4)\right]$ $= 2\times\left[\frac{8(5)^3}{25(3)} - \frac{8(0)^3}{25(3)}\right]$ $= 26\frac{2}{3}$	K1 K1 K1 N1 K1 K1 K1 N1
	(c)		$V = \pi\int_0^8 \frac{25y}{8}dy$ $= \pi\left[\frac{25}{8}\left(\frac{y^2}{2}\right)\right]_0^8$ $= \pi\left[\frac{25}{8}\left(\frac{8^2}{2}\right) - \frac{25}{8}\left(\frac{0^2}{2}\right)\right]$ $= 100\pi$	K1 N1
				8

SOALAN 15 : SOALAN PERCUBAAN SPM NEGERI SELANGOR SET 2 2023 (KERTAS 2)

2	(a)	$\int_3^1 2f(y)\ dy = (-2) \times (-5)$ 10	P1 N1
	(b)	Kamirkan $\int px^2 - 2x\ dx$ $y = \frac{px^3}{3} - x^2 + c$ Ganti (1,6) @ $(-2,-15)$ dalam $*y = \frac{px^3}{3} - x^2 + c$ $6 = \frac{p(1)^3}{3} - (1)^2 + c$ @ $-15 = \frac{p(-2)^3}{3} - (-2)^2 + c$ Selesaikan untuk p @ c $p = 6$ @ $c = 5$ $y = 2x^3 - x^2 + 5$	K1 K1 K1 K1 N1

SOALAN 16 : SOALAN PERCUBAAN SPM NEGERI PERAK 2023 (KERTAS 1)

5	(a)	(i)	$\int \frac{m}{x^n} dx$	y	y	1 1	6
			$\int \frac{m}{x^2} dx$	$-\frac{m}{x} + c$	$-\frac{m}{(2-1)x^{2-1}} + c$		
			$\int \frac{m}{x^3} dx$	$-\frac{m}{2x^2} + c$	$-\frac{m}{(3-1)x^{3-1}} + c$		
			$\int \frac{m}{x^4} dx$	$-\frac{m}{3x^3} + c$	$-\frac{m}{(4-1)x^{4-1}} + c$		
			$\int \frac{m}{x^5} dx$	$-\frac{m}{4x^4} + c$	$-\frac{m}{(5-1)x^{5-1}} + c$		
	(a)	(ii)	Jika $y = \frac{m}{x^n}$, maka $\int \frac{m}{x^n} dx = -\frac{m}{(n-1)x^{n-1}} + c$			1	
	(b)		$\int \frac{2}{x^2}\ dx = -\frac{2}{(2-1)x^{2-1}} + c$ $\int \frac{2}{x^2}\ dx = -\frac{2}{x} + c$			1,1 1	

SOALAN 17 : SOALAN PERCUBAAN SPM NEGERI PERAK 2023 (KERTAS 1)

12	Use the formula V = $\pi\int y^2 dx$		
	V = $\int_0^h (\frac{R^2x^2}{h^2})dx$		
	V = $[\frac{R^2x^3}{3h^2}]_0^h$	**1**	
	V = $[\frac{R^2(h)^3}{3h^2} - \frac{R^2(0)^3}{3h^2}]$	**1**	**5**
	V = $\frac{1}{3}\pi R^2 h$ Sub h = R and Volume = 9 π		
	$\frac{1}{3}\pi R^2(R) = 9\pi$	**1**	
	$\frac{1}{3}R^3 = 9$ $R^3 = 27$		
	$R = \sqrt[3]{27}$	**1**	
	R = 3	**1**	

SOALAN 18 : SOALAN PERCUBAAN SPM NEGERI PERAK 2023 (KERTAS 2)

2			$P = \int \frac{dP}{dV} dV$		
			$P = \frac{3V^3}{3} - \frac{12V^6}{6} + c$	1	
			$P = V^3 - 2V^6 + c$	1	
			Sub the values of V = 1 and P = 8	1	6
			$(8) = (1)^3 - 2(1)^6 + c$		
			$P = V^3 - 2V^6 + 9$	1	
			Sub the value of V = 0.2	1	
			$P = (0.2)^3 - 2(0.2)^6 + 9$ $P = 9.008$	1	