

TINGKATAN 4

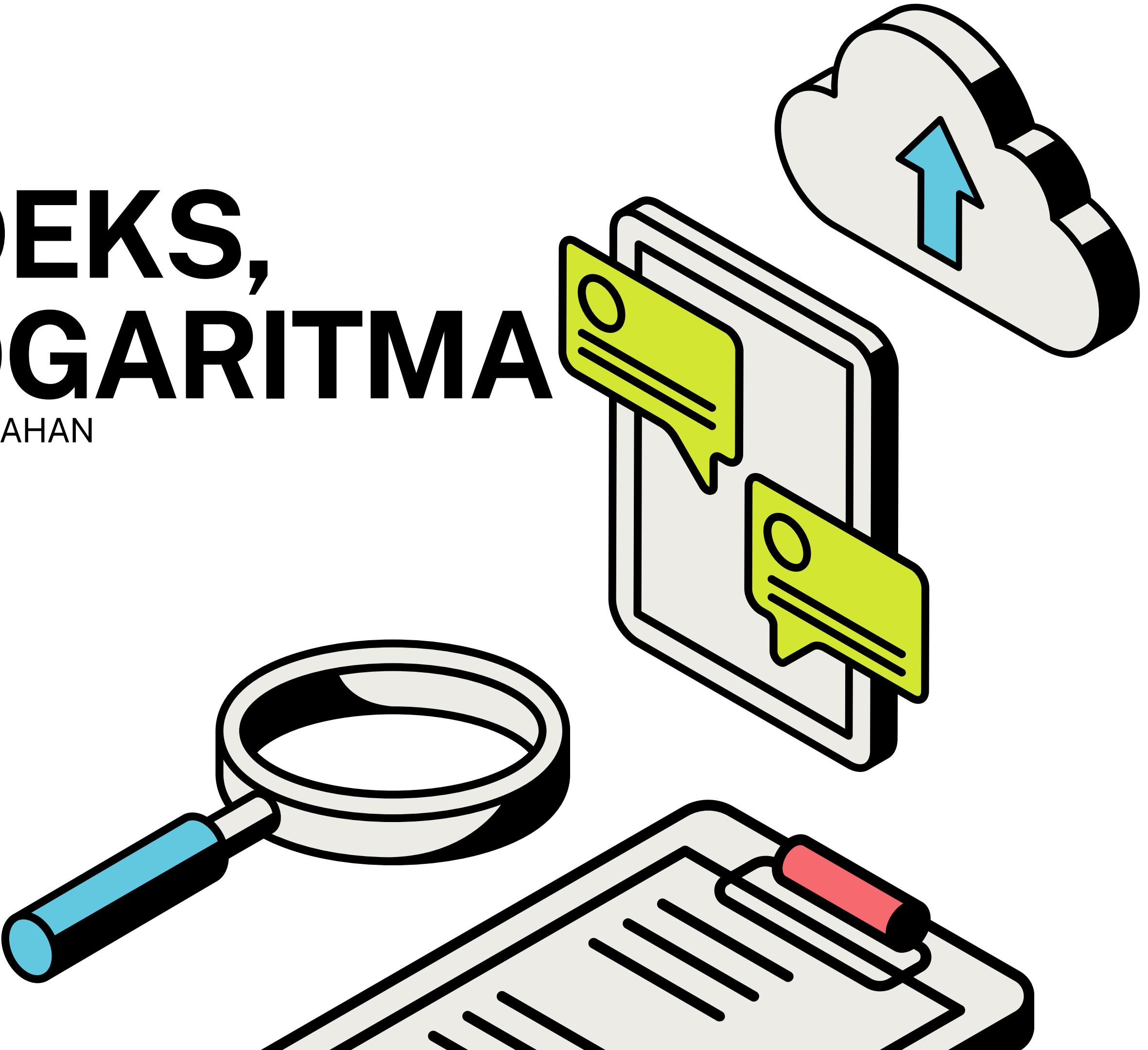
BAB 4: INDEKS, SURD & LOGARITMA

KOMPILASI SOALAN MATEMATIK TAMBAHAN
PERCUBAAN SPM 2023

SKEMA PEMARKAHAN

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KELANTAN (K1)

INDEKS, SURD & LOGARITMA

5. (a) Tunjukkan bahawa $27^{y+1} + 3^{3y}$ boleh ditulis dalam bentuk $a(b^y)$, dengan a dan b adalah pemalar. Seterusnya, cari nilai a dan nilai b . [2 markah]

Show that $27^{y+1} + 3^{3y}$ can be written in the form $a(b^y)$, where a and b are constants. Hence, find the values of a and of b . [2 marks]

- (b) Selesaikan persamaan yang berikut

Solve the following equation

$$\log_p(5+x^2) = \log_{\sqrt{p}}(3-x)$$

[3 markah]

[3 marks]

6. (a) Tukarkan $0.168168168\dots$ kepada pecahan tanpa menggunakan kaedah janjang. [2 markah]

Convert $0.168168168\dots$ to a fraction without using progression method. [2 marks]

- (b) Penyelesaian bagi persamaan $\sqrt{12}x - \sqrt{5}x = \sqrt{6}$ adalah $\frac{p\sqrt{2} + \sqrt{q}}{7}$, dengan keadaan

p dan q adalah pemalar. Tanpa menggunakan kalkulator, cari nilai bagi p dan q . [3 markah]

The solution of the equation $\sqrt{12}x - \sqrt{5}x = \sqrt{6}$ is $\frac{p\sqrt{2} + \sqrt{q}}{7}$, where p and q are

constants. Without using a calculator, find the values of integer p and q . [3 marks]

5 (a)	$27^{y+1} + 27^y$ atau $3^{3y+3} + 3^{3y}$ atau $(27+1)27^y$ $a = 28$ dan $b = 27$	K1 N1
5 (b)	$\frac{\log_p(3-x)}{\log_p \sqrt{p}}$ $5+x^2 = (3-x)^2$ $x = \frac{2}{3}$	P1 K1 N1

6 (a)	$1000x - x = 168$ $x = \frac{56}{333}$	K1 N1
6 (b)	$\frac{\sqrt{6}}{\sqrt{12} - \sqrt{5}} \times \frac{\sqrt{12} + \sqrt{5}}{\sqrt{12} + \sqrt{5}}$ $\frac{\sqrt{6 \times 12} + \sqrt{6 \times 5}}{\sqrt{12^2} - \sqrt{12 \times 5} + \sqrt{12 \times 5} - \sqrt{5^2}}$ $p = 6$ dan $q = 30$	K1 K1 N1

MELAKA (K1)**INDEKS, SURD & LOGARITMA**

5. (a) Selesaikan persamaan :
Solve the equation:

$$16^{2x}(64) = 4^x$$

[2 markah]
[2 marks]

- (b) Diberi bahawa $\log_3 p + \log_9 k = 3$, bentukkan satu persamaan dalam sebutan p dan k.

Given that $\log_3 p + \log_9 k = 3$, form an equation in terms of p and k.

[3 markah]
[3 marks]

5	(a)	$4^{4x+3} = 4^x$ dan $4x + 3 = x$ atau $2^{4(2x)+6} = 2^{2x}$ dan $4(2x) + 6 = 2x$ $x = -1$	1	
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(b)	$\frac{\log_3 k}{\log_3 9}$ $\log_3 p^2 k = 6$ $p^2 k = 729$ atau setara	1	1	5
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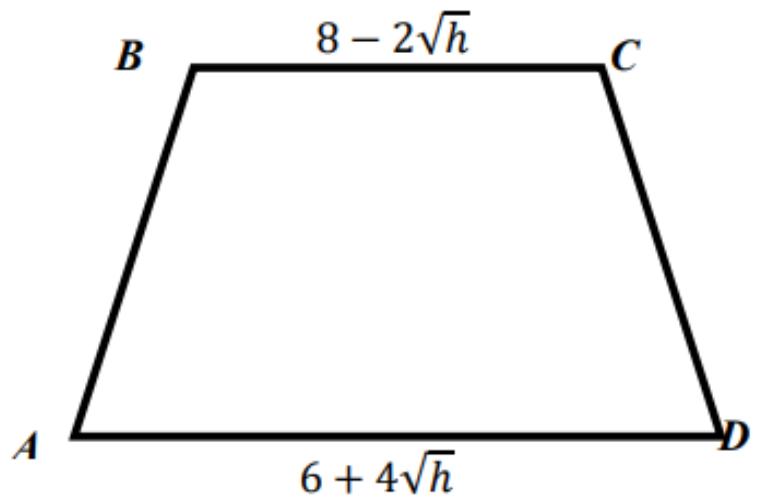
MELAKA (K1)

INDEKS, SURD & LOGARITMA

10. (a) Permudahkan $(3 + 2\sqrt{h})(14 + 2\sqrt{h})$.
Simplify $(3 + 2\sqrt{h})(14 + 2\sqrt{h})$.

[2 markah]
[2 marks]

- (b) Rajah 7 menunjukkan sebuah trapezium ABCD
Diagram 7 shows a trapezium ABCD.



Rajah 7
Diagram 7

Diberi panjang AD dan BC ialah $6 + 4\sqrt{h}$ dan $8 - 2\sqrt{h}$ masing-masing.
Tinggi trapezium itu ialah separuh daripada panjang AD dan luas trapezium ABCD ialah $27 + 17\sqrt{h}$. Cari nilai h .

[3 markah]

*Given the length of AD and BC are $6 + 4\sqrt{h}$ and $8 - 2\sqrt{h}$ respectively.
The height of the trapezium is half of the length of AD and the area of trapezium ABCD is $27 + 17\sqrt{h}$. Find the value of h.*

[3 marks]

10. (a)	$42 + 6\sqrt{h} + 28\sqrt{h} + 4h$ $42 + 34\sqrt{h} + 4h$	1 1
(b)	$\frac{1}{2}[(8 - 2\sqrt{h}) + (6 + 4\sqrt{h})][\frac{1}{2}(6 + 4\sqrt{h})] = 27 + 17\sqrt{h}$ $21 + 2h = 27$ $h = 3$	1 1 1

PAHANG (K1)

INDEKS, SURD & LOGARITMA

14 (a) (i) Terbitkan bahawa $\log_a \frac{x}{y} = \log_a x - \log_a y$.

Derive that $\log_a \frac{x}{y} = \log_a x - \log_a y$.

(ii) Seterusnya, tanpa menggunakan kalkulator, cari nilai bagi $\log_3 54 - \log_3 6 - \log_3 3$.

Hence, without using a calculator, find the value of $\log_3 54 - \log_3 6 - \log_3 3$.

[5 markah]

[5 marks]

(b) Pada awal tahun 2022, populasi bagi sejenis haiwan di dalam sebuah hutan simpan dianggarkan berjumlah 20 000. Bilangan haiwan tersebut, N menurun selepas t tahun diberi oleh $N = 20000e^{-0.03t}$. Hitung bilangan tahun minimum di mana populasi haiwan tersebut kurang daripada separuh populasi haiwan pada awal tahun 2022.

At the beginning of the year 2022, the population of a species of the animals in a forest was estimated at 20 000. This number of the animals, N decreased after t years, is given by $N = 20000e^{-0.03t}$.

Calculate the minimum number of years in which the animal's population is less than half the animal's population at the beginning of the year 2022.

[3 markah]

[3 marks]

14	(a)	(i)	$\log_a x = p$, $x = a^p$ atau $\log_a y = q$, $y = a^q$	1
			$\frac{x}{y} = a^{p-q}$	1
			$\log_a \frac{x}{y} = \log_a x - \log_a y$	1
		(ii)	Guna hukum hasil bagi: $\log_3 \left(\frac{54}{6} \div 3 \right)$	1
			1	1
	(b)		$20000e^{-0.03t} < \frac{1}{2}(20000)$	1
			$t > \frac{\ln \frac{1}{2}}{-0.03}$	1
			24	1

PERLIS (K1)

INDEKS, SURD & LOGARITMA

4 Diberi bahawa $x = a^p$ dan $y = a^q$.

Given that $x = a^p$ and $y = a^q$.

(a) Buktikan hukum hasil bagi logaritma iaitu $\log_a \frac{x}{y} = \log_a x - \log_a y$.

Prove that the division law of logarithm is $\log_a \frac{x}{y} = \log_a x - \log_a y$.

[3 markah / marks]

(b) Seterusnya, tunjukkan $\log_2 P - \log_2 Q = 2 \log_4 \frac{P}{Q}$.

Hence, show that $\log_2 P - \log_2 Q = 2 \log_4 \frac{P}{Q}$.

[3 markah / marks]

4

a)

$$\log_a x = p \quad \text{atau} \quad \log_a y = q$$

P1

$$\frac{x}{y} = a^{p-q}$$

K1

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

N1

3

b)

Guna hukum bagi / tukar asas

K1

$$\log_4 P - \log_4 Q \quad \text{atau} \quad \log_2 \frac{P}{Q} \quad \text{atau} \quad \frac{\log_2 \frac{P}{Q}}{\log_2 4} \quad \text{atau} \quad \frac{\log_4 \frac{P}{Q}}{\log_4 2}$$

Guna hukum kuasa

K1

$$2 \log_4 \frac{P}{Q}$$

N1

3

SABAH (K1)**INDEKS, SURD & LOGARITMA**

- 2.** a) Permudahkan $-\log_m a - \log_m b - \log_m c$ sebagai sebutan tunggal.

Simplify $-\log_m a - \log_m b - \log_m c$ as a single term.

[1 markah/mark]

- b) Diberi $49m^5n^c = \frac{2401 m^{10} n^4 \times m^b n^2}{am^7 n}$. Cari nilai bagi a, b dan c .

Given $49m^5n^c = \frac{2401 m^{10} n^4 \times m^b n^2}{am^7 n}$. Find the values of a, b and c .

[3 markah/marks]

- 2** a) $-\log_m abc$

N1

b) $49m^5n^c = \frac{2401}{a} m^{10+b-7} n^{4+2-1}$

K1

$$49 = \frac{2401}{a} \text{ or } 5 = 3 + b \text{ or } c = 4 + 2 - 1$$

K1

$$a = 49, b = 2 \text{ and } c = 5$$

N1

- 3.** a) Selesaikan persamaan $e^{3x+2} = 10$.

Solve the equation $e^{3x+2} = 10$.

[2 markah/marks]

- b) Selesaikan persamaan

Solve the equation

$$\frac{\log_3(6+5x)}{\log_9(2+x)} = 4$$

[4 markah/marks]

3

a) $3x+2 = \log_e 10$ OR $3x+2 = \ln 10$
 $x = 0.1009$

K1
N1

b) $\frac{\log_3(2+x)}{\log_3 9}$ (Change base)

K1

$$(6+5x) = (2+x)^2$$

K1

$$(x+1)(x-2) = 0$$

K1

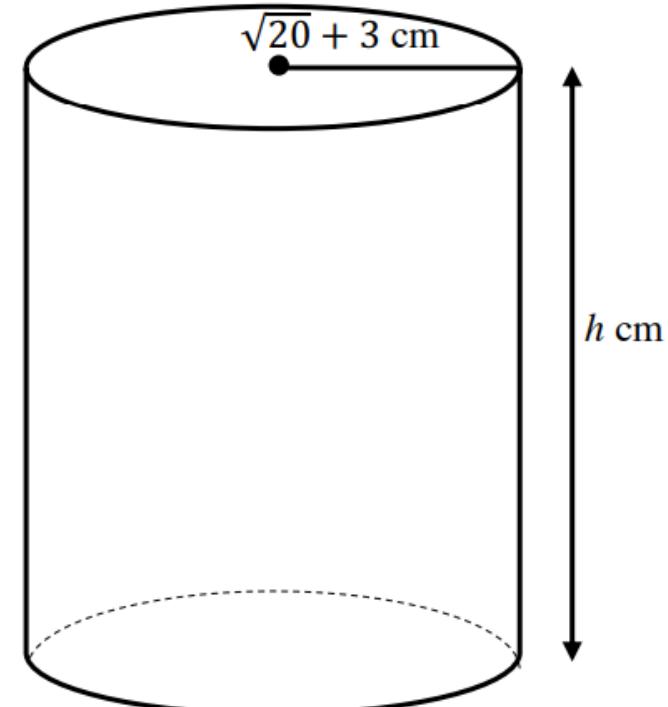
$$x = 2, \quad x = -1 \text{ (rejected)}$$

N1

SABAH (K1)

11. Rajah 11 menunjukkan sebuah silinder berisipadu $(23 - 3\sqrt{5})\pi \text{ cm}^3$. Jejari silinder itu ialah $(\sqrt{20} + 3) \text{ cm}$. Cari tinggi silinder, h dan beri jawapan anda dalam bentuk $a + b\sqrt{c}$, dengan keadaan a, b dan c ialah integer.

Diagram 11 shows a cylinder with a volume of $(23 - 3\sqrt{5})\pi \text{ cm}^3$. The radius of the cylinder is $(\sqrt{20} + 3) \text{ cm}$. Find the height of the cylinder, h and give your answer in the form $a + b\sqrt{c}$, where a , b and c are integers.



Rajah 11/Diagram 11

INDEKS, SURD & LOGARITMA**11**

$$(23 - 3\sqrt{5})\pi = \pi(3 + \sqrt{20})^2 h$$

P1

Nampak mana-mana satu pengembangan surd:

$$-3 \times \sqrt{20} \text{ atau } -3 \times 2\sqrt{5} \text{ atau } -6 \times \sqrt{4 \times 5} \text{ atau } -6 \times 2\sqrt{5} \text{ atau } 6\sqrt{20} \text{ atau } 12\sqrt{5} \text{ atau } -138 \times \sqrt{4 \times 5} \text{ atau } -138 \times 2\sqrt{5} \text{ atau } -138 \times 2\sqrt{5} \text{ atau } 276 \times \sqrt{5} \text{ atau } 363\sqrt{5} \text{ atau } -138\sqrt{20} \text{ atau setara}$$

K1

$$\frac{23-3\sqrt{5}}{29+6\sqrt{20}} \times \frac{29-6\sqrt{20}}{29-6\sqrt{20}} \text{ or } \frac{23-3\sqrt{5}}{29+12\sqrt{5}} \times \frac{29-12\sqrt{5}}{29-12\sqrt{5}}$$

K1

$$\frac{667-138\sqrt{20}-87\sqrt{5}+18\sqrt{100}}{841-36(20)} \text{ or } \frac{667-276\sqrt{5}-87\sqrt{5}+180}{841-144(5)}$$

K1

$$h = 7 - 3\sqrt{5}$$

N1

[5 markah/marks]

SELANGOR SET 1 (K1)

INDEKS, SURD & LOGARITMA

- 3 (a) Tunjukkan bahawa $6^{k+2} + 6^{k+1} - 18(6^k)$ boleh dibahagikan tepat dengan 24 bagi semua nilai integer positif k .

Show that $6^{k+2} + 6^{k+1} - 18(6^k)$ is divisible by 24 for all positive integer values of k .

[3 markah]

[3 marks]

- (b) Diberi bahawa $5^a = s$ dan $5^b = t$. Ungkapkan $\log_{25} \frac{s^2}{5t}$ dalam sebutan a dan b .

It is given that $5^a = s$ and $5^b = t$. Express $\log_{25} \frac{s^2}{5t}$ in terms of a and b .

[3 markah]

[3 marks]

3	(a)	Tukar 6^{k+2} kepada $6^k(6^2)$ atau 6^{k+1} kepada $6^k(6^1)$ $6^k(6^2 + 6 - 18)$ $6^k(24)$	K1 K1 N1
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	(b)	Alternatif 1: Gunakan mana-mana hukum log dengan betul $\frac{\log_5 t}{\log_5 25} @ \log_{25} s^2 - \log_{25} t - \log_{25} 5 @ \log_5 s @$ $\log_5 t @ \frac{\log_5 5}{\log_5 25} @ \frac{\log_5 s^2}{\log_5 25}$ Tukar asas $-\frac{\log_5 5}{\log_5 25} + \frac{2 \log_5 s}{\log_5 25} - \frac{\log_5 t}{\log_5 25}$ $-\frac{1}{2} + a - \frac{b}{2} \text{ atau } \frac{2a-1-b}{2}$ Alternatif 2: Tukar kepada bentuk log $a = \log_5 s, b = \log_5 t$ $\log_{25} \frac{s^2}{5t}$ $2\log_{25} s - \log_{25} 5t$ $2\left(\frac{\log_5 s}{\log_5 5^2}\right) - \frac{\log_5 5 + \log_5 t}{\log_5 5^2}$ $\frac{2a-1-b}{2}$	K1 K1 N1 K1 N1
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- 15 (a) Permudahkan ungkapan berikut.

Simplify the following expression.

$$(1 + \sqrt{3})(4 - 2\sqrt{3})$$

[2 markah]

[2 marks]

- (b) Suhu suatu larutan, selepas t minit disejukkan dalam ${}^\circ\text{C}$, ialah $30\left(\frac{4}{5}\right)^t$.

The temperature of a solution, in ${}^\circ\text{C}$, after t minutes being cooled is $30\left(\frac{4}{5}\right)^t$.

- (i) Cari suhu awal larutan itu.

Find the initial temperature of the solution.

- (ii) Cari nilai t apabila suhu larutan ialah $15.36 {}^\circ\text{C}$.

Find the value of t when the temperature of the solution is $15.36 {}^\circ\text{C}$.

- (iii) Hitung selepas berapa minit suhu larutan itu kurang daripada $5 {}^\circ\text{C}$ buat pertama kali.

Calculate after how many minutes the temperature of the solution be less than $5 {}^\circ\text{C}$ for the first time.

[6 markah]

[6 marks]

15	(a)	$4\sqrt{3} - 2(3) + 4 - 2\sqrt{3}$ $2\sqrt{3} - 2$	K1 N1
	(b) (i)	30	P1
	(ii)	$30\left(\frac{4}{5}\right)^t = 15.36$ $t \log_{10}\left(\frac{4}{5}\right) = \log_{10}\left(\frac{15.36}{30}\right)$ $t = \frac{\log \frac{15.36}{30}}{\log \frac{4}{5}}$ $t = 3$	K1 N1
	(iii)	$30\left(\frac{4}{5}\right)^t < 5$ $t \log_{10} 0.8 < \log_{10} \frac{1}{6}$ $t > 8.0296$ $t = 9$	K1 K1 N1

SELANGOR SET 2 (K1)

INDEKS, SURD & LOGARITMA

N9 (K2)

4 (a) Permudahkan $8^{x-1} \div 32^x$ ke dalam bentuk $\frac{1}{p(2^{nx})}$ dengan keadaan p dan n ialah integer.

Simplify $8^{x-1} \div 32^x$ into the form of $\frac{1}{p(2^{nx})}$ such that p and n are integers.

[3 markah]

[3 marks]

(b) Sebuah silinder mempunyai jejari $\frac{1}{(\sqrt{2}-1)}$ cm dan tinggi h cm. Diberi bahawa isi padu silinder itu ialah $\pi(7 + 5\sqrt{2})$ cm³. Hitung nilai bagi h dalam sebutan $a + \sqrt{b}$.

A cylinder has a radius of $\frac{1}{(\sqrt{2}-1)}$ cm and height of h cm. It is given that the volume of the cylinder is $\pi(7 + 5\sqrt{2})$ cm³. Calculate the value of h in terms of $a + \sqrt{b}$.

[3 markah]

[3 marks]

3 (a) Diberi $2\log_2 y = \log_2 x + 4$. Ungkapkan y dalam sebutan x .

Given $2\log_2 y = \log_2 x + 4$. Express y in terms of x .

[3 markah]

[3 marks]

(b) Selesaikan persamaan
Solve the equation

$$4^x = 80 - 4^{x-1}$$

[3 markah]

[3 marks]

Jawapan / Answers :

4	(a)	Lihat $2^3 @ 2^5$ Guna $(a^m)^n = a^{mn}$ & $a^m \div a^n = a^{m-n}$ $2^{3(x-1)-5x}$ $\frac{1}{8(2^{2x})}$	P1 K1 N1
	(b)	Kembang $(\sqrt{2}-1)^2 @ (7+5\sqrt{2}) \times (3-2\sqrt{2})$ dalam 4 sebutan $h = \frac{(7+5\sqrt{2}) \times (3-2\sqrt{2})}{1+\sqrt{2}}$	K1 K1 N1

3 (a)	$\log_2 \left(\frac{y^2}{x} \right) = 4$	K1
	$\frac{y^2}{x} = 2^4$	K1
	$y = 4\sqrt{x}$	N1
(b)	$4^x (4^{-1})$ atau $\frac{4^x}{4}$	K1
	$\left(\frac{5}{4}\right)4^x = 80$ atau $\frac{5}{4}y = 80$	K1
	$x = 3$	N1

PAHANG (K2)

INDEKS, SURD & LOGARITMA

- 2 (a) Diberi $\log_2 3 = h$ dan $\log_2 5 = k$, ungkapkan dalam sebutan h dan k

Given $\log_2 3 = h$ and $\log_2 5 = k$, express in terms of h and k

(i) $\log_2 3.6$,

(ii) $\log_2 270$.

[4 markah]
[4 marks]

- (b) Selesaikan persamaan $\sqrt{2} p - 8 = \sqrt{24} - \sqrt{8} p$.

Solve the equation $\sqrt{2} p - 8 = \sqrt{24} - \sqrt{8} p$.

[3 markah]
[3 marks]

2	(a)	(i)	$\log_2\left(\frac{2 \times 3^2}{5}\right)$	1
			$1 + 2h - k$	1
		(ii)	$\log_2(3^2 \times 2 \times 5)$	1
			$1 + k + 3h$	1

	(b)	$\frac{8 + \sqrt{24}}{\sqrt{2} + \sqrt{8}} \times \frac{\sqrt{2} - \sqrt{8}}{\sqrt{2} - \sqrt{8}}$ @ setara	1
		$\frac{8\sqrt{2} - 8\sqrt{8} + \sqrt{48} - \sqrt{192}}{2 - \sqrt{16} + \sqrt{16} - 8}$ @ setara	1
		$\frac{4\sqrt{2} + 2\sqrt{3}}{3}$	1

PERLIS (K2)

INDEKS, SURD & LOGARITMA

- 7 (a) Diberi $3 - 2 \log_3 x = \log_3 y$, ungkapkan y dalam sebutan x .

Given that $3 - 2 \log_3 x = \log_3 y$, express y in terms of x .

[5 markah / marks]

- (b) Jika $\frac{3^{2x}}{9^y} = 243$ dan $2^{x-1} \times 4^y = 64$, cari nilai x dan y .

If $\frac{3^{2x}}{9^y} = 243$ and $2^{x-1} \times 4^y = 64$, find the value of x and of y .

[3 markah / marks]

7
(a)

Guna hukum kuasa
 $\log_3 3^3$ atau $\log_3 x^2$

K1

Guna hukum bahagi

$$\log_3 \frac{3^3}{x^2}$$

$$y = \frac{27}{x^2}$$

N1

(b)

$$3^{3x} \times 3^{-2y} = 3^5 \quad \text{atau} \quad 2^{x-1} \times 2^{2y} = 2^6$$

P1

$$3x - 2y = 5 \quad \text{atau} \quad x - 1 + 2y = 6$$

P1

Selesaikan persamaan linear serentak
 $3x = 12$

K1

$$x = 4$$

N1

$$y = \frac{3}{2}$$

N1

SELANGOR SET 1 (K2)

INDEKS, SURD & LOGARITMA

SELANGOR SET 2 (K2)

3 (a) Ringkaskan:

Simplify:

$$\frac{(-8x^4y^3)^2}{16x^5y}$$

[2 markah]
[2 marks]

(b) Selesaikan persamaan:

Solve the equation:

$$2^{4x} = 16 + 2^{4x-1}$$

[3 markah]
[3 marks]

3	(a)		$\frac{64x^8y^6}{16x^5y}$ $4x^3y^5$	K1 N1
	(b)		$2^{4x-1} = 2^4$ $4x - 1 = 4$ $x = \frac{5}{4}$	K1 K1 N1

1 (a) Diberi bahawa $\log_m k + \log_m \frac{1}{k} = t$. Cari nilai t .It is given that $\log_m k + \log_m \frac{1}{k} = t$. Find the value of t .[3 markah]
[3 marks](b) Permudahkan $\log_2(2x+1) - 5\log_4x^2 + 4\log_2x$.Simplify $\log_2(2x+1) - 5\log_4x^2 + 4\log_2x$.[4 markah]
[4 marks]

1	(a)	$\log_m \left(k \times \frac{1}{k}\right) = t$ $m^t = 1$ $t = 0$	K1 K1 N1
	(b)	Guna $\log_a b = \frac{\log_c b}{\log_c a}$ $\log_2(2x+1) - \frac{\log_2 x^{10}}{\log_2 4} + \log_2 x^4$. Guna hukum $\log_a m^n = n \log_a m$ $\log_2(2x+1) - \log_2 x^5 + \log_2 x^4$. Guna hukum $\log_a mn = \log_a m + \log_a n$ & $\log_a \frac{m}{n} = \log_a m - \log_a n$ $\log_2 \frac{(2x+1)x^4}{x^5}$ $\log_2 \left(2 + \frac{1}{x}\right) @ \log_2 \frac{2x+1}{x}$	K1 K1 K1 K1 N1

TERENGGANU (K2)

INDEKS, SURD & LOGARITMA

3 Diberi bahawa $\sqrt{a+b\sqrt{2}} = \frac{7}{3-\sqrt{2}}$, dimana a dan b ialah pemalar.

It is given that $\sqrt{a+b\sqrt{2}} = \frac{7}{3-\sqrt{2}}$, where a and b are constants.

(a) Cari nilai a dan b . [4 markah]

Find the value of a and of b . [4 marks]

(b) Seterusnya, selesaikan $e^{2\ln a} + e^{2\ln b}$. [2 markah]

Hence, solve $e^{2\ln a} + e^{2\ln b}$. [2 marks]

3	<p>(a) Pendaraban dengan surd konjugat & selesaikan</p> $\frac{7}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} \quad \text{dan} \quad \frac{21+7\sqrt{2}}{9-2}$ $\left(\sqrt{a+b\sqrt{2}}\right)^2 = (3+\sqrt{2})^2$ <p>K1</p> <p>$a = 11$ N1</p> <p>$b = 6$ N1</p> <p>(b) $e^{\ln(11*)^2} + e^{\ln(6*)^2}$ & Selesaikan @ $*(11)^2 + *(6)^2$ K1</p> <p>157 N1</p>
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